To the positive miscut influence on the crystal collimation efficiency

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Abstract

The paper concerns the crystal based collimation suggested to upgrade the Large Hadron Collider collimation system. The issue of collimation efficiency dependence on the muscut angle characterizing nonparallelity of the channeling planes and crystal surface is mainly addressed. It is shown for the first time that even the preferable positive miscut could severely deteriorate the channeling collimation efficiency in the crystal collimation UA9 experiment. We demonstrate that the positive miscut influence can increase the nuclear reaction rate in the perfectly aligned crystal collimator by a factor of 4.5. We also discuss the possible miscut influence on the future LHC crystal collimation system performance as well as suggest simple estimates for the beam diffusion step, average impact parameter of particle collisions with the collimator and angular divergence of the colliding particle beam portion.

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I. INTRODUCTION

Crystal based collimation was proposed to facilitate the beam halo cleaning at large accelerators long ago. Its application to the LHC upgrade becomes more and more topical [1–4]. The basic idea is to use a bent crystal in channeling mode to deflect halo particles by relatively large angles to high impact parameters of particle collisions with an absorber [1–5].

If the first particle collision with the crystal collimator occurs at sufficiently small particle incidence angle w.r.t. the crystal planes (at pure alignment, $\theta_c = 0$), the probability of particle capture into the channeling regime reaches its maximum [6]. However even a small nonparallelity of the lateral crystal surface with atomic planes, characterized by the muscut angle θ_m, is able to severely disturb the motion of particles hitting the crystal with small impact parameters. In particular, if the miscut angle is negative, the channeling motion can be interrupted before the particle reaches the exit crystal face [5]. Since a considerable number of such particles will not reach an absorber fast, the negative miscut is recommended to be avoided [5]. By this reason the positive one (see Fig. 1) was chosen for the recent UA9 experiments [2] aimed to demonstrate the viability of crystal collimation. However the nuclear reaction rate in the perfectly aligned crystal collimator about five time exceeding the theoretically predicted [3] value was observed. In this paper we for the first time investigate

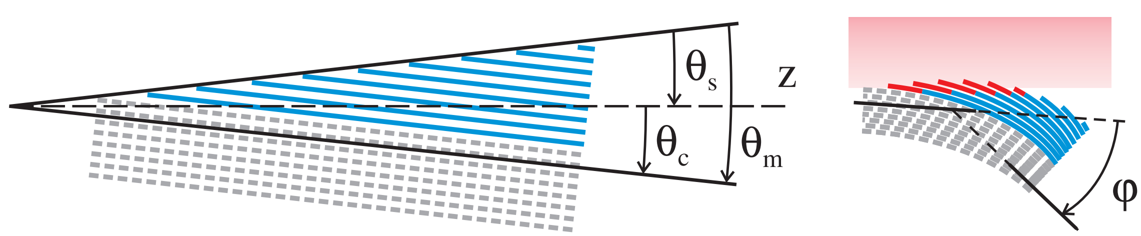

FIG. 1: A crystal with positive miscut angle before (left) and after (right) bending with angle φ. θ_c (positive), θ_m (**positive**) and $\theta_s = \theta_c - \theta_m$ (negative) are, respectively, the crystal plane misalignment angle, miscat angle and crystal surface misorientation angle, all measured in the direction of crystal bending, at that the angles θ_c and θ_s – from the z axis, parallel to the velocity of the particle just touching the crystal, and the angle θ_m – from the crystal surface direction. Particles, moving from the left to the right with small impact parameters, enter the crystal through the lateral (upper) crystal surface.

the influence of positive miscut on collimation efficiency and demonstrate that it gives rise to up to 4.5 time increase of the nuclear reaction rate in the collimator. We also predict the low influence of the positive miscut on the efficiency of the future LHC crystal collimation system but first suggest simple estimates for the beam diffusion step, impact parameter of particle collisions with the collimator and angular divergence of the part of the beam particles colliding with the latter for the first time.

II. PARTICLE DIFFUSION IN THE ACCELERATOR RING

When positively charged particles strike a modestly bent crystal moving strictly parallel to its planes, they are captured into the regime of stable channeling motion with a probability of 80-85% [6]. However if the incident particle beam possesses an angular divergence, the "channeling probability" decreases by the value $\Delta P_{ch} \propto \langle \vartheta^2 \rangle$, where $\langle \vartheta^2 \rangle$ is the mean square of the incident beam divergence angle, assumed here to be considerably smaller than the critical channeling angle ϑ_{ch}. This decrease remains negligible only if $\langle \vartheta^2 \rangle \leq 0.01\vartheta_{ch}^2$. Fortunately, the angular divergence of the beam portion striking a primary collimator first time often satisfies this rigorous condition. However if a particle has not been captured at the first passage through the collimator, its deflection can reach significant values. This actually pertains to the 15-20% of particles escaping channeling at the crystal penetration through its normal entrance (transverse to the beam) face. Particles entering a crystal with negative miscut [5] at small enough impact parameters are angularly dispersed even stronger. Because of this and also since a miscut can not be avoided in practice, the positive miscut crystal orientation is commonly preferred [5]. In this case, however, the particles with the small enough impact parameters enter the crystal through its lateral face. Most of them avoid capture even in the case of pure crystal alignment. The uncaptured particles are scattered nearly the same way as the ones in amorphous matter, acquiring the average deflection angle squared proportional to the length Δz of particle path through the crystal. Besides the angles of miscut θ_m, crystal bending $\varphi = l/R$ and crystal plane misalignment θ_c, the Δz value depends on the particle impact parameter Δ with the crystal collimator, the value of which needs special consideration.

Having limited access to the parameters of particle motion in the accelerators, not mentioning the tools allowing to simulate a number of them, we need to suggest simple estimates

TABLE I: Beam diffusion parameter.

Accelerator	ε	τ	$\sigma(\mu m)$	ρ_c/σ	δ
SPS UA9	120GeV	10h	1010	3.5	0.086 nm
SPS UA9	120GeV	4min	1010	3.5	13 nm
LHC	7TeV	10h	200	6	$5.4\mu m$
LHC	7TeV	10h	420	6	$11.4\mu m$

for the former. To start with, recall that particle collisions with crystal collimator originate from the particle diffusion in the radial beam direction caused by intra beam collisions, scattering by residual gas, elastic scattering at the interaction point, etc. [4]. Since a joint description of all these processes is hardly available, we will proceed from a simple estimate based on the accelerator beam lifetime τ, particle revolution period T, r.m.s. beam radius σ and collimator radial coordinate ρ_c. We will also assume, as in [4], for simplicity, that the beam is axially symmetric and possesses normal distribution

$$\frac{dN}{d\rho} = \frac{N\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \tag{1}$$

in particle radial number density integrated over the particle revolution period T. Here N is a total number of particles in the ring. Introducing a *diffusion step* δ, an average increase of radial coordinate acquired during one revolution by particles reaching the collimator position, one can express the particle loss rate in two ways:

$$\frac{dN}{dt} = \frac{N}{\tau} = \left(\frac{dN}{d\rho}\right)_{\rho_c} \frac{\delta}{T}, \tag{2}$$

coming directly to an estimate

$$\delta = \frac{\sigma^2 T}{\tau \rho_c} \exp\left(\frac{\rho_c^2}{2\sigma^2}\right). \tag{3}$$

Table I illustrates Eq. (3) application to the cases of both the UA9 experiment and IR7 beta collimation region at the LHC [7]. Note that the exponential dependence on the collimator aperture squared leads to a really drastic δ difference in the cases of collimation dedicated UA9 experiment with low intensity beam and $\rho_c \simeq 3.5\sigma$ and the intensive LHC beam and $\rho_c = 6\sigma$.

III. PARTICLE IMPACT PARAMETER AND DEFLECTION ANGLE

To simulate both the particle impact parameter and angular deflection at the moment of the first collision with the collimator we will proceed from the usual pseudoharmonic representation

$$x(\psi) = x_0 cos\psi \tag{4}$$

of the betatron oscillations. Here ψ and $x_0 = \sqrt{\varepsilon\beta}$ are, respectively, the oscillation phase and amplitude, the latter of which is determined by the beam emittance ε and accelerator beta function β. Particle collisions with the collimator become possible since the amplitude x_0 reaches the transverse collimator coordinate $x_c = \rho_c$ (from here on we consider betatron oscillations in some transverse plane parallel to x axis). Collision really occurs if $x(\psi) \geq x_c$ or $|\psi| \leq \psi(x_0)$, where

$$\psi(x_0) = arccos\left(\frac{x_c}{x_0}\right) \simeq \sqrt{2(x_0 - x_c)/x_c} \tag{5}$$

(we assumed that $x(\psi) - x_c \ll x_c$). The particle direction of incidence on the crystal will be described by the angle

$$\theta(\psi) = -\frac{x_0}{\beta}\left[sin\psi - \frac{1}{2}\frac{d\beta}{ds}cos\psi\right] \tag{6}$$

of the velocity deviation from the direction of motion of the particle just touching the collimator having $x_0 = x_c$.

The process of increase of the betatron oscillation amplitude was simulated using the formula

$$x_0(n) = x_0(n-1) + 2\delta\xi_1, \tag{7}$$

where $n = 1, 2, ..$ is the number of particle revolution in the ring after the moment when $x_0(0) = x_c$. ξ_1 (as well as ξ_2 and ξ_3 below) are random numbers uniformly distributed through the interval $(0, 1)$. Note that in average $\langle x_0(n)\rangle = \langle x_0(n-1)\rangle + \delta$, in agreement with the δ definition.

In absence of phase correlations between betatron oscillations on different revolution periods the phase can be sampled by the formula $\psi = (2\xi_2 - 1)\pi$. A collision occurs with the probability

$$p_n = \frac{\psi(x_0(n))}{\pi} \simeq \frac{1}{\pi}\sqrt{\frac{2n\delta}{x_c}} \tag{8}$$

at the n-th revolution if

$$|\psi| \leq \psi(x_0(n)). \tag{9}$$

If, otherwise, $|\psi| > \psi(x_0(n))$, no collision occurs and one should continue the simulations with $n = n+1$ and so on. Since the cumulative collision probability

$$P_N = \sum_{n=1}^{n=N} p_n \simeq \frac{2}{3\pi}\sqrt{\frac{2\delta}{x_c}} N^{3/2} \tag{10}$$

increases faster and faster with the revolution number N, the "collision condition" (9) inevitably becomes fulfilled at some revolution N with some random values of $x_0(N)$ and ψ allowing to evaluate both the collision coordinate (4) and angle (6) of particle deflection at the moment of collision. Particle distributions in the impact parameter $\Delta \equiv x - x_c$ and deflection angle θ, simulated for the UA9 and LHC cases, are represented in Figs. 3-6. A three order in value difference in the impact parameter in the UA9 and LHC cases is directly related to that in the diffusion step – see Table I.

Assuming that the collisions, naturally, occur at $P_N \sim 1$, Eq. (10) can be reversed to

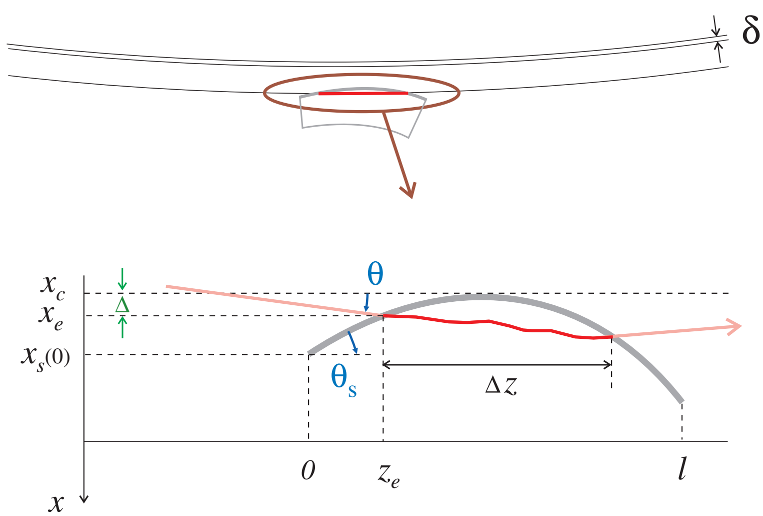

FIG. 2: A particle entering the crystal through the lateral surface. The crystals extends from $z = 0$ to $z = l$, x_c is the collimator radial coordinate, $(x_e = x_s(z_e), z_e)$ is the particle enter point while $(x_s(0), 0)$ is the left upper corner of the crystal, θ and θ_s are the deflection particle angle and crystal surface misalignment one, respectively, Δz is the length of particle trajectory inside the crystal, δ and Δ are, respectively, the particle diffusion step and impact parameter.

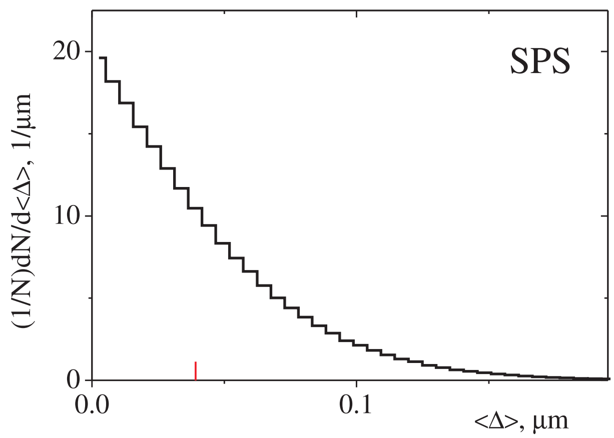

FIG. 3: Particle distribution in impact parameter for the UA9 case.

estimate the typical revolution number before the collision

$$N \simeq \frac{(3\pi P_N)^{2/3}}{2}\sqrt[3]{\frac{x_c}{\delta}} \sim 2\sqrt[3]{\frac{x_c}{\delta}}, \tag{11}$$

the average impact parameter

$$\langle\Delta\rangle = P_N^{-1}\sum_{n=1}^{n=N} p_n[x_0(n)cos(\psi) - x_c] \simeq \frac{3(3\pi P_N')^{2/3}}{10}\sqrt[3]{x_c\delta^2} \sim 1.3\sqrt[3]{x_c\delta^2} \tag{12}$$

and absolute value of the deflection angle

$$\langle|\theta|\rangle = P_N^{-1}\sum_{n=1}^{n=N} p_n x_0(n) sin(\psi)/\beta \simeq \frac{3(3\pi P_N'')^{1/3}}{8}\frac{\sqrt[3]{x_c^2\delta}}{\beta} \sim 0.8\frac{\sqrt[3]{x_c^2\delta}}{\beta}. \tag{13}$$

The right hand sides of Eqs. 11-13 contain numerical factors found from the simulations

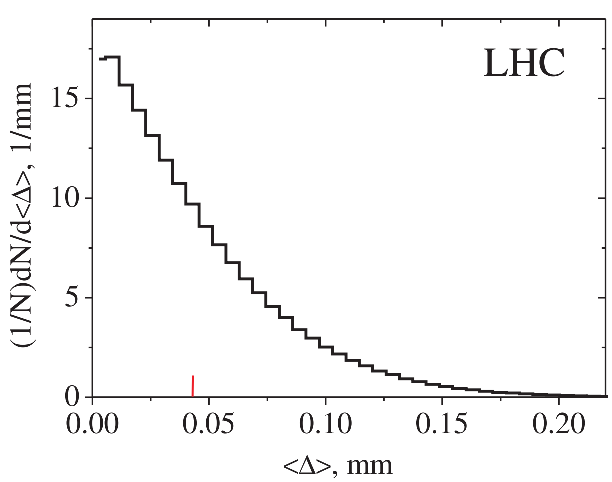

FIG. 4: The same for the LHC case.

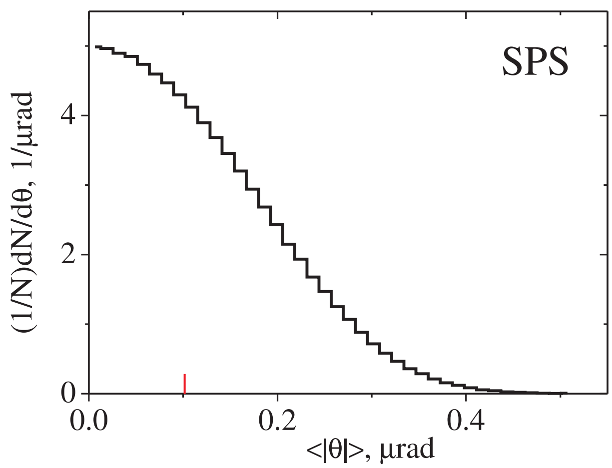

FIG. 5: Particle distribution in deflection angle for the UA9 case.

and determining some effective collision probabilities $P_N, P'_N,\ P''_N \sim 1$ which should be considered as the parameters compensating a slight model inconsistency. Despite the latter Eqs. 11-13 provide a sufficient base to compare the conditions of the UA9 crystal collimation experiment with the possible LHC crystal collimator performance as well as to estimate the perspectives of possible crystal collimation development [8]. Fig. 7 again demonstrates that the radical difference in diffusion steps results in the drastic difference in average impact parameters in the UA9 and LHC cases.

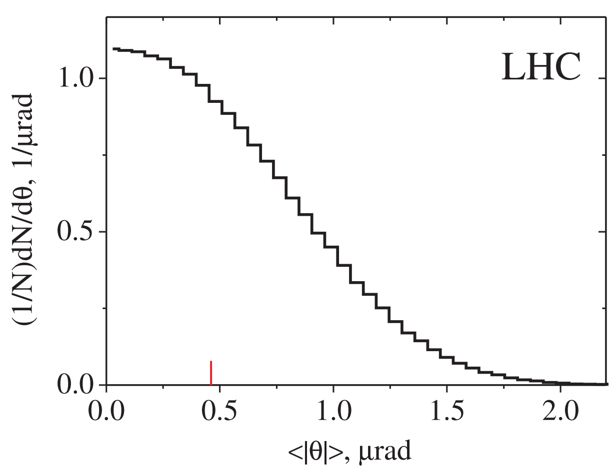

FIG. 6: The same for the LHC case.

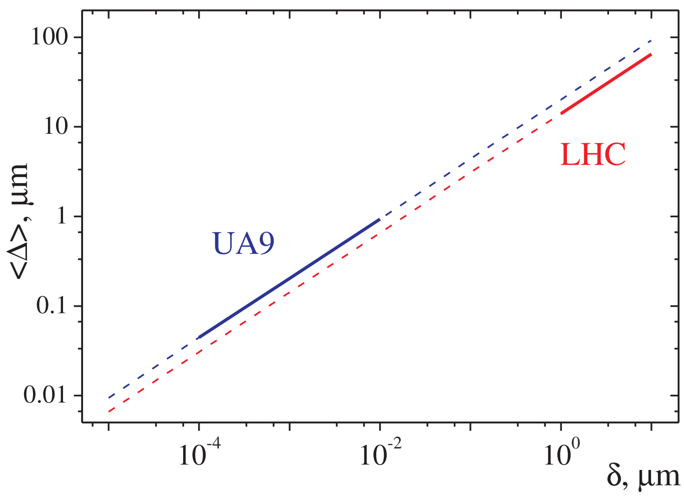

FIG. 7: Average impact parameter *vs* average beam diffusion step for the SPS UA9 (upper curve) and the LHC (lower one). Solid parts mark the actual parameter regions.

IV. COLLIMATION EFFICIENCY FOR THE UA9 EXPERIMENTS

A simulated value of the impact parameter Δ can be directly used to evaluate both the entrance transverse coordinate (4) and angle (6), the knowledge of both of which is necessary to simulate the particle trajectory inside the crystal in order to obtain the angle of particle deflection by the latter. Recall that we measure the angles from the direction of motion of the particle just touching the collimator at its maximum displacement $x(\psi = 0) = x_0 = x_c$ from the beam axis – see Fig. 2. Note *de bene esse* that the chosen zero direction forms the angle

$$\frac{dx(\psi = 0)}{ds} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\beta}}\frac{d\beta}{ds} = -\frac{x_c}{\beta}\alpha \tag{14}$$

w.r.t. the beam axis. Here s is the beam longitudinal coordinate and $\alpha = -d\beta/ds/2$ is the conventional Twiss parameter. In the ideal case a crystal has no miscut and its planes form zero angle w.r.t. the chosen zero angle direction. Particles neither enter the crystal through its lateral surface no leave the one through it if they are channeled in this case.

The real situation is complicated by the inevitable presence of both crystal miscut and crystal plane misalignment at the entrance surface, characterized by the angles θ_m and θ_c, respectively. The crystal misalignment angle is assumed to be positive if the planes are rotated in the direction of crystal bending. If one determines the miscut angle as the one of crystal plane rotation in the direction of crystal bending w.r.t. the crystal lateral surface, the misorientation angle of the latter, measured from the same zero angle direction, will be equal to $\theta_{s0} = \theta_s(0) = \theta_c - \theta_m$. If the crystal is bent with radius R, the surface tangential direction will vary like $\theta_s(z) = \theta_{s0} + z/R$ with the longitudinal coordinate $z \simeq s - s_c$. A behavior of the surface coordinate

$$x_s(z) = x_s(0) + \int_0^z \theta_s(z)dz = x_s(0) + \theta_{s0}z + z^2/2R, \tag{15}$$

also measured in the crystal bending direction, considerably differs if $\theta_{s0} > 0$ and $\theta_{s0} < 0$ and, in the latter case, if $|\theta_{s0}| > \varphi$ and $|\theta_{s0}| < \varphi$, where $\varphi = l/R$ is the bending angle of the crystal with length l. Namely, if $\theta_{s0} < 0$ a particle can enter the crystal through the lateral surface and, if $-\varphi < \theta_{s0} < 0$, also leave it through the latter. On the opposite, if $\theta_{s0} > 0$ particles always enter the crystal through the entrance face, while leave it either through the lateral or exit ones. In all the cases the actual situation is determined by the impact parameter Δ, the random nature of which allows for diverse trajectory types at any choice of θ_s and φ. To get simple formulae for all possible situations we first determined the minimal crystal surface coordinate $x_{min} \equiv x_c$. After the Monte Carlo sampling of the impact parameter value $\Delta = x(\psi) - x_c$ corresponding transverse collision coordinate $x(\psi)$ was used to evaluate the longitudinal one z_e from the equation $x_s(z_e) = x(\psi)$. Then both the particle entrance point coordinates $(x(\psi), z_e)$ and initial deflection angle (6) were used as the initial conditions for its trajectory simulation inside the crystal. A possibility to leave the crystal through the lateral surface at some $z < l$ was permanently monitored using Eq. (15). The particle transverse coordinate and deflection angle at the exit together with the crystal position s_c in the ring became the initial conditions for the particle motion simulation in the accelerator ring, for which the simplest model of betatron motion was applied.

However since the simulation of thousands of trajectories for tens of different miscut angle and diffusion step values takes considerable time, more rationally was first to use a simplified fast approach allowing to elaborate a general view on the influence of positive miscut on the collimation efficiency. The idea originates from the mentioned proportionality of the decrease of the channeling efficiency at the second crystal penetration to the squared angle of multiple scattering of particles entering the crystal first through the lateral surface. Since the latter, in turn, is proportional to the scattering length Δz, one can conclude that simply

$$\Delta P_{ch} \propto \Delta z \tag{16}$$

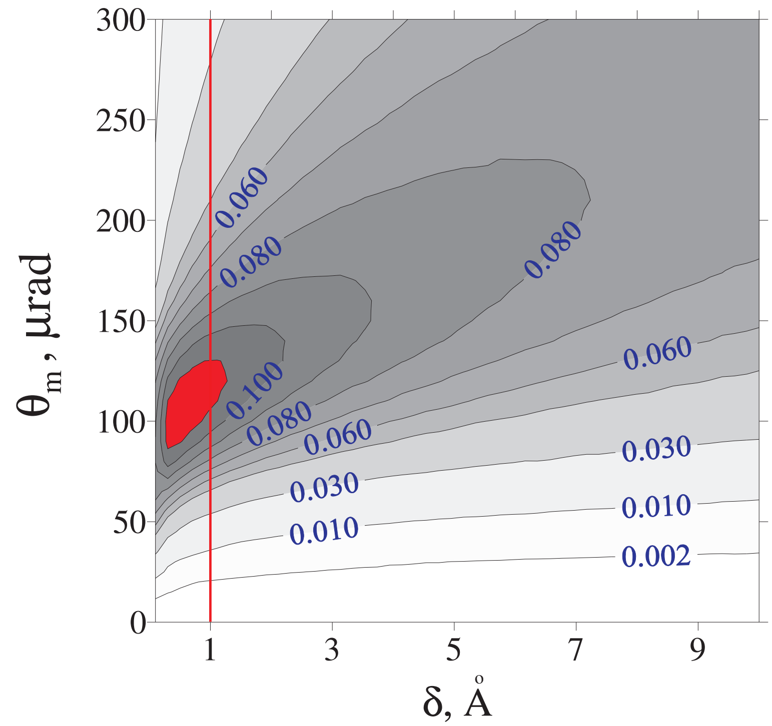

FIG. 8: Measure in centimeters average length $\langle \Delta z \rangle$ of scattering of particles entering the crystal through the lateral crystal surface *vs* both miscut angle and diffusion step at perfect crystal plane alignment.

and reduce the issue to the analysis of the behavior of the averaged length $\langle \Delta z \rangle$ of the first pass through the crystal of the particles entering the latter exclusively crossing its lateral surface. Fig. 8 illustrates the simulated behavior of $\langle \Delta z \rangle$ in the typical UA9 case of $l = 2mm$ and $\varphi = l/R = 150\mu rad$. Surprisingly, the simulations unambiguously point to the region $\theta_m \sim 100\mu rad$ and $\delta \sim 1$ Å of the UA9 experiment parameters as to the one of the greatest possible miscut influence on the collimation.

Since the scattering length determines the decrease of the channeling probability and since nonchanneled particles induce more nuclear reactions that the channeled ones, Fig. 8 has to simultaneously reflect the behavior of the rate of the nuclear reactions induced in the crystal collimator. To illuminate the possible role of positive miscut in the UA9 experiment we, according to Ref. [3] and Table I, had put $\delta = 1$ Å and conducted more detail Monte Carlo simulations of the nuclear reactions in the miscut angle interval $-300\mu rad \leq \theta_m \leq 300\mu rad$ taking now into detail consideration also the particle motion in the crystal collimator. The dependence obtained (see Fig. 9) demonstrates an evident agreement with that of $\langle \Delta z \rangle$

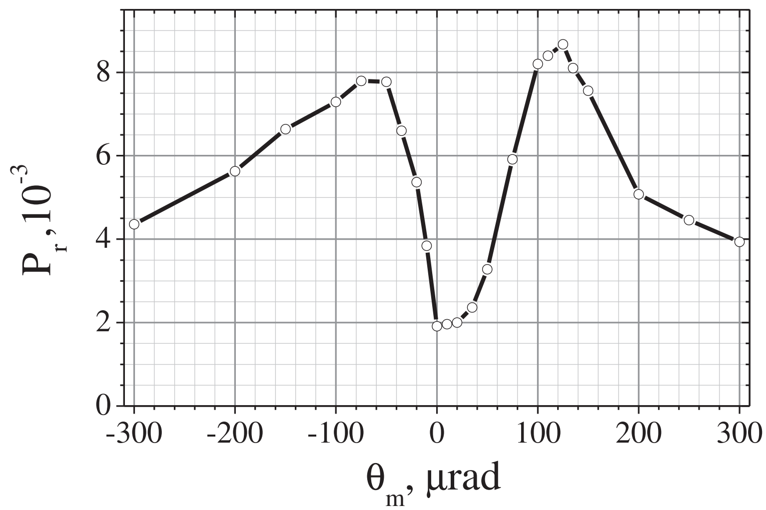

FIG. 9: Probability of nuclear reactions in the crystal collimator *vs* miscut angle at perfect crystal plane alignment.

along the vertical (red) line drawn at $\delta = 1$ Å in Fig. 8, confirming thus the strong influence of the positive miscut on the collimation process. In principle, positive miscut causes even slightly larger increase in nuclear reaction rate (the right peak) than the negative one (the left one). At this the increase of the reaction probability caused by the positive miscut with $\theta_m \simeq 125\mu rad$ reaches $8.6/1.9 \simeq 4.5$. Thus, the miscut influence, for sure, should be taken into consideration for the full interpretation of the UA9 experiments [2].

V. MISCUT INFLUENCE AT THE LHC

For the future possible application at the LHC it is important is to clarify how the deteriorating miscut influence can be avoided. A joint consideration of the particle motion in both the ring and the crystal results in the encouraging conclusion that the observed undesirable increase in nuclear reaction rate can be easily avoided in both UA9 and LHC cases. In fact, some of the conditions of the UA9 experiment prove to be practically optimal for the demonstration of the maximum miscut role. The point is a perfect matching of the average impact parameter (12) $\langle\Delta\rangle \simeq 0.039\mu m$ with the width $x_s(0) - x_c = \theta_m^2 R/2 \simeq 0.067\mu m$ of the impact parameter region allowing particle entrance through the lateral crystal surface. This matching made possible both the lateral entrance of the majority of particles and their relatively continuous path inside the crystal, the average value of which $\langle\Delta z\rangle \simeq$

$1.2mm$ exceeds a half of the crystal length $l = 2mm$. It is namely the nearly "amorphous" scattering at such a length which gave the origin to the angular dispersion of particle beam causing the decrease of the capture probability to the channeling regime at their subsequent passages through the crystal collimator.

At least two ways to decrease the miscut role by fulfilling the condition $\Delta \gg x_s(0) - x_c < \theta_m l$ can be readily suggested. The most evident, though, probably, more difficult, is to lessen the miscut angle down to about $10\mu rad$, as Figs. 8 and 9 suggest. The second is to increase the collision parameter (12) $\langle\Delta\rangle \propto \delta^{2/3}$ by means of beam diffusion acceleration. While the diffusion step could be nearly freely chosen in the collimation UA9 experiment, the actual set of the LHC parameters solves this problem automatically. Indeed, if $R = 100m$ and $l = 4mm$, one obtains $x_s(0) - x_c \simeq 0.32\mu m$, or more than a *hundred times* less than $\langle\Delta\rangle \simeq 43\mu m \simeq 130(x_s(0) - x_c)$ without special measures. Thus, only a negligible portion of the LHC protons will enter the crystal collimator through the lateral crystal surface even at the typical miscut angles of $\theta_m \sim 100\mu rad$.

It also should be noted that despite the relatively large value of the diffusion step δ, the angular divergence (13) of the colliding beam portion, as Fig. 6 demonstrates, is low enough to provide the probability of capture into the regime of channeling motion comparable to the maximum one. Nevertheless some decrease in divergence remains desirable. The sharp dependence (3) of the diffusion step on the collimator aperture x_c/σ allows to decrease δ by means of a slight decrease of the latter. At this, if the divergence of the colliding beam portion is decreased by several times, it will become possible to sharply rise the probability of particle capture into the channeling regime up to 99% by the method of the crystal cut [8].

In conclusion, the positive miscut influence indeed could increase the nuclear reaction probability in the crystal collimator up to 4.5 times. Nevertheless if the crystal collimator system based on the channeling particle deflection is realized at the LHC, its functioning will not be considerably disturbed by the influence of crystal miscut. In addition, the performance of the crystal collimator can be drastically improved by the method [8] of the crystal cut.

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