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Channeling, Radiation and Reactions in Crystals under High Energy

Author-made translation of the book
published in Russian in 1982

2016

Contents

Chapter 1

Channeling of High-Energy Particles in Crystals

1.1 Channeling and Diffraction of Particles

A fast particle passing through a single crystal undergoes elastic and inelastic scattering due to the interaction with electrons and nuclei and causes various reactions. From a quantum mechanical viewpoint, scattering processes and reactions excite secondary (scattered) waves in a crystal. One should bear in mind that secondary waves, which describe elastic scattering, interfere with one another and with the incident wave. This leads to the formation of a sum coherent wave in a crystal. Since the formation of a coherent wave is caused by the processes of elastic scattering, its transmission through the crystal can be described by introducing the effective periodic potential $V(\vec{r})$ averaged over temperature oscillations of the atoms (nuclei). The expansion of $V(\vec{r})$ into the Fourier series has the form [Hirsch *et al.* (1965); Baryshevsky (1976); Kagan and Kononets (1970)]

$$V(\vec{r}) = \sum_{\vec{\tau}} V(\vec{\tau}) e^{i\vec{\tau}\vec{r}}, \tag{1.1}$$

where $\vec{\tau}$ is the reciprocal lattice vector of the crystal;

$$V(\vec{\tau}) = \frac{1}{\Omega} \sum_{j} V_{j0}(\vec{\tau}) e^{-w_{\vec{j}}(\vec{\tau})} e^{-i\vec{\tau}\vec{r}_j}$$

is the Fourier component of the potential; here Ω is the crystal unit cell volume; $\vec{r}_j$ is the coordinate of the j-type atom (nucleus) in the unit cell; the square of $e^{-w_j(\vec{\tau})}$ is equal to the thermal factor, or the Debye-Waller factor, known from X-ray and neutron scattering; $V_{j0}(\vec{\tau})$ is the Fourier component of the interaction potential between the particle and the atom whose center of gravity rests in the origin of coordinates.

When a particle of charge $\pm e$ (e is the value of the electron charge) passes through a crystal, $V(\vec{r})$ represents the ordinary Coulomb interaction, while

V_{j0} is determined by the expression

$$V_{j0}(\vec{\tau}) = \pm \frac{4\pi e^2}{(\vec{\tau})^2}[z_j - F_j(\vec{\tau})]\,,$$

where z_j is the charge of the nucleus located at point $\vec{r}_j$ in the unit cell; $F_j(\vec{\tau})$ is the form factor of the atom located at point $\vec{r}_j$ [Landau and Lifshitz (1977)].

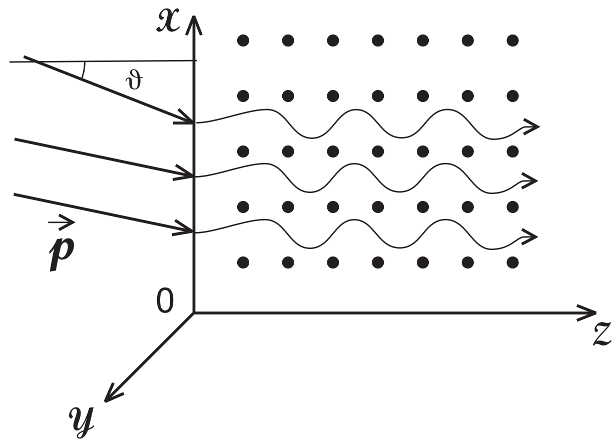

Fig. 1.1 Particle channeling in a crystal

Consider in more detail the case when a particle enters a single crystal at a certain small angle ϑ with respect to the crystallographic planes (axes) of the crystal (Fig. 1). If this angle is smaller than the so-called Lindhard angle, the particle in the crystal moves in the channeling regime [Thompson (1968); Lindhard (1965); Gemmell (1974)].

Theoretical analysis of the channeling effect should take into account that when a high-energy particle, for which the wavelength λ is much smaller than the interatomic distance, is incident on a crystal at a small angle ϑ, the periodicity of chains and planes of the crystal along the direction of particle motion has almost no influence on the nature of particle motion [Kagan and Kononets (1970); Baryshevskii and Dubovskaya (1977d); Kalashnikov and Strikhanov (1975)]. As a result, the particle behavior is determined by the averaged potential of the crystal axes (planes), which is constant along the direction of particle incidence and periodic in the transverse plane.

Direct the z-axis of the coordinate system along the crystal axes (planes), relative to which the particle moves at a small angle. In this case the periodic along the x-axis potential of planes, which describes planar channeling, can be written as follows [Kagan and Kononets (1970); Baryshevskii and Dubovskaya (1977d); Kalashnikov and Strikhanov (1975)]:

$$V(x) = \sum_{\tau_x} V(2\pi\tau_x) e^{-i2\pi\tau_x x} (\tau_y = \tau_z = 0). \tag{1.2}$$

Axial channeling is described in terms of the two-dimensional periodic in a transverse plane potential

$$V(\vec{\rho}) = \sum_{\vec{\tau}_\perp} V(2\pi\vec{\tau}_\perp) e^{-i2\pi\vec{\tau}_\perp\vec{\rho}}, \tag{1.3}$$

where $\vec{\rho} = (x, y)$; $\vec{\tau}_\perp = (\tau_x, \tau_y)$; $\tau_z = 0$.

To determine the influence of a single crystal on a passing relativistic particle in the general case, it is necessary to study the solution of the Dirac equation. With this aim in view, it is convenient to convert it into a second–order equation, very much similar in form to the Schrödinger equation [Berestetsky *et al.* (1968)]:

$$\left[-\hbar^2\Delta_r - p^2 + \frac{2}{c^2}EV(\vec{r}) - \frac{1}{c^2}V^2(\vec{r}) - i\frac{\hbar}{c}\vec{\alpha}\vec{\nabla}V(\vec{r})\right]\psi(\vec{r}) = 0\,, \tag{1.4}$$

where $p^2 = (E^2 - m^2c^4)/c^2$ is the momentum of the particle entering the crystal; E is its energy; $\vec{\alpha}$ are the Dirac matrices; ψ is the bispinor.

According to (1.4), the effective potential acting on a relativistic particle is the sum of three terms, one of which increases with the growth of particle energy E. For these reason, the terms including V^2 and $\vec{\alpha}$ can be dropped when analyzing spatial and angular distribution of the particles which have interacted with the crystal. However, when analyzing the polarization properties of particles transmitted through a crystal, it is crucial that the term containing the matrices $\vec{\alpha}$ should be taken into account [Baryshevsky (1980d)].

Dropping the terms proportional to V^2 and $\vec{\alpha}$, from (1.4) we obtain the following equation

$$\left[-\hbar^2\Delta_r - p^2 + \frac{2}{c^2}EV(\vec{r})\right]\psi(\vec{r}) = 0\,. \tag{1.5}$$

Upon dividing (1.5) first by $2m$ and then by $2m\gamma$ ($\gamma = E/mc^2$ is the particle Lorentz factor), we can recast it in two forms:

$$\left[-\frac{\hbar^2}{2m}\Delta_r + \gamma V(\vec{r})\right]\psi(\vec{r}) = \varepsilon\psi(\vec{r})\,, \tag{1.6}$$

and

$$\left[-\frac{\hbar^2}{2m\gamma}\Delta_r + V(\vec{r})\right]\psi(\vec{r}) = \varepsilon'\psi(\vec{r})\,, \tag{1.7}$$

where $\varepsilon = p^2/2m$; $\varepsilon' = p^2/2m\gamma = \varepsilon\gamma^{-1}$. Recall that $p^2 = (E^2 - m^2c^4)/c^2$.

Equation (1.6) coincides with the nonrelativistic Schrodinger equation for a particle moving in a potential growing with the increase in the particle energy. Equation (1.7) coincides with the Schrödinger equation for a particle with a relativistic mass $m\gamma$.

The eigenfunctions of the Dirac (Schrodinger) equations with a periodic potential are known to be the Bloch functions [Callaway (1964)]. Hence, an arbitrary solution of equations (1.4)-(1.7) is described by the superpositions of the Bloch functions. This fact makes it possible to draw some general conclusions about the nature of the particle-crystal interaction. Further we follow the line of reasoning given in [Sommerfeld and Bethe (1938)] for the case of the interaction between non-relativistic electrons and a crystal, which is also suitable for our case due to the mathematical equivalence of equations (1.4) (1.7) and the non-relativistic Schrodinger equation.

Let a beam of particles with the momentum $\vec{p}$ fall on a plane–parallel crystal plate bounded by the planes $z = 0$ and $z = L$ (see Fig. 1.1). The corresponding plane wave that describes the incident particle is determined by the expression [1]

$$\psi_0(\vec{r}) = \exp(i\vec{p}\vec{r}) = \exp(i\vec{p}_\perp\vec{\rho} + p_z z)\,, \tag{1.8}$$

where $\vec{\rho}$ is the vector with the components x and y; the z-axis is directed into the interior of the crystal perpendicular to its entrance surface. For simplicity, we shall further assume that the crystal lattice is rectangular and has the lattice constants a, b, c in the directions x, y, z, respectively.

The interaction between the wave ψ_0 and a crystal gives rise to secondary waves. The potential V equals zero outside the crystal, and the secondary waves can be represented as a superposition of the eigenfunctions of (1.6), (1.7) when $V = 0$, i.e., as a superposition of plane waves. Therefore outside the crystal on the side of incidence, i.e. at $z > 0$, there are the waves reflected from the crystal, which have the form

$$\psi_{\text{ref}} = \sum_n A_n \exp[i(\vec{p}_{\perp n}\vec{\rho} - p_{zn}z)]\,. \tag{1.9}$$

The transversal components of the momentum $\vec{p}_{\perp n}$ are still arbitrary. As (1.9) should describe the particle flow moving to the left of the crystal,

[1] Unless otherwise stated, assume that $\hbar = c = 1$.

the values of p_{zn} are always positive. Moreover, since the energy of a scattered particle equals the energy of an incident particle, the momentum is

$$p_{zn} = \sqrt{E^2 - p_{\perp n}^2 - m^2} = \sqrt{p^2 - p_{\perp n}^2}\,.$$

So, within the range $z < 0$, the wave function

$$\psi = \psi_0 + \psi_{\text{ref}}\,. \tag{1.10}$$

On the other side of the crystal, at $z > L$, there is a transmitted wave alone

$$\psi = \sum_n A'_n \exp[i(\vec{p}'_{\perp n}\vec{\rho} + p'_{zn}z)]\,, \tag{1.11}$$

where $p'_{zn} = \sqrt{p^2 - p'^2_{\perp n}}$.

Inside the plate, the potential $V(\vec{r})$ differs from zero. The eigenfunctions are the Bloch waves, and the general solution inside the crystal is described by the superposition of the Bloch waves. It is known that the Bloch wave for band n can be written accurate to the normalization factor in the form:

$$\psi_{\kappa n}(\vec{r}) = e^{i\vec{\kappa}\vec{r}} u_{\kappa n}(\vec{r})\,, \tag{1.12}$$

where κ is the reduced quasimomentum; $u_{\kappa n}(\vec{r})$ is the periodic function with the period of the crystal. The function $u_{\kappa n}(\vec{r})$ is likely to be expanded into a Fourier series. As a result, (1.12) can be represented as

$$\psi_{\kappa n}(\vec{r}) = \sum_\tau a_{n\kappa}(\vec{\tau}) \exp[i(\vec{\kappa} + \vec{\tau})\vec{r}]\,, \tag{1.13}$$

where $\vec{\tau}$ is the reciprocal lattice vector with the components $\tau_x = \lambda/a$, $\tau_y = \mu/b$, $\tau_z = \nu/c$ (λ, μ, ν are integral numbers, running over the integers from $-\infty$ to $+\infty$).

Near the plate surface the wave function and its first derivative in the z direction should be continuous. The continuity condition implies that the superposition of the functions (1.3), which describes the wave in a crystal should only contain such Bloch functions for which the sum of vector $\kappa_\perp$ and a certain reciprocal lattice vector $2\pi\vec{\tau}_{0\perp}$ equals $\vec{p}_\perp$, i.e., $\vec{\kappa}_\perp + 2\pi\vec{\tau}_{0\perp} = \vec{p}_\perp$.

Thus, the arbitrary solution of (1.6), (1.7) inside a crystal can also be represented as the superposition of plane waves.

Since inside the plate the wave function contains plane waves with transversal momenta $\vec{\kappa}_\perp + 2\pi\vec{\tau}_\perp$, due to the boundary conditions at the $z = L$ surface, the waves having the same transversal momenta should propagate from a crystal into vacuum. As a consequence behind the plate ($z > L$))

$$\psi = \sum_{\vec{\tau}_\perp} A'(\vec{\tau}_\perp) \exp[i(\vec{p}_\perp + 2\pi\vec{\tau}_\perp)\vec{\rho}] \exp[ip_z(\vec{\tau}_\perp)z], \tag{1.14}$$

where $p_z(\vec{\tau}_\perp) = \sqrt{p^2 - (\vec{p}_\perp + 2\pi\vec{\tau}_\perp)^2}$; before the plate ($z < 0$)

$$\psi = e^{i\vec{p}\vec{r}} + \sum_{\vec{\tau}_\perp} A(\vec{\tau}_\perp) \exp[i(\vec{p}_\perp + 2\pi\vec{\tau}_\perp)\vec{\rho}] \times \exp[-ip_z(\vec{\tau}_\perp)z]. \quad (1.15)$$

An important result (which was already emphasized in [Sommerfeld and Bethe (1938)]) follows from equalities (1.14), (1.15): the direction of scattered waves leaving a plane–parallel plate is uniquely determined by the incident direction and the magnitude of the momentum (energy, wavelength) of the incident particles in the same way as in the elementary kinematic Laue theory of interference developed for thin plates, when the effects of wave refraction may be neglected, namely the projection of the momentum of each scattered wave onto the crystal surface differs from the corresponding value for the incident wave by a reciprocal lattice vector $2\pi\vec{\tau}_\perp$. The possible refraction only leads to the redistribution of intensity among the scattered waves.

This conclusion is valid for any particles interacting with a single–crystal plate and any angles of particle entrance into the crystal (even for those smaller than the Lindhard angle). It means that the channeling phenomenon is just a particular case of diffraction by a periodically arranged set of scatterers (see also [Thompson (1968)]).

1.2 Principles of the Quantum Theory of Channeling

The possibility to describe the interaction of fast particles with a crystal in terms of an averaged potential (1.2), (1.3) enables carrying out a more detailed analysis of the peculiarities of their transmission through a crystal. A thorough quantum mechanical study of this problem on the basis of a time-dependent density matrix (temporal) is given by Kagan and Kononetz in [Kagan and Kononets (1970, 1973, 1974)]. A similar problem in a stationary representation of wave scattering by a crystal was examined by Kalashnikov and Strikhanov in [Kalashnikov and Strikhanov (1975); Kalashnikov *et al.* (1985)] who scrutinized the extreme case of particle scattering by a one plane (axis). Further we will follow the analysis performed by the author together with Dubovskaya [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d)].

First of all, we shall make use of the fact that in the case of axial channeling, due to the constant character of the potential (1.2), (1.3) along the z-axis (in the case of planar channeling, along the y- and z-axes), the particle motion in these directions is free and can be characterized by a well–defined momentum. As a consequence, it is possible to separate variables in (1.6) and (1.7) and then analyze the equations, which depend only on the coordinates relative to which the potential V is periodic. Thus, in the axial case from (1.6) we obtain

$$\left[-\frac{1}{2m}\Delta_\rho + \gamma V(\vec{\rho})\right]\psi_{n\kappa}(\vec{\rho}) = \varepsilon_{n\kappa}\psi_{n\kappa}(\vec{\rho})\,. \tag{1.16}$$

The two–dimensional Bloch functions $\psi_{n\kappa}(\rho)$ (one–dimensional in the planar case) are the eigenfunctions of (1.16). The corresponding eigenvalues are $\varepsilon_{n\kappa}$.

Expand the function $\psi(\vec{r})$ into the eigenfunctions which are determined by equation (1.16):

$$\psi(\vec{r}) = \sum_{n'}\int d^2\kappa' b_{n'\kappa'}(z)\psi_{n'\kappa'}(\vec{\rho}). \tag{1.17}$$

Substitution of (1.16) into (1.6), further multiplication of (1.6) by $\psi^*_{n\kappa}(\vec{\rho})$ and its integration with respect to $\vec{\rho}$ with due account of the orthogonality condition

$$\int \psi^*_{n\kappa}(\vec{\rho})\psi_{n'\kappa'}(\vec{\rho})d^2\rho = \delta_{nn'}\delta(\vec{\kappa}-\vec{\kappa}'), \tag{1.18}$$

gives the equation determining the quantities $b_{n\kappa}(z)$:

$$-\frac{1}{2m}\frac{\partial^2}{\partial z^2}b_{n\vec{\kappa}}(z) = (\varepsilon - \varepsilon_{n\vec{\kappa}})b_{n\vec{\kappa}}(z). \tag{1.19}$$

The solutions of (1.19) are plane waves

$$b_{n\vec{\kappa}}(z) \sim \exp[\pm i p_{zn}(\vec{\kappa})z], \tag{1.20}$$

where the momentum $p_{zn}(\vec{\kappa}) = \sqrt{2m(\varepsilon-\varepsilon_{n\vec{\kappa}})}$, i.e., $p_{zn}(\vec{\kappa}) = \sqrt{p^2 - 2m\varepsilon_{n\vec{\kappa}}}$. (Recall that p is the momentum of a particle incident on a crystal.) So, the general solutions of (1.6) in a crystal can be written as the superposition:

$$\psi(\vec{r}) = \sum_n \int \tilde{c}_{n\kappa}\psi_{n\kappa}(\vec{\rho})\exp[ip_{zn}(\vec{\kappa})z]d^2\kappa\,. \tag{1.21}$$

The waves of the form $\exp[-ip_{zn}(\vec{\kappa})z]$ are not included into the superposition because they describe the mirror reflected waves whose amplitudes for particles incident at not a very small angle relative to the crystal surface are negligible. At the entrance surface of the crystal ($z = 0$), it is necessary to join the superposition of (1.19) and the solution of (1.10), where the mirror reflected waves are also neglected. Thus, at $z = 0$, we have the equality

$$\exp(i\vec{p}_\perp\vec{\rho}) = \sum_n \int d^2\kappa \tilde{c}_{n\vec{\kappa}}\psi_{n\vec{\kappa}}(\vec{\rho})\,. \tag{1.22}$$

Multiplying (1.22) by $\psi^*_{n'\kappa'}(\vec{\rho})$ and integrating it with respect to d^2p, we directly find the expansion coefficients

$$\tilde{c}_{n\vec{\kappa}} = \int \exp(i\vec{p}_\perp\vec{\rho})\psi^*_{n\vec{\kappa}}(\vec{\rho})d^2\rho\,. \tag{1.23}$$

Now make use of the fact that the Bloch function can be written in the form

$$\psi_{n\vec{\kappa}}(\vec{\rho}) = \exp(i\vec{\kappa}\vec{\rho})u_{n\vec{\kappa}}(\vec{\rho})\,,$$

where $u_{n\vec{\kappa}}(\vec{\rho})$ is the function periodic in a transverse plane. The integration with respect to $\vec{\rho}$ in (1.23) is split into the sum of integrals over the unit cells and then (1.23) can be written as follows

$$\tilde{c}_{n\vec{\kappa}} = \sum_{\vec{\tau}_\perp} \delta(\vec{p}_\perp - \vec{\kappa} - 2\pi\vec{\tau}_\perp)c_{n\vec{\kappa}}; \tag{1.24}$$

$$c_{n\vec{\kappa}} = \frac{(2\pi)^2}{S}\int_s e^{i\vec{p}_\perp\vec{\rho}}\psi^*_{n\kappa}(\rho)d^2\rho\,. \tag{1.25}$$

As $\vec{\kappa}$ is the reduced momentum, (1.24) means the requirement of the equality of vector $\vec{\kappa}$ and the reduced part of the transversal momentum of the incident particle $\vec{p}_\perp - 2\pi\vec{\tau}_\perp$. It follows from (1.24) and (1.25) that the wave function of a particle inside a crystal (1.21) can be written in the form

$$\psi(\vec{r}) \sum_n c_{n\vec{\kappa}}\psi_{n\vec{\kappa}}(\vec{\rho})\exp(ip_{\vec{z}n}z), \tag{1.26}$$

where $\vec{\kappa} = \vec{p}_\perp - 2\pi\vec{\tau}_\perp$ ($\vec{\tau}_\perp$ is chosen from the condition of reduction of $p_\perp$ to the first Brillouin zone); $p_{zn} = \sqrt{p^2 - 2m\varepsilon_{n\vec{\kappa}}}$.

If a particle moves in a regime of planar channeling, then the motion along the y-axis is also free (the x-axis is directed perpendicular to the family of planes, along which the particle is channeled). In this case

$$\psi(\vec{r}) = \sum_n c_{n\kappa}\psi_{n\kappa}(x)e^{ip_y y}e^{ip_{zn}z}; \tag{1.27}$$

$$c_{n\kappa} = \frac{2\pi}{a}\int_0^a e^{ip_x x}\psi^*_{n\kappa}(x)dx, \tag{1.28}$$

where a is the lattice spacing along the x-axis; $\kappa = p_x - 2\pi\vec{\tau}_x$.

Let us present the expressions relating the Bloch functions to the localized Wannier functions, which come in handy when analyzing the behavior of a channeled particle. A detailed treatment of their properties for a three–dimensional case is given in [Callaway (1964)]. In the one–and two–dimensional cases, which are of interest for us, the Wannier function centered in a well with the coordinate of its center $\vec{\rho}_m(x_m)$ is determined as follows:

$$W_n(\vec{\rho}-\vec{\rho}_m) = \frac{\sqrt{S}}{2\pi}\int e^{-i\vec{\kappa}\vec{\rho}_m}\psi_{n\vec{\kappa}}(\vec{\rho})d^2\kappa \tag{1.29}$$

or

$$W_n(x-x_m) = \sqrt{\frac{a}{2\pi}}\int e^{-i\kappa x_m}\psi_{n\kappa}(x)d\kappa\,.$$

Integration with respect to κ is made within the first Brillouin zone.

The Bloch functions expressed in terms of the Wannier functions have the form

$$\psi_{n\vec{\kappa}}(\vec{\rho}) = \frac{\sqrt{S}}{2\pi}\sum_m e^{i\vec{\kappa}\vec{\rho}_m}W_n(\vec{\rho}-\vec{\rho}_m);$$

$$\psi_{n\vec{\kappa}}(x) = \sqrt{\frac{a}{2\pi}}\sum_m e^{i\kappa x_m}W_n(x-x_m)\,. \tag{1.30}$$

In normalizing in a finite volume, the factor $\sqrt{S}/2\pi$ $(\sqrt{a}/2\pi)$ should be replaced by $1/\sqrt{N}(1/\sqrt{N_x})$; $N(N_x)$ is the number of unit cells (the number of crystal spacings) in the $(x,\,y)$ plane (along the x-axis). Equalities (1.26)–(1.28) enable analyzing the features of behavior of fast particles in a crystal in the general case.

Now consider a wave produced by a particle behind a crystal. According to (1.14) it is necessary to find the explicit form of the coefficients $A'(\vec{\tau}_\perp)$. For this purpose join the solutions of (1.26) and (1.14) in the $z=L$ plane:

$$\sum_n c_{n\vec{\kappa}}\psi_{n\vec{\kappa}}(\vec{\rho})e^{ip_{zn}L} = \sum_{\vec{\tau}_\perp} A'(\vec{\tau}_\perp)e^{i(\vec{p}_\perp+2\pi\vec{\tau}_\perp)\rho}e^{ip_z(\tau_\perp)L}. \tag{1.31}$$

Substitute the expansion of the Bloch function $\psi_{n\vec{\kappa}}(\vec{\rho})$ into Fourier series into (1.31) (see (1.13)):

$$\psi_{n\vec{\kappa}}(\vec{\rho}) = \sum_{\vec{\tau}_\perp} a_{n\vec{\kappa}}(\vec{\tau}_\perp)e^{i(\kappa+2\pi\vec{\tau}_\perp)\vec{\rho}};$$

$$a_{n\vec{\kappa}}(\vec{\tau}_\perp) = \frac{1}{S}\int_S u_{n\vec{\kappa}}(\vec{\rho})e^{-i\pi\vec{\tau}_\perp\vec{\rho}}.$$

Since (1.31) should be fulfilled at an arbitrary point ρ, it will hold if the coefficients of the identical exponents on the right and left sides of (1.31) are equal. As a result, we have

$$A'(\vec{\tau}_\perp) = \sum_n c_{n\vec{\kappa}} a_{n\vec{\kappa}}(\vec{\tau}_{0\perp} + \vec{\tau}_\perp) e^{i(p_{zn} - p_z(\tau_\perp))L}, \quad (1.32)$$

where $\vec{\tau}_{0\perp} = \vec{p}_\perp - \vec{\kappa}$.

The coefficients $A'(\vec{\tau}_\perp)$ have the meaning of the probability amplitudes to find a particle with the transversal momentum $\vec{p}_\perp + \vec{\tau}_\perp$ in the wave which has passed through a crystal, and, hence, they actually determine the angular distribution of particles behind the crystal.

The expressions obtained above make it possible to determine in the general case all the required characteristics of particles in a crystal and outside it and study the features of the reactions they initiate.

1.3 The Energy–Band Spectrum of Electrons and Positrons Channeled in a Single Crystal

Give a more detailed treatment of the structure of a particle wave function $\psi(\vec{r})$ in a crystal, which is described by equality (1.26). According to (1.26) $\psi(\vec{r})$ is represented as the superposition of the Bloch functions corresponding to a potential periodic in a transverse plane. The contribution of each wave is determined by the coefficient $c_{n\kappa}$, whose squared absolute value defines the probability to find a particle in the state of the band energy spectrum $n\vec{\kappa}$. The formation of the superposition (1.26) at the particle entrance into the crystal is due to the fact that there is a quasi-momentum, not a momentum remaining in a crystal. As a consequence the state with the defined momentum, which describes the particle falling upon the crystal is not stationary inside the crystal. As a result, upon entering into the crystal a particle (for instance, a muon) does not appear to be in some specified band state, but populates the whole set of such states.

To understand the character of the band energy spectrum of channeled particles and the features of its population, it is important to have quantitative results for the stated quantities, which have been obtained by using real interplanar potentials. As demonstrated in [Baryshevsky and Chevganov (1979)], such quite exact calculations for channeled particles can be carried out, using a quasi-classical WKB (Wentzel-Kramer-Brillouin) method. A detailed treatment of the band spectrum theory in the quasi-classical approximation for planar channeling was given by I.D. Feranchuk and B.A.

Chevganov.

First, pay attention to the fact that the general dispersion equation, which defines the band spectrum of a particle in a one-dimensional periodic potential can be obtained without any approximations [Dykhne (1961)]. Indeed, in the range where $\varepsilon' > V(x)$, two linearly independent solutions of equation (1.16) correspond to every value of ε'. Let f and f^* denote these solutions, respectively. Then the general solution of equation (1.16) at $ld < x < (l+1)d$, $l = 0, 1$ may be represented as

$$\varphi_l(x) = c_1 f(x) + c_2 f^*(x). \tag{1.33}$$

Translation to the range $(l+1)d \le x \le (l+2)d$ transforms both function f and f^* into a linear superposition of the same functions, i.e.,

$$\begin{aligned} f(x+d) &= Df(x) + Rf^*(x)\,, \\ f^*(x+d) &= D^* f^*(x) + R^* f(x)\,. \end{aligned} \tag{1.34}$$

From the the periodicity condition follows $|D|^2 = 1 + |R|^2$. Then

$$\varphi_{l+1}(x) = \varphi_l(x+d) = c_1(Df + Rf^*) + c_2(D^* f^* + R^* f)\,, \tag{1.35}$$

and in accordance with the Bloch theorem, $\varphi_{l+1} = e^{i\kappa d}\varphi_l(x)$, where κ is the quasimomentum.

As the functions f and f^* are independent, (1.35) yields the system of equations

$$\begin{aligned} c_1 D + c_2 R^* &= c_1 e^{i\kappa d}\,, \\ c_1 R + c_2 D^* &= c_2 e^{i\kappa d}\,. \end{aligned} \tag{1.36}$$

The condition of the existence of a nontrivial solution of the system (Eq. (1.36)) leads to the desired dispersion equation.

$$\cos\kappa d = |D|\cos\varphi(\varepsilon)\,, \qquad \text{where} \qquad D(\varepsilon) = |D(\varepsilon)|e^{i\varphi(\varepsilon)}\,. \tag{1.37}$$

For convenience sake, instead of the quantities R and D, we shall further use the coefficients of reflection R_1 and transmission D_1 related to them, which can be determined in a conventional manner. For example, for a wave passing through a potential barrier from the right to the left, we obtain

$$f(x) + R_1 f^*(x) = D_1 f(x+d)\,,$$

i.e.,

$$\begin{aligned} &D = \frac{1}{D_1}, \quad R = \frac{R_1}{D_1}, \quad D_1 = |D_1|e^{i\varphi_1(\varepsilon)}\,, \\ &\cos\kappa d = \frac{1}{|D_1|}\cos\varphi_1(\varepsilon)\,. \end{aligned} \tag{1.38}$$

To determine the explicit form of the coefficient D_1, it is necessary to turn to a certain approximation and find the functions f and f^*. Here the quasiclassical approximation is applied with the accuracy for the given equation determined by the parameter [Pokrovskii and Khalatnikov (1961)] $\xi \simeq 1/n^2$, where n is the number of bound levels in an isolated potential well coinciding with the channel potential. In the stated approximation, the functions f and f^* have the form

$$f(x) = \frac{e^{i\int p(x)dx}}{\sqrt{p(x)}}, \quad f^*(x) = \frac{e^{-i\int p(x)dx}}{\sqrt{p(x)}},$$
$$p(x) = \sqrt{2E(\varepsilon' - V(x))}\,. \tag{1.39}$$

Within the range $\varepsilon' < V_{\max}$, the coefficient D_1 is determined by a well–known expression which is true when $|D_1| \ll 1$:

$$|D_1| \approx e^{-\tau_1}, \quad R_1 \approx 1, \quad \varphi(\varepsilon') = \sigma_1(\varepsilon')\,.$$

Here

$$\sigma_1 = \int_{x_1}^{x_0} \sqrt{2E(\varepsilon' - V(x))}dx\,, \quad \tau_1 = \int_{x_2}^{x_3} \sqrt{2E(V(x) - \varepsilon')}dx\,.$$

Location of the turning points x_1, x_2, x_3 is presented in Fig. 1.2, the energy is counted off from the minimum of the potential.

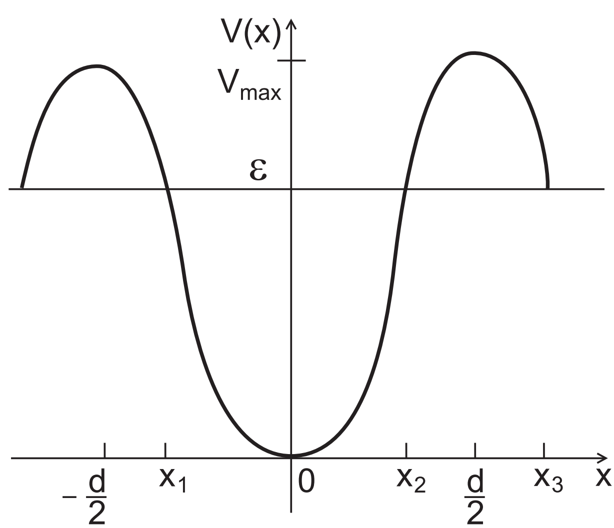

Fig. 1.2 Location of the turning points

In contrast to the classical case, the reflection coefficient is nonzero in the range $\varepsilon' > V_{\max}$ too. The method for calculating D_1 in the quasiclassical approximation at $\varepsilon' > V_{\max}$ developed in [Dykhne (1961); Pokrovskii and Khalatnikov (1961)] is based on application of the path of integration, which passes through the complex turning points defined by the following equalities

$$V(z_0^l) = \varepsilon' , \quad z_0^l = ld + iz_0 , \quad l = 0, 1, 2, \dots . \tag{1.40}$$

Without reproducing the calculations carried out in [Pokrovskii and Khalatnikov (1961)] (see also [Dykhne (1961)]), we only present the expression for the reflection coefficient

$$D_1 = \exp\left\{ i \int_{z_0^l}^{z_0^{l+1}} \sqrt{2E(\varepsilon' - V(x))} dz \right\} . \tag{1.41}$$

If the function $V(x)$ is symmetrical with respect to the line $x = d/2$ as it usually occurs for real potentials, then the path of integration can be chosen so that the quantities $|D_1|$ and $\varphi_1(\varepsilon')$, which we are concerned with, would be represented as real integrals:

$$\varphi_1(\varepsilon') = \sigma_0(\varepsilon') = \int_{-d/2}^{d/2} \sqrt{2E(\varepsilon' - V(x))} dx , \quad |D_1| = e^{-\tau_2} , \tag{1.42}$$

where

$$\tau_2 = 2 \int_0^{y_0} \sqrt{2E\left(\varepsilon' - V\left(\frac{d}{2} + iy\right)\right)} dy , \quad z_0 = \frac{d}{2} + iy_0 .$$

When the energy of transverse motion is close to the top of the potential barrier, (1.41) and (1.42) for D_1 are not applicable. We intend to obtain the formulas, which are valid at $\varepsilon' \approx V_{\max}$ and go over to (1.41) or (1.42) in the corresponding limiting cases.[2]

At the top of the barrier,

$$V(x) \approx V_{\max} - kx_1^2 , \quad k = 2V''(\frac{d}{2}) , \quad x_1 \equiv x - \frac{d}{2} ,$$

and the Schrödinger equation in this case has the analytical solution

$$\varphi(x) = A_1 \mathcal{D}_m(x_1 e^{-i\frac{\pi}{4}} \sqrt[4]{4\lambda}) + B_1 \mathcal{D}_{-m-1}(x_1 e^{i\frac{\pi}{4}} \sqrt[4]{4\lambda}) , \tag{1.43}$$

where $\mathcal{D}_m(z)$ is the parabolic cylinder function;

$$\lambda = \sqrt{2kE}; \quad m = -\frac{1}{2} - \frac{i}{2}\sqrt{\lambda} x_0^2 , \quad x_0^2 = -\frac{\varepsilon' - V_{\max}}{k} .$$

[2]A similar investigation was performed in [Fok (1948)] for a different problem and only for the case when $\varepsilon' > V_{\max}$.

On the right of the potential barrier, i.e., at $x_1 > 0$, the functions $\mathcal{D}_m$ and $\mathcal{D}_{-m-1}$ asymptotically go into functions f and f^*, correspondingly, which are determined by (1.39). As we are concerned with the coefficients of reflection and transmission at the singular barrier alone, let us assume that $B_1 = 0$. Then at $x_1\sqrt[4]{4\lambda} \gg 1$,

$$\varphi(x) \simeq \frac{A_1}{\sqrt[4]{4\lambda x_1^2}} \exp i\left(\frac{\sqrt{\lambda}}{2}x_1^2 - \frac{\sqrt{\lambda}x_0^2}{2}\ln\sqrt[4]{4\lambda}x_1 \right.$$
$$\left. + i\frac{\pi}{8} - \frac{\pi}{8}\sqrt{\lambda}x_0^2\right) \simeq \frac{A'}{\sqrt{p}}\exp\left(i\int_0^{x_1} pdx\right). \tag{1.44}$$

Upon translation to the region $x_1 < 0$, the function $\varphi(x)$ is transformed as follows [Gradstein and Ryzhik (1980)]:

$$\varphi(x) = A_1\mathcal{D}_m(\sqrt[4]{4\lambda}x_1e^{i3\pi/4}) = A_1\left[\mathcal{D}_m(\sqrt[4]{4\lambda}x_1e^{-i\pi/4})\right.$$
$$\left. -\frac{\sqrt{2\pi}}{\Gamma(-m)}e^{i\pi m}\mathcal{D}_{-m-1}(\sqrt[4]{4\lambda}x_1e^{i\pi/4})\right] \tag{1.45}$$

and (at $|x_1\sqrt[4]{4\lambda}| \gg 1$) it should asymptotically go into a linear combination

$$\varphi(x) \approx A\frac{\exp(i\sigma_0 - i\int_0^{x_1}pdx)}{\sqrt{p}} + B\frac{\exp(-i\sigma_0 + i\int_0^{x_1}pdx)}{\sqrt{p}},$$

and $A_1A^{-1} = D_1$, $BA^{-1} = R_1$.

Using the known asymptotics of the functions $\mathcal{D}_m$ and $\mathcal{D}_{-m-1}$ [Gradstein and Ryzhik (1980)], we find

$$D_1 = (\sqrt{2\pi})^{-1}\Gamma\left(\frac{1}{2} + \frac{i}{2}\sqrt{\lambda}x_0^2\right)\exp\left(i\sigma_0 - \frac{\pi}{4}\sqrt{\lambda}x_0^2\right),$$

$$R_1 = (\sqrt{2\pi})^{-1}\Gamma\left(\frac{1}{2} + \frac{i}{2}\sqrt{\lambda}x_0^2\right) \times \exp\left(2i\sigma_0 - i\frac{\pi}{2} + \frac{\pi}{4}\sqrt{\lambda}x_0^2\right).$$

where $\Gamma(z)$ is the gamma function.

Thus, near the top of the barrier

$$|D_1| = (\sqrt{2\pi})^{-1}\left|\Gamma\left(\frac{1}{2} + \frac{i}{2}\sqrt{\lambda}x_0^2\right)\right|\exp\left(-\frac{\pi}{4}\sqrt{\lambda}x_0^2\right)$$
$$= \frac{\exp\left(-\frac{\pi}{2}\sqrt{\lambda}x_0^2\right)}{2\cosh\frac{\pi}{2}\sqrt{\lambda}x_0^2},$$
$$\varphi(\varepsilon') = \arg D_1 = \sigma_0 + \delta; \delta = \arg\Gamma\left(\frac{1}{2} + \frac{i}{2}\sqrt{\lambda}x_0^2\right). \tag{1.46}$$

Now take into account that in the domain of applicability of the solution of (1.43), the following equalities hold

$$\pi\sqrt{\lambda}x_0^2 = \int_{x_2}^{x_3} \sqrt{2E[V(x)-\varepsilon']}dx = \tau_1\,, \quad \text{at} \quad \varepsilon' < V_{\max}\,, \tag{1.47}$$

$$\pi\sqrt{\lambda}x_0^2 = -2\int_0^{y_0} \sqrt{2E\left[\varepsilon' - V\left(\frac{d}{2}+iy\right)\right]}dy = -\tau_2\,, \quad \text{at} \quad \varepsilon' > V_{\max}\,.$$

As a result, with the considered accuracy, the dispersion equation takes the form

$$\cos\kappa d = 2\cosh\frac{\tau_1}{2}e^{\tau_1/2}\cos\sigma\,, \quad \text{at} \quad \varepsilon' < V_{\max}\,,$$

$$\cos\kappa d = 2\cosh\left(\frac{1}{2}\tau_2\right)e^{-\tau_2/2}\cos\sigma_0 \quad \text{at} \quad \varepsilon' > V_{\max} \tag{1.48}$$

and enables plotting a band spectrum within the whole energy range of a particle.

Consider the limiting cases for which the analytical solution of the equation can be constructed:
(1) $\varepsilon' < V_{\max}, \tau_1 \gg 1$. In this case [Feranchuk (1979a)]

$$\sigma = \ pi\left(n+\frac{1}{2}\right) + e^{-\tau_1}\cos\kappa d, n = 0,1,2,\dots\,,$$

$$\varepsilon'_{n\kappa} \ = \varepsilon_n^{(0)} + \Delta\varepsilon_{n\kappa}\,, \tag{1.49}$$

where $\varepsilon_n^{(0)}$ coincide with the energy levels of the discrete spectrum in the isolated potential well;

$$\int_{x_1}^{x_2} \sqrt{2E[\varepsilon_n^{(0)} - V(x)]}dx = \pi\left(n+\frac{1}{2}\right)\,,$$

and the quantities

$$\Delta\varepsilon_{n\kappa} \approx e^{-\tau_1(\varepsilon_n^{(0)})}\cos\kappa d \ll \varepsilon_{n+1}^{(0)} - \varepsilon_n^{(0)}$$

determine the energy levels of the allowed band with the width considerably smaller than the distance between the bands;
(2) $\varepsilon' > V_{\max}, \tau_2 \gg 1$:

$$\cos\kappa d = (1+e^{-\tau_2})\cos\sigma,$$

$$\int_{-d/2}^{d/2} \sqrt{2E(\varepsilon'_{n\kappa} - V(x))}dx = \pi n + \kappa d + \delta_n(\kappa)\,,$$

$$\delta_n \ll 1, 0 \le \kappa d \le \pi\,, \tag{1.50}$$

$$\delta_n(\kappa) = (-1)^n c \pm \sqrt{\kappa^2 + 2\exp[-\tau_2(\varepsilon'_{n\kappa})]}\,, \quad c = \pi - \kappa d\,,$$

in this case the energy spectrum consists of wide allowed bands and narrow forbidden bands with the width

$$\Delta\varepsilon_{\rm forb} = 2\sqrt{2\exp[-\tau_2(\varepsilon'_{n\kappa})]}\,, \tag{1.51}$$

and for $\varepsilon' \gg V_{\max}$, $\varepsilon'_{n\kappa} = (\kappa d + \pi n)^2/2E$.

The exact solutions of equation (1.48) were numerically found for a silicon crystal. The potential obtained in the Moliere approximation and averaged over temperature oscillations [Gemmell (1974)] was used as an interplanar potential.The potential obtained in the Moliere approximation and averaged over temperature oscillations [Gemmell (1974)] was used for computation. This potential has the form:

for positrons

$$\begin{aligned} V_{e^+} = \; & \psi_a^2 \sum_{i=1}^{3} \frac{1}{2}\gamma_i e^{\tau_i} \left\{ e^{-\beta_i x/a} erfc\left[\frac{1}{\sqrt{2}}\left(\frac{\beta_i u_1}{a} - \frac{x}{u_1}\right)\right] \right. \\ & \left. + e^{\beta_i x/a} erfc\left[\frac{1}{\sqrt{2}}\left(\frac{\beta_i u_1}{a} + \frac{x}{u_1}\right)\right]\right\}\,, \end{aligned}$$

for electrons

$$V_{e^-}(x) = -V_{e^+}\left(x - \frac{d}{2}\right) + V_{\max}\,.$$

Here ψ_a, γ_i, β_i, τ_i, u_1 are the potential parameters determined in [Gemmell (1974)].

It should be noted that when using the programme of numerical solution of equation (1.48) arranged in the optimum way, plotting of the whole energy spectrum for electrons and antielectrons (Fig. 3) with 1% accuracy takes a few minutes on the EC 1030 computers.

Now proceed to considering normalization of the wave functions and the occupancy coefficients for the energy levels.

According to (1.33) and (1.36), the stationary particle wave function in a channel $\varphi(x) = c_1 f(x) + c_2 f^*(x)$, and $c_2 = q(\varepsilon)c_1$, $q(\varepsilon) = R_1^{-1}[D_1(\varepsilon)e^{i\kappa d} - 1]$, the coefficient c_1 is determined from the normalizing condition

$$|c_1|^2(1 + |q|^2)J = 1,\; J = \int_{-d/2}^{d/2} |f|^2 dx\,,$$

where the integrals of rapidly oscillating function f^2 and f^{*2} are dropped.

For the energy levels lying far from the top of the potential barrier:

$$J = \frac{1}{E}T(\varepsilon') = \begin{cases} \int_{x_1}^{x_2} \frac{dx}{p(x)} & \text{at} \quad \varepsilon' < V_{\max} \\ \int_{-d/2}^{d/2} \frac{dx}{p(x)} & \text{at} \quad \varepsilon' > V_{\max} \end{cases}\,, \tag{1.52}$$

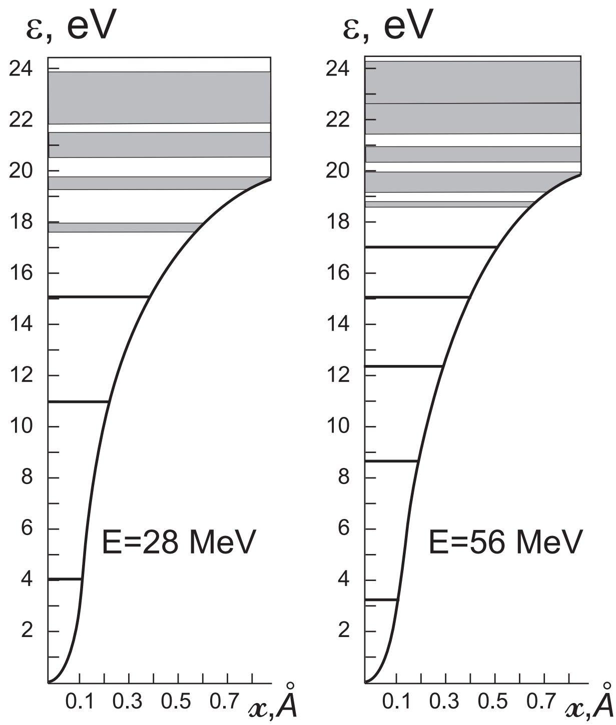

Fig. 1.3 Energy bands calculated for electrons with the energies $E = 28$ and 56 MeV. Channeling along the plane (110) in a silicon crystal. Solid horizontal lines - the positions of the energy levels in the Moliere potential

where $T(\varepsilon')$ is the classical flight time of particles between the planes. From this the normalizing constant is

$$c_1 = \frac{1}{\sqrt{1+|q|^2}}\sqrt{\frac{E}{T}}\,. \tag{1.53}$$

If the quantum effects of tunneling and over-barrier reflection are neglected, then $|q|^2$ takes on only two values: $|q|^2 = 0$ at $\varepsilon' > V_{\max}$ and $|q|^2 = 1$ at $\varepsilon' < V_{\max}$. The normalizing constant abruptly changes by a factor of $\sqrt{2}$ when ε' goes from the subbarrier to the over-barrier range, whereas in reality the quantity $|q|^2$ smoothly changes from 0 to 1.

This fact also manifests itself within the classical approach to the problem: in this case when a particle passes through a barrier, the cycle of particle motion changes abruptly. The false opposition of particles executing infinite motion and channeled particles appears.

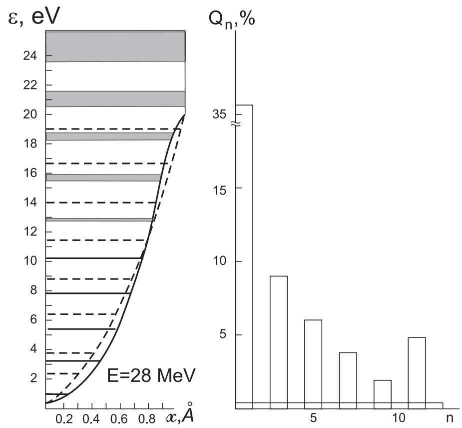

Fig. 1.4 Energy bands (a) and occupancy coefficients of the energy levels (b) for positrons at zero entrance angle. Dashed curve - the parabolic potential, dashed horizontal line - the position of levels in it

Formula (1.53) becomes unsuitable in the vicinity of the top of the barrier when a classical cycle T becomes infinite. However, using the analytical solution of (1.43) enables one to regularize the expressions for the normalization integral (1.53). Indeed, let us introduce a certain passing point with the coordinate a satisfying the conditions compatible in the case of quasi-classical motion:

$$\frac{d}{2} - a \ll \frac{d}{2}\,,$$

but

$$\sqrt[4]{4\lambda}\left(\frac{d}{2} - a\right) \gg 1\,.$$

Then we obtain for a potential symmetrical with respect to the $d/2$-axis

$$\begin{aligned} J &\approx 2\int_{x_2-a}^{x_2} |f|^2 dx + 2\int_0^{x_2-a} \frac{dx}{p(x)} \\ &\simeq 2\left[c(\varepsilon')\int_0^{x_2} |\mathcal{D}_m(x\sqrt[4]{4\lambda}e^{-i\frac{\pi}{4}})|^2 dx \right. \\ &\qquad \left. + \int_0^{x_2}\left(\frac{1}{p(x)} - \frac{1}{p_0(x)}\right) dx\right]\,, \end{aligned} \tag{1.54}$$

where

$$p_0(x) = \sqrt{2E(\varepsilon' - V_{\max} + kx^2)}\,,$$

$$c(\varepsilon') = \begin{cases} e^{-\frac{1}{4}\tau_1}\,, & \text{at} \quad \varepsilon' < V_{\max} \\ e^{-\frac{1}{4}\tau_2}\,, & \text{at} \quad \varepsilon' > V_{\max}\,, \end{cases}$$

and the asymptotics of (1.44) is used for the function $\mathcal{D}_m(z)$.

Transform the formula for $|q|^2$ allowing for the dispersion equation:

$$|q|^2 = \begin{cases} \dfrac{A^2 + 1 - 2\cos^2\sigma - 2\sin\sigma\sqrt{A^2 - \cos^2\sigma}}{1 + A^2}\,, & \varepsilon' < V_{\max} \\[2ex] \dfrac{B^2 + 1 - 2\cos^2\sigma_0 - 2\sin\sigma_0\sqrt{B^2 - \cos^2\sigma_0}}{1 + B^2}\,, & \varepsilon' > V_{\max}\,, \end{cases} \tag{1.55}$$

and

$$A = \left(2\cosh\frac{\tau_1}{2}e^{\frac{\tau_1}{2}}\right)^{-1}, \quad B = \left(2\cosh\frac{\tau_2}{2}e^{-\frac{\tau_2}{2}}\right)^{-1}.$$

Formulas (1.54) and (1.55) enable one to calculate the normalization constant c_1 at all possible values of ε'. It should be pointed out that for the integral of $|\mathcal{D}_m|^2$ in (1.54), it is possible to obtain the analytical expression in terms of the G-function of Meyer [Gradstein and Ryzhik (1980)]. But it is more reasonable to find this integral numerically, using the integral representation of $\mathcal{D}$-functions.

Now go over to calculating the occupancy coefficients $Q_{n\kappa} = |c_{n\kappa}|^2$ of the energy levels. Consider first those values of E, at which the quantum mechanical effects are insignificant, as we did when calculating the normalization integral. Then

$$c_{n\kappa} = c_1 \int_{-d/2}^{d/2} \exp\left[-i\int_{x_1}^{x} p(x')dx' + ip_{0x}x\right] \frac{dx}{\sqrt{p(x)}}\,, \tag{1.56}$$

$$p_{0x} = p_{0z}\theta\,, \tag{1.57}$$

and to calculate the integral (1.56) in the given approximation, one can use the saddle–point method. As a result, at

$$0 < \varepsilon'_{n\kappa} - \frac{p_{0x}^2}{2E} < V_{\max}\,,$$

we find the expression derived in [Ryabov (1970)]:

$$c_{n\kappa} = \frac{\sqrt{\pi}c_1}{\sqrt{E|V'(x_0)|}} \exp\left[ip_{0x}x - i\int_{x_1}^{x} p(x')dx'\right], \tag{1.58}$$

and the saddle point x_0 is determined by the condition

$$p_{0x} = p(x_0)\,, \quad \text{i.e.}\,, \quad V(x_0) = \varepsilon'_{n\kappa} - \frac{p_{0x}^2}{2E}\,. \tag{1.59}$$

In the case when

$$\varepsilon'_{n\kappa} - \frac{p_{0x}^2}{2E} > V_{\max} \qquad \text{or} \qquad \varepsilon'_{n\kappa} - \frac{p_{0x}^2}{2E} < 0\,,$$

(1.59) does not have real roots (recall that for electrons, as well as for positrons, the energy is counted off from the potential minimum), but the solution $z_0 = x_0 + iy_0$ always exists in a complex plane. Appearance of the imaginary part of the saddle point coordinate means, in fact, the exponential attenuation of the occupancy coefficients in these energy bands. Analytical continuation of quasiclassical wave functions makes it possible to obtain the following expression for the occupancy coefficients:

$$Q_{n\kappa} = \begin{cases} \dfrac{\pi|c_1|^2}{E|V'(x_0)|}\,, \quad \text{at} \quad 0 < \varepsilon'_{n\kappa} - \dfrac{p_{0x}^2}{2E} < V_{\max}\,, \\[2ex] \dfrac{\pi|c_1|^2}{E|V'(z_0)|} e^{-2\int_0^{y_0}\sqrt{2E[\varepsilon'_{n\kappa} - V(\frac{d}{2}+iy)]}dy + 2p_{0x}y_0}\,, \\[2ex] \qquad \text{at} \quad \varepsilon'_{n\kappa} - \dfrac{p_{0x}^2}{2E} > V_{\max}\,, \\[2ex] \dfrac{\pi|c_1|^2}{E|V'(z_0)|} e^{2\int_0^{y_0}\sqrt{2E[\varepsilon'_{n\kappa} - V(0+iy)]}dy - 2p_{0x}y_0}\,, \\[2ex] \qquad \text{at} \quad \varepsilon'_{n\kappa} - \dfrac{p_{0x}^2}{2E} < 0\,, \end{cases} \tag{1.60}$$

and at any p_{0x}, the complex turning point z_0 is defined by the equality

$$\varepsilon'_{n\kappa} - \frac{p_{0x}^2}{2E} = V(z_0)\,.$$

Expression (1.60) becomes inapplicable at $|V'(z_0| \approx 0$, i.e., when the saddle point is located either near the top of the barrier or near the bottom of the potential well. In the vicinity of these points, the real potential is approximated by a parabola, and the values of $Q_{n\kappa}$ in this energy band can be calculated, using analytical solutions of the Schrödinger equation:

$$\begin{aligned} &\varphi_1 = A_1[\mathcal{D}_{m_1}(x\sqrt[4]{4\lambda_1}) + \mathcal{D}_{-m_1-1}(ix\sqrt[4]{4\lambda_1})]\,, \\ &\varepsilon'_{n\kappa} - \frac{p_{0x}^2}{2E} \simeq 0\,, \\ &\varphi_2 = A_2\mathcal{D}_{m_2}(x_1 e^{-i\frac{\pi}{4}}\sqrt[4]{4\lambda_2})\,, \quad \varepsilon'_{n\kappa} - \frac{p_{0x}^2}{2E} \simeq V_{\max}\,, \end{aligned} \tag{1.61}$$

where

$$m_1 = -\frac{1}{2} + i\frac{\sqrt{\lambda_1}}{2}\frac{(\varepsilon' - V_{max})}{k_1}\,,$$

$$m_2 = -\frac{1}{2} + \frac{\sqrt{\lambda_2}}{2}\frac{\varepsilon'}{k_2}\,, \quad \lambda_{1,2} = \sqrt{2Ek_{1,2}}\,,$$

$$k_1 = 2V''(0)\,, \quad k_2 = 2V''\left(\frac{d}{2}\right)\,, \quad x_1 = x - \frac{d}{2}\,.$$

The coefficients in linear combinations of the parabolic cylinder functions in (1.61) are chosen on condition that at $|m_{1,2}| \gg 1$, the functions $\varphi_{1,2}$ should go over to a quasi-classical solution

$$\frac{1}{\sqrt{p(x)}}\exp[\pm i\int p(x)dx]\,.$$

Equation (1.60) enables one to calculate the occupancy coefficients of the energy levels within the whole energy range (Fig. 1.4).

Chapter 2

A Channeled Fast Particle as a Two-Dimensional (One-Dimensional) Relativistic Atom

2.1 Spontaneous Photon Radiation in Radiation Transitions Between the Bands of Transverse Energy of Channeled Particles

Transverse motion of a channeled particle is characterized by a distinct band energy spectrum (see Fig. 3). The bands deep in the wells are very narrow. In this case it is possible to speak of discrete levels in a well. Kalashnikov, Koptelov and Ryazanov in [Kalashnikov *et al.* (1972, 1975)] put forward the idea that the emission of X-ray and γ-radiation may occur through radiative capture of the electron entering a crystal at the levels of transverse motion in a well formed by the axis (plane). According to [Vorobiev *et al.* (1975)] at the transition of a channeled electron with the energy of the order of a few mega electron-volts between the levels in a well, one should expect the emission of optical radiation. A detailed treatment carried out by the author together with Dubovskaya in [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d, 1976a)] demonstrated that the stated above effects of photon formation are a particular case of the general mechanism of γ-quantum emission at radiative transitions between the energy bands of the transverse motion of particles passing through a crystal, which occurs for both electrons and positrons.

Within the framework of the quantum mechanical correspondence principle every radiative transition may be described as the radiation of a certain classical oscillator. Since a particle has a transversal momentum, we shall deal with a moving one- or two-dimensional "atom" whose radiation spectrum is considerably influenced by the Doppler effect [Baryshevskii and Dubovskaya (1976a)]. From the viewpoint of the classical theory the possibility of the induction of γ-radiation by channeled electrons and positrons and the importance of the Doppler effect in this process were pointed out by Kumakhov [Kumakhov (1976)]. Note, however, that the idea of the induction of X-ray and γ-radiation at radiative transitions between the energy bands of transverse motion of relativistic particles in crystals still was not articulated in this work.

Interestingly enough, that the concept of the possibility of optical and soft X-ray radiation of diffracted particles in crystals at interband transition was expressed in [Hirsch *et al.* (1965)]. But only in [Baryshevsky and Dubovskaya (1976a); Kumakhov (1976); Baryshevskii and Dubovskaya (1976a)]the authors came to clear awareness of the crucial role of the Doppler effect causing the transformation of relatively low-frequency particle oscillations in a crystal (characteristic frequencies are in the optical and soft X-ray spectral regions) into hard X-ray and γ-radiation, whose frequency increases with the growth of particle energy.

The major characteristics of the radiation produced by channeled particles may be deduced by the simple reasoning given below [Baryshevskii and Dubovskaya (1977d, 1976a)].

Let a particle with the momentum $\vec{p}$ and the energy E fall upon a plane-parallel crystal plate. Its collision with the crystal results in the emission of a photon with the energy ω and momentum k. In the final state the particle energy and momentum take on the values E_1 and $\vec{p}_1$. It is important to remember that if the reaction proceeds in an arbitrary constant field, the energy (not the momentum) of the system is conserved. Thus, for particle energies we have the equality

$$E = E_1 + \omega. \tag{2.1}$$

Due to the periodicity in a transverse plane of the crystal potential responsible for channeling, the transversal component of the momentum retains accurate to the reciprocal lattice vector of the crystal (see Chapter I),

$$\vec{p}_\perp = \vec{p}_{1\perp} + \vec{k}_\perp + \vec{\tau}_\perp \,. \tag{2.2}$$

In the longitudinal direction, the potential responsible for channeling is constant, and the particle has a certain longitudinal momentum p_{zn} (see Chapter I), so

$$p_{zn} = p_{1zn} + k_z n(k_z) \,, \tag{2.3}$$

where $n(k_z)$ is the refractive index of the crystal, still considered to be real.

According to the analysis made in [Ginzburg (1940)] the photon momentum in a medium is kn. In the representation of (2.2), (2.3) we have taken into account the fact that at radiation in a finite plate the transversal component of the momentum does not change through refraction at the boundary, but the longitudinal component of the photon momentum undergoes an abrupt change. Equalities (2.2), (2.3) follow from the rigorous theory of radiation in a plate of thickness L (see Chapter I).

Consider thoroughly equality (2.3) determining the change in the particle longitudinal momentum through photon emission. Write the explicit form of (2.3) in terms of the particle energy. According to Chapter I, section 2

$$p_{zn} = \sqrt{p^2 - 2m\varepsilon_{n\kappa}(E)}; \qquad p_{1zf} = \sqrt{p_1^2 - 2m\varepsilon_{f\kappa_1}(E_1)}\,,$$

κ is the reduced quasi-momentum corresponding to the transversal momentum of the particle in the initial state $\vec{p}_\perp$; κ_1 is the quasi-momentum of the particle in the final state, which is obtained from (2.2) by reduction of $p_{1\perp}$ to the first Brillouin zone. Using the equalities for p_{zn} and p_{1zn}, equation (2.3) can be written in the form

$$\sqrt{E^2 - m^2 - 2m\varepsilon_{n\kappa}(E)} = \sqrt{E_1^2 - m^2 - 2m\varepsilon_{f\kappa_1}(E_1)} + k_z n(k_z). \qquad (2.4)$$

As the total particle energy is much greater than the energy associated with the transversal motion of a particle in a crystal, it is possible to expand the square roots in equality (2.4).

In the most interesting case in consideration of radiation under channeling of particles with the energy less than a few gigaelectronvolts $\omega \ll E, E_1$. As a result (2.4) can be recast as

$$\omega[1 - \beta n(\omega)\cos\vartheta] - \frac{m}{E}(\varepsilon_{n\kappa} - \varepsilon_{f\kappa_1}) = 0. \qquad (2.5)$$

In writing (2.5), it is assumed that $\cos\vartheta$ in the expression for $n(k_z) = n(\omega\cos\vartheta) \simeq n(\omega)$ is equal to unity due to the fact that for relativistic particles the effective angle of photon radiation is

$$\vartheta \sim \frac{m}{E} = \frac{1}{\gamma} \ll 1\,, \quad \beta = v_z\,,$$

From (2.5) follows

$$\omega = \frac{(\varepsilon_{n\kappa} - \varepsilon_{f\kappa_1})\gamma^{-1}}{1 - \beta n(\omega)\cos\vartheta}\,. \qquad (2.6)$$

To clarify the meaning of equality (2.6), let us compare it with the expression determining the frequency of photons emitted by an oscillator moving in a medium:

$$\omega = \frac{\Omega}{1 - \beta n(\omega)\cos\vartheta}, \qquad (2.7)$$

where Ω is the oscillator frequency in the laboratory frame of reference; $\Omega = \Omega_0\sqrt{1-\beta^2} = \Omega_0\gamma^{-1}$; Ω_0 is the oscillator frequency in its rest frame. Comparing (2.6) and (2.7), one can notice that a particle under channeling conditions can be considered as a moving in a medium oscillator having the following frequency in its rest frame (i.e., the frame with a zero longitudinal particle velocity)

$$\Omega_{0nf} = \varepsilon_{n\kappa} - \varepsilon_{f\kappa_1} \,. \tag{2.8}$$

Thus, the frequency Ω_{0nf} is determined by the difference of energies between the discrete zones (levels) of particle transverse motion [Baryshevskii and Dubovskaya (1976a)]. In the laboratory frame the frequency of such an oscillator is

$$\Omega_{nf} = (\varepsilon_{n\kappa} - \varepsilon_{f\kappa_1})\gamma^{-1} = \varepsilon'_{n\kappa} - \varepsilon'_{f\kappa_1} \,. \tag{2.9}$$

It should be pointed out that unlike a conventional oscillator, the frequency of the oscillator correlated with a channeled particle in the rest frame depends on the particle energy owing to the fact that the value of the potential $u_c(\vec{\rho})$, produced by crystal axes (planes) depends on the particle energy $u_c(\vec{\rho}) = \gamma u(\vec{\rho})$ ($u_c(\vec{\rho})$ is the potential of axes (planes) in the laboratory frame). In this regard it is interesting that equation (1.16) can be treated as the equation describing the spectrum of a particle transverse motion in the coordinate system where its longitudinal momentum is equal to zero.

To be more specific, suppose that a particle undergoes transitions between the zones of transverse motion located inside the well (see **Figure** Channeling Figure 3). In this case the energy zones may be treated as discrete levels. Their dependence on the particle energy can be found explicitly for the simplest cases. Let, for example, a potential well be rectangular. Then $\varepsilon_n = \pi^2 n^2/2md^2$ ($n = 1, 2, 3, ... : d$ is the well width). At the transition between the levels with specified values of (n, f), the frequency $\Omega_{nf} \sim \gamma^{-1}$ and the frequency of a forward-emitted photon (without regard to the refraction effect) is

$$\omega = 2(\varepsilon_n - \varepsilon_f)\gamma = \frac{\pi^2}{m^2 d^2}(n^2 - f^2)E \,. \tag{2.10}$$

Thus, the radiation frequency increases linearly with the increase in the particle energy, and for $\varepsilon_n - \varepsilon_f \ll m$ it is always $\omega \ll E$.

If we consider the transition between the level located at the well edge ($\varepsilon_n \sim \gamma u$, u is the well depth) and the lower state (for example, $\varepsilon_f = \pi^2/2md^2$), then $\omega = 2u\gamma^2$, and the maximum photon frequency in this case increases quadratically with increasing energy. At the energies close to m^2/u, the frequency ω is comparable to E, and in (2.4) it is important that a significant change in E_1 should be taken into account. At the transitions between the levels located at the well edge $n \sim f \sim \sqrt{\gamma}$, $\varepsilon_n - \varepsilon_f \sim \sqrt{\gamma}$. As a consequence, $\omega \sim \gamma^{3/2}$. The frequency of a forward-emitted photon exhibits the same energy-dependent behavior pattern when moving in an oscillatory well [Kumakhov (1977)], as well as in a quasi-classical approximation for the transition between neighboring levels [Zhevago (1978)].

However, the stated energy-dependent behavior of the frequency ω holds true only in the absence of refraction and absorption of photons (the refractive index is $n = 1$). If recall that n is different from unity, equality (2.6) in fact turns into the equation determining the value of the frequency ω. As a result, it is possible that additional frequencies determined by the dependence of the refractive index of a medium on the frequency of a produced photon appear in the radiation spectrum of a channeled particle, i.e., the complex and anomalous Doppler effect may arise [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1976a)]. In the case under study, due to the above mentioned similarity of the laws governing the process of photon emission by a channeled particle and those concerning the process of the photon emission by a moving atom, the theory of the complex and anomalous Doppler effects is formed in a perfect analogy with the that given by Frank in [Frank (1942, 1959, 1969, 1979)] for the case of moving atoms. According to [Frank (1942)], the region of the complex photon spectrum existence is determined by the condition

$$\frac{v\cos\vartheta}{W(\omega)} \geq 1,$$

where $W(\omega) = d\omega/dk$ is the photon group velocity.

In the X-ray and harder spectral ranges $n - 1 < 10^{-5}$. Hence, W is close to the velocity of light in a vacuum. In order to observe the manifestation of a few frequencies within the stated spectral range, the oscillator in a medium is to be started up to achieve very high energies. For instance, if we are concerned about the emission of photon with the energy $\omega \geq 1\,\text{keV}$, then at $\vartheta = 10^{-3}$ rad the particle velocity should satisfy the condition $v \geq 1 - 10^5$, which corresponds to the energies $E \geq 3 \cdot 10^2\, m$. Such energies are really difficult to achieve for atoms and nuclei, but at the same time they are attainable for a channeled electron (positron). Thus, the study of radiation of channeled particles enables us to investigate the complex and anomalous Doppler effects even within the X-ray spectrum [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1976a)].

Using (2.6) and the following explicit expression for the refractive index in the X-ray spectrum far from the characteristic atomic frequencies

$$n(\omega) = 1 - \frac{\omega_L^2}{2\omega^2}$$

($\omega_L^2 = 4\pi z N_A e^2/m$ is the plasma frequency of the medium; N_A is the number of atoms per 1 cm^3), we get the explicit expression for the possible frequencies of the emitted photons at the transitions $n \to f$ inside a well, when the zone width may be neglected, being considered as a discrete level in the form

$$\omega_{nf}^{(1,2)} = \frac{2m(\varepsilon_n - \varepsilon_f) \pm [4m^2(\varepsilon_n - \varepsilon_f)^2 - 8E^2\omega_L^2(1 - \beta\cos\vartheta)]^{1/2}}{4E(1 - \beta\cos\vartheta)} . \quad (2.11)$$

According to (2.11) in the spectral range in question, two frequencies of the emitted photons the difference between which depends on the energy of the incident particle and the observation angle ϑ correspond to the given transition. If the difference $(\varepsilon_n - \varepsilon_f)$ changes with the energy growth slower than E^2, then for the given nonzero value of the angle ϑ, the difference between the frequencies $\omega^{(1)}$ and $\omega^{(2)}$ decreases, vanishing at a certain value of $E = E_{nf}$.

At $E > E_{nf}$ the frequencies (2.11) become complex. This means that the radiation of hard photons at a selected angle ϑ is impossible. As there is a one–to–one correspondence between the frequencies of the emitted photons and the angle of radiation of γ-quanta, it is obvious that for a given value of ω_{nf}, we obtain the constraints for the possible angles of observation of this frequency. At $\vartheta \to 0$ the threshold energy value grows, and at $\vartheta = 0$ the upper limit (threshold) disappears. In this case (compare with [Frank (1969)])

$$\omega_{nf}^{(1,2)} = \left[(\varepsilon_n - \varepsilon_f) \pm \sqrt{(\varepsilon_n - \varepsilon_f)^2 - \omega_L^2}\right] \frac{E}{m} . \quad (2.12)$$

It follows from (2.12) that certain restrictions are also imposed on the possible values of the difference of the energies of transitions $(\varepsilon_n - \varepsilon_f)$. Namely, it is necessary that $|\varepsilon_n - \varepsilon_f| > \omega_L$. At $|(\varepsilon_n - \varepsilon_f)| < \omega_L$, the frequencies characterizing the transitions between the discrete levels n and f are not observed in the radiation spectrum. The restrictions obtained agree well with the criterion of the appearance of the Doppler effect for an oscillator moving in a medium [Frank (1959)]. If $\varepsilon_n - \varepsilon_f \gg \omega_L$, then

$$\omega_{nf}^{(1)} \simeq 2(\varepsilon_n - \varepsilon_f)\frac{E}{m}; \qquad \omega_{nf}^{(2)} \simeq \frac{\omega_L^2}{2(\varepsilon_n - \varepsilon_f)} \frac{E}{m} . \quad (2.13)$$

This makes it clear that the medium has practically no influence on hard radiation. Soft radiation is totally dependent on the refractive properties of the medium. If $(\varepsilon_n - \varepsilon_f) \sim \sqrt{\gamma}$ (the oscillatory well, the transitions between neighboring levels in the quasiclassical case), the frequency $\omega_{nf}^{(2)} \sim \sqrt{\gamma}$, i.e., the frequency goes up slowly with the growth of energy. If the difference $(\varepsilon_n - \varepsilon_f) \sim \gamma$, then $\omega_{nf}^{(2)} = \text{const}$. It also follows from the apparent requirement $\omega_{nf}^{(1,2)} \geq 0$ that in view of (2.11), the transitions to lower energy levels $\varepsilon_f < \varepsilon_n$ are only possible.

2.2 Complex and Anomalous Doppler Effects in an Absorption Medium

Now consider the how the radiation spectrum changes of in an absorbing medium [Baryshevsky and Dubovskaya (1978)]. In this case the refractive index is complex, and equality (2.3), which in fact shows that the momentum transmitted to the medium $q_{znf} = p_{zn} - p_{1zf} - k_{zn}$ is zero, does not hold. However, the smaller q_{znf}, the greater the probability of photon emission is. The radiation probability will have its peak value at the γ-quantum frequencies ω_{nf}, for which the longitudinal transmitted momentum has a minimum value.

With the presence of the imaginary part of n and the fulfillment of the condition $\omega \ll E$, the longitudinal transmitted momentum may be written as follows:

$$q_{znf} = \omega - \omega\beta n'(\omega)\cos\vartheta - \frac{m}{E}(\varepsilon_{n\vec{\kappa}} - \varepsilon_{f\vec{\kappa}_1}) - i\omega\beta n''(\omega)\cos\vartheta, \qquad (2.14)$$

where $n = n' + in''$; n' is the real part of n; n'' is the imaginary part of n. According to (2.14), the minimum value of q_{znf} is limited by its imaginary part.

$$\mathtt{Im}q_{znf} = \omega\beta n''(\omega)\cos\vartheta \equiv \delta(\omega). \qquad (2.15)$$

The corresponding photon frequencies for which the quantity q_{znf} is minimum, and, hence, the radiation probability is maximum, are determined from the condition

$$\mathtt{Re}q_{znf} = \omega - \omega\beta n'\cos\vartheta - \frac{m}{E}(\varepsilon_{n\vec{\kappa}} - \varepsilon_{f\vec{\kappa}_1}) = |\varepsilon|\delta(\omega), \qquad (2.16)$$

where $|\varepsilon| \leq 1$.

At $\varepsilon = 0$, the condition (2.16) determines the frequencies corresponding to the frequencies in the center of the given intensity maximum. All other frequencies at $\varepsilon \neq 0$ are located in some vicinity on either side of the central frequency. The radiation intensity corresponding to them is comparable with the radiation intensity of the central frequency. Therefore equation (2.16) in fact determines the radiation spectrum and may be recast as follows

$$\omega_{nf} = \frac{\varepsilon'_{n\vec{\kappa}} - \varepsilon'_{f\vec{\kappa}_1} + |\varepsilon|\delta(\omega_{nf})}{1 - \beta n'(\omega_{nf})\cos\vartheta}. \tag{2.17}$$

Note that solving equation (2.17), one should bear in mind that the reduced quasi-momentum $\vec{\kappa}_1$ depends on the frequency $\omega_{\vec{n}f}$. If $|\vec{k}_\perp| \ll \frac{\pi}{a}$, $\varepsilon_{f\vec{\kappa}_1}$ can be expanded into a series: $\varepsilon_{f\vec{\kappa}_1} = \varepsilon_{f\vec{\kappa}} + \vec{k}_\perp \vec{\nabla}_{\vec{\kappa}}\varepsilon_{f\vec{\kappa}} + \dots$ ($\vec{k}_\perp = \omega_{nf}\vec{n}_\perp$; $\vec{n}_\perp = \vec{k}_\perp/|\vec{k}_\perp|$; $\vec{\nabla}_{\vec{\kappa}}\varepsilon_{f\vec{\kappa}}$) is the particle velocity in the state $f_{\vec{\kappa}}$). Near the extremums of the bands the first expansion term is zero, and it is necessary to allow for the following terms of the series. Taking account of the stated dependence is crucial when analyzing the formation of photons through intraband transitions, when the difference $\varepsilon_{n\vec{\kappa}} - \varepsilon_{f\vec{\kappa}_1}$ is determined just by the correction terms $\vec{k}_\perp \vec{\nabla}_{\vec{\kappa}}\varepsilon_{f\vec{\kappa}} + \dots$.

Expressions (2.16), (2.17) are obtained without using the explicit form of the refractive index $n(\omega)$, so they are also applicable for the analysis of the radiation spectrum of γ-quanta in the frequency range, where a large contribution to $n(\omega)$ comes from the crystal nuclei with, for example, low resonances. As in this case the medium under consideration is strongly absorbing, the entire frequency spectrum is given by (2.17) which allows for the imaginary part of the refractive index. Though, as it has already been pointed out, in order to find central frequencies in the intensity maxima, it is sufficient to make use of equation (2.5).

The refractive index within the X-ray frequency range for Mossbauer crystals can be represented in the form

$$n' = 1 - \frac{\omega_L^2}{2\omega^2} - \mu\frac{(\omega - \omega_0)}{(\omega - \omega_0) + \Gamma^2/4}, \tag{2.18}$$

where $\mu \equiv \frac{\pi N}{2\omega_0^2}\frac{2I+1}{2I_0+1}\frac{\Gamma}{1+\alpha_\gamma}f_M$; f_M is the Lamb-Mossbauer factor; α_γ is the internal conversion coefficient; I and I_0 are the spins of the initial and final states of the nucleus, respectively; ω_0 is the resonant frequency of the nuclear γ-transition; Γ is the nuclear level width.

It was stated in [Kolpakov (1973)] that the refractive index in a Mossbauer crystal (2.18) may become greater than unity. This enables observation of the Vavilov-Cherenkov effect, and hence, the anomalous Doppler effect for short-wavelength photons at which the emitting particle moves to a higher energy level. Substituting (2.18) into (2.5), we obtain the following expression for central frequencies in the maximum;

$$\omega - \omega\beta\cos\vartheta + \frac{\beta\omega_L^2}{2\omega} + \frac{\mu\beta(\omega-\omega_0)\omega}{(\omega-\omega_0)^2+\Gamma^2/4} - \Omega_{nf} = 0. \tag{2.19}$$

According to (2.18) $n'(\omega)$ can become greater than unity only in a narrow range near the resonant frequency ω_0 (for instance, for $^{57}Fe\Delta\omega \equiv \omega-\omega_0 = 10\Gamma$ [Kolpakov (1973)]. Using this fact, the frequencies determined by the anomalous Doppler effect can be sought in the form $\omega = \omega_0 - \Delta$, where $\Delta \ll \omega_0$. It is clear from (2.19) that in this range the equation is solvable, when Ω_{nf} is less than zero, which corresponds to the system transition to a higher energy level through radiation. Consequently, from (2.19) we may obtain the following expression for anomalous Doppler frequencies corresponding to the central frequencies in the intensity maximum

$$\omega_{nf}^{(1,2)} \equiv \omega_0 - \Delta_{nf}^{(1,2)} = \omega_0 - \frac{\mu}{2A} \mp \left[\left(\frac{\mu}{2A}\right)^2 - \frac{\Gamma^2}{4}\right]^{1/2}, \tag{2.20}$$

where

$$A \equiv \frac{\vartheta^2}{2} + \frac{m^2}{2E^2} + \frac{\omega_L^2}{2\omega_0^2} + \frac{|\Omega_{nf}|}{\omega_0}.$$

Far from the frequency ω_0 the contribution of the resonance term in the refractive index may be neglected. Finally we turn back to the case of radiation considered above, which is described by formula (2.11), from which we obtain the other two solutions of equation (2.19) corresponding to the normal Doppler frequencies. In view of (2.20) at

$$E < E' = m|\Omega_{nf}|\left[\sqrt{2}\omega_0\left(\frac{\mu}{\Gamma} - \frac{\vartheta^2}{2} - \frac{\omega_L^2}{2\omega_0^2}\right)\right]^{-1}$$

the frequencies $\omega_{nf}^{(1,2)}$ become complex. As a result, at such energies the anomalous Doppler effect is impossible.

Interestingly enough, the phenomenon of photon emission accompanied by the excitation of the emitting system itself does not only arise as a result of the anomalous Doppler effect, or when the velocity of the source is higher than the velocity of light in a vacuum. This process also occurs when the oscillator moves at subluminal velocity in a medium with $n < 1$, if the coherent radiation length is limited (for example, due to the photon absorption in the medium, the presence of the crystal boundaries, multiple scattering [Baryshevsky and Dubovskaya (1978)]). Indeed, in the case of absorbing medium there is a whole set of frequencies for which $q_{znf} \sim \omega \mathtt{Im}\, n$. As a result the radiation intensities for these frequencies are comparable with one another, so from (2.21) we may obtain the following expression for a photon spectrum

$$\omega_{nf}^{(1,2)} = \frac{[\Omega_{nf} + |\varepsilon|\delta(\omega_{nf})] \pm \left\{[\Omega_{nf} + |\varepsilon|\delta(\omega_{nf})]^2 - 2\omega_L^2(1-\beta\cos\vartheta)\right\}^{1/2}}{2(1-\beta\cos\vartheta)} \tag{2.21}$$

It follows from (2.21) that in the case of absorbing medium the photon radiation accompanied by the excitation of the emitting system itself becomes possible. Indeed, in view of (2.21) the following conditions should be fulfilled to make this process possible:

$$\begin{gathered}\omega_{nf} > 0; |\varepsilon|\delta(\omega_{nf}) > |\Omega_{nf}|; \\ [\Omega_{nf} + |\varepsilon|\delta(\omega_{nf})]^2 - 2\omega_L^2(1-\beta\cos\vartheta) \geq 0.\end{gathered} \tag{2.22}$$

The conditions (2.22) may be reduced to one

$$|\varepsilon|\delta(\omega_{nf}) > |\Omega_{nf}| + \sqrt{2}\omega_L(1-\beta\cos\vartheta)^{1/2}. \tag{2.23}$$

It is seen from the expression for frequency Ω_{nf} that with the increase in the energy of the channeled particle the requirement (2.23) becomes less strict and proves to be feasible for a larger number of levels n and f of the discrete spectrum of particle transverse motion. If the condition (2.23) is not satisfied, the radiation corresponding to the transition between the given energy levels n and f of the transverse motion will only occur when the system moves to a lower energy level.

As it has already been mentioned, the presence of the target boundaries and multiple scattering of a channeled particle along with absorption, lead to limitation of the minimum value of the longitudinal component of the momentum transmitted to the medium, and hence, to limitation of the coherent length. Thus, for instance, for thin crystal plates with $L < (\omega n'' \cos\vartheta)^{-1}$ the maximum coherent length $l \sim 1/q_{znf}$ determining the process of radiation cannot exceed L. The frequency spectrum in this case is described by formula (2.21) with $\delta(\omega)$ replaced by L^{-1}.

Note also that with the presence of boundaries, the momentum transmitted along the normal to the crystal surface, should no longer be zero (or $2\pi\vec{\tau}_z$) even for a thick nonabsorbing medium. In this case the frequency spectrum can be written in the form

$$\omega_{nf}^{(1,2)} = \frac{(l_{nf}^{-1}(\omega) - \Omega_{nf}) \pm [(l_{nf}^{-1}(\omega) - \Omega_{nf})^2 - 2\omega_L^2(1-\beta\cos\vartheta)]^{1/2}}{2(1-\beta\cos\vartheta)} \tag{2.24}$$

where $l_{nf}(\omega) = (p_{zn} - p_{1zf} - k_z n)^{-1}$.

So, radiation of a channeled particle accompanied by the excitation of the emitting system is possible in a medium with $n < 1$ not only for a source moving at the velocity greater than the velocity of light in vacuum but also for an oscillator moving with subluminal velocity. From the viewpoint of physics the phenomenon in question can be understood, taking into account the fact that the limitation of the coherent length, and hence, the magnitude of the longitudinal momentum transmitted to the medium gives rise to uncertainty in the real part of such a momentum. From the conservation laws follows that this is equivalent to the appearance of uncertainty in the value of the energy of the particle transverse motion. If the uncertainty in the energy which results from the limitation of the coherent length exceeds the distance between the discrete levels of transverse motion in a laboratory system, it will cause virtual elimination of the distinction between the levels in the given interval of changes in the transverse momentum. Consequently, the system through radiation can move to both lower and higher levels of transverse motion.

Until now we considered a crystal as an optically isotropic medium for photons. Note, however, that a crystal can exhibit optical anisotropy in both optical and X-ray (and shorter wavelength) spectral ranges [Baryshevsky (1976)]. When analyzing the radiation process, the refractive index in conservation laws means one of the major target refractive indices [Baryshevskii and Dubovskaya (1976a)]. In a short wavelength range the optical anisotropy of crystals is manifest in the case diffraction of γ-quanta in them. Then both real and imaginary parts of the crystal refractive index strongly depend on the direction of photon propagation, which results in a significant change in spectral, angular and polarization characteristics of all types of radiation excited by a charged particle in a crystal [[Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1976a); Baryshevsky *et al.* (1978, 1980a, 1979); Baryshevsky *et al.* (1980c)]] (see (3.3)).

It should be pointed out that radiation associated with the transitions between the levels of discrete spectrum of the particle transverse motion in a crystal may be treated as spontaneous radiation of a channeled particle. When a crystal is put in the area occupied by an electromagnetic field (for, example, light radiation), one can stimulate induced transitions between the stated levels, which will give rise to induced radiation [Baryshevskii and Dubovskaya (1977d); Beloshitsky and Kumakhov (1977)].

Chapter 3

The Foundations of the Theory of γ-quanta Emission in Crystals under Channeling Conditions

3.1 The Cross Section of Photon Generation by Particles in an External Field

Theoretical study of the process of photon production by channeled particles has been carried out from various viewpoints. The emission of γ-quanta in crystals with the thicknesses smaller than the length of transformation of the wave function of an incident particle from a plane wave to a superposition of the Bloch waves was examined in [Kalashnikov and Koptelov (1979); Kalashnikov and Olchak (1979); Kalashnikov and Strikhanov (1980)]. According to [Kalashnikov and Koptelov (1979); Kalashnikov and Olchak (1979); Kalashnikov and Strikhanov (1980)] the process of electron emission for such thicknesses can be analyzed in terms of the concept of radiative capture of a particle incident on a crystal into the channeling regime. In [Kumakhov (1976)], there considered radiation in an infinite crystal within the framework of the classical model of a particle motion in a parabolic potential. Within the framework of this model there is only one radiation frequency corresponding to the Doppler shifted frequency of particle oscillation in a harmonic well involved in the formation of the radiation spectrum. Further analysis of the problem given in [Kumakhov (1977); Zhevago (1978); Beloshitsky and Kumakhov (1978); Bazylev and Zhevago (1977)] was also performed for an infinite crystal.

At the same time as far back as in our early works [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d, 1976a)], devoted to the problem of photon radiation under channeling conditions, it was shown that in a real potential the radiation spectrum is produced by frequencies corresponding to a wide range of the particle transitions between the levels of transverse motion. Such transitions result in the fact that when exploring the radiation spectrum at a given angle to the direction of a particle motion, a discrete set of spectral lines is to be observed. That spectrum was experimentally revealed in [Swent *et al.* (1979); Alguard *et al.* (1979); Cue *et al.* (1980)]. Moreover, according to [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d); Kagan and Kononets (1973, 1974)], when a particle enters the crystal, the whole set of transverse motion levels is necessarily populated. As a result, not only sub-barrier transitions (inside a well) but also the over-barrier transitions, as well as the transitions from over-barrier to sub-barrier states take part in the spectrum formation. Over-barrier states located near the barrier edge are characterized by a wide regions of transverse motion, which was completely ignored in [Kumakhov (1977); Zhevago (1978); Beloshitsky and Kumakhov (1978); Bazylev and Zhevago (1977)], and it was only in [Bazylev *et al.* (1980, 1981)] where this fact was taken into consideration.

The population of all the above-mentioned states depends on the type of a particle, the angle at which it enters the crystal, and the shape of a well. This fact has a considerable impact on the shape of the spectrum formed by particles during radiative transitions between the levels of transverse motion, which was convincingly demonstrated by Bayer, Katkov and Strakhovenko by using numerical calculations [Baier *et al.* (1979)]. In [Podgoretsky (1980); Akhiezer *et al.* (1979)] the important role of the radiation produced through over-barrier transitions has also been emphasized recently.

According to [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d, 1976a)], refraction, absorption, and diffraction of photons in crystals also considerably affect the radiative spectrum. Below we presented the results obtained in our investigations [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d, 1976a); Baryshevsky *et al.* (1978, 1980a)].

Let a beam of charged particles with the momentum $\vec{p}$ and energy E fall on a crystal of volume V. As a result of collision with the crystal, the particle momentum changes and the particle, undergoing acceleration, emits radiation. Theoretical analysis of the process under study implies that each particle corresponds to a wave packet, produced in a generator (particle accelerator) at a certain moment t_0. Due to the particle interaction with a medium, at long distances from the crystal in addition to a primary wave packet diverging spherical waves which describe the scattered and newly produced particles (in this case - photons) also appear (Fig. 5). To calculate the cross-section, it is necessary to know the transition probability per unit time for the process, when one particle scattered in a constant field produces in its final state a certain number of other particles. In view of the quantum mechanical theory of reactions it may be represented by the general formula of the form (see, for example, [Berestetsky *et al.* (1968)]) ($\hbar = c = 1$):

$$dW = 2\pi\delta(E_f - E)\overline{|M_{fi}|^2}\frac{1}{2E\mathcal{L}^3}\prod_a \frac{d^3p_a}{(2\pi)^3 2E_a}\,, \tag{3.1}$$

where E is the energy of the initial particle; E_f is the energy of the final state; $\vec{p}_a$ and E_a are the momenta and energies of the final particles; $\mathcal{L}^3$ is the normalization volume; M_{fi} is the amplitude of scattering from the initial i to the final f state; the overline means averaging over the spin states of the particles involved in the reaction. The scattering cross section $d\sigma$ is obtained by dividing dW by the incident particle flux density $j = v/\mathcal{L}^3$, where $v = |\vec{p}|/E$ is the velocity of the primary particle. as a result, we get

$$d\sigma = 2\pi\delta(E_f - E)\overline{|M_{fi}|^2}\frac{1}{2|\vec{p}|}\prod_a \frac{d^3p_a}{(2\pi)^3 2E_a}\,. \tag{3.2}$$

In the case of interest there is a particle and a photon in the final state. Write the radiation cross-section as

$$d\sigma = 2\pi\delta(E_1 + \omega - E)\overline{|M(\vec{p}_1, \vec{k}; \vec{p})|^2}\frac{d^3p_1 d^3k}{8(2\pi)^6 pE_1\omega}\,, \tag{3.3}$$

where $\vec{p}$ is the momentum of the primary particle (electron, positron); $\vec{k}$ is the photon momentum; ω is the photon frequency; E_1 and $\vec{p}_1$ are the energy and momentum of the particle in the final state.

The matrix element M describing the process of photon emission in an arbitrary external field can be represented in the form

$$M(\vec{p}_1, \vec{k}; \vec{p}) = e\int \Psi^{(-)*}_{p_1}(\vec{r})\vec{\alpha}\vec{A}^{(-)*}_{\vec{k}}(\vec{r})\Psi^{(+)}_{p}(\vec{r})d^3r\,, \tag{3.4}$$

where $\Psi^+_p(\vec{r})$, $\Psi^-_{p_1}(\vec{r})$ are the exact solutions of the Dirac equation for particle scattering in the external field, having different asymptotics far from the crystal: $\Psi^+_p(\vec{r})$, for the primary particle (asymptotics type — an incident plane wave plus diverging spherical waves), $\Psi^-_{p_1}(\vec{r})$, for the final particle (the asymptotics type — an incident plane wave plus converging spherical waves); $\vec{A}^{(-)}_{\vec{k}}(\vec{r})$ is the vector potential of the emitted photon, being the exact solution of Maxwell equations and describing photon scattering by a crystal (the asymptotic type — an incident plane wave plus converging spherical wave) [Baryshevskii and Dubovskaya (1977d); Baryshevskii and Feranchuk (1974)].

The wave functions of all the particles are normalized to one particle within the volume $\mathcal{L}^3$. T he terms $1/\sqrt{2E\mathcal{L}^3}$ $1/\sqrt{2E_1\mathcal{L}^3}$ and $1/\sqrt{2\omega\mathcal{L}^3}$ appearing in them are shown explicitly,and they are included in the definitions of dW and $d\sigma$ (see(3.3)). Thus, if the photon-crystal interaction is ignored, the vector potential $\vec{A}^{(-)}_{\vec{k}}(\vec{r})$ has the form [Berestetsky *et al.* (1968)]

$$\vec{A}^{(-)}_{\vec{k}}(\vec{r}) = \sqrt{4\pi}\vec{e}_s e^{i\vec{k}\vec{r}}, \tag{3.5}$$

where $\vec{e}_s$ is the photon polarization vector. Then the matrix element in (3.4) is written as follows:

$$M(\vec{p}_1, \vec{k}; \vec{p}) \equiv e\sqrt{4\pi}\mathcal{M}_{fi} = e\sqrt{4\pi}\int \Psi^{(-)*}_{p_1}(\vec{r})\vec{\alpha}\vec{e}^{\,*}_s e^{-i\vec{k}\vec{r}}\Psi^{(+)}_{\vec{p}}(\vec{r})d^3r. \tag{3.6}$$

To find the explicit form for $d\sigma$, one should know the wave functions $\psi^{(\pm)}$. (The general analysis of the characteristics of the functions describing particle scattering by a crystal was given in (). Considering the photon radiation in a crystal, when solving the Dirac equation it is necessary (as well as in bremsstrahlung by a screened Coulomb potential [Olsen and Maximon (1959)]) to take into account the terms proportional to α. This occurs through the fact that for fast particles the matrix element $\vec{\alpha}$ involved in (3.6) is the vector, whose direction is close to $\vec{k}$. Therefore the major term $\vec{\alpha}\vec{e}$ proves to be small, and the correction terms have the same order of magnitude [Berestetsky *et al.* (1968)].

Write the Dirac equation (1.4) in the form:

$$[\Delta_r + p^2 - 2EV(\vec{r})]\Psi(\vec{r}) = -i\vec{\alpha}\nabla V(\vec{r})\Psi(\vec{r})\,. \tag{3.7}$$

The term proportional to V^2 may still be ignored. Dividing both sides of equation (3.7) by $2m\gamma$, we obtain

$$\left[\frac{1}{2m\gamma}\Delta_r + \varepsilon' - V(\vec{r})\right]\Psi(\vec{r}) = -\frac{i}{2m\gamma}(\vec{\alpha}\nabla V(\vec{r}))\Psi(\vec{r})\,. \tag{3.8}$$

Remember that (see (1.7))

$$\varepsilon' = \frac{p^2}{2m\gamma}$$

and $\Psi(\vec{r})$ is sought as

$$\Psi(\vec{r}) = \Psi^{(0)}(\vec{r}) + \Psi^{(1)}(\vec{r})\,, \tag{3.9}$$

where $\Psi^{(0)}(\vec{r})$ satisfies equation (3.8) with a zero right–hand side, and $\Psi^{(1)}(\vec{r})$ is the desired correction. The equation for $\Psi^{(0)}(\vec{r})$ does not include spin matrices. Therefore the spin state of a particle passing through a crystal cannot change in this approximation: it coincides with the spin state of a particle in a plane wave incident on a crystal. It is convenient to extract a bispinor amplitude describing the particle spin state in its explicit form, and represent $\Psi(\vec{r})$ as follows [Berestetsky *et al.* (1968)]:

$$\Psi(\vec{r}) = e^{i\vec{p}\vec{r}}[u(\vec{p})\varphi_p(\vec{r}) + \varphi_p^{(1)}(\vec{r})] \,, \tag{3.10}$$

where $u(\vec{p})$ is the constant bispinor amplitude of the plane wave incident on the crystal normalized by the condition

$$\bar{u}(\vec{p})u(\vec{p}) = 2m \,. \tag{3.11}$$

Substituting (3.10) into (3.8) and retaining the first–order terms over $\vec{\alpha}\nabla V$, we come to the equation

$$\begin{aligned}\left[\frac{1}{2m\gamma}\Delta + \frac{i}{m\gamma}\vec{p}\nabla - V(\vec{r})\right] \varphi_p^{(1)}(\vec{r}) \\ = -\frac{i}{2m\gamma}u(\vec{p})(\vec{\alpha}\nabla V(\vec{r}))\varphi_p(\vec{r}) \,.\end{aligned} \tag{3.12}$$

To solve (3.12), make use of the fact that the function $\varphi_p(\vec{r})$ satisfies the equation

$$\left[\frac{1}{2m\gamma}\Delta + \frac{i}{m\gamma}\vec{p}\nabla - V(\vec{r})\right] \varphi_p(\vec{r}) = 0 \,. \tag{3.13}$$

Application of the operation ∇ to equation (3.13) gives

$$\left[\frac{1}{2m\gamma}\Delta + \frac{i}{m\gamma}\vec{p}\nabla - V(\vec{r})\right] \nabla\varphi_p(\vec{r}) = \varphi_p(\vec{r})\nabla V(\vec{r}) \,. \tag{3.14}$$

Upon multiplying (3.14) by

$$-\frac{i}{2m\gamma}u(\vec{p})\vec{\alpha}$$

and comparing the result with (3.12), we obtain immediately

$$\varphi_p^{(1)}(\vec{r}) = -\frac{i}{2m\gamma}\vec{\alpha}\nabla\varphi_p(\vec{r})u(\vec{p}) \,. \tag{3.15}$$

Thus, finally

$$\Psi(\vec{r}) = e^{i\vec{p}\vec{r}}\left(1 - \frac{i}{2m\gamma}\vec{\alpha}\nabla\right) \varphi_p(\vec{r})u(\vec{p}) \,. \tag{3.16}$$

It should be emphasized that, as demonstrated by the direct comparison between the expansion of (3.16) and the exact solution of the Dirac equation for the Kronig-Penney model [Baryshevskii and Dubovskaya (1977d)], relation (3.16) in the case of not very thick crystals is always suitable (applicable) (for example, for $E = 1$ GeV the thickness is $l \leq 1$ cm, for $E = 500$ keV $\div 1$ MeV $l \sim 10^{-3} \div 10^{-2}$ cm). In fact, the stated expansion holds true, when the parameter $\Omega l/v$ is small (Ω is the characteristic energy of spin-orbit interaction between the incident particle spin and the crystal axis; v is the particle velocity).

Substitute the wave functions (3.16) into the expression for the matrix element $\mathcal{M}_{fi}$ (see (3.6)):

$$\mathcal{M}_{fi} = \int d^3 r e^{-i(\vec{p}_1 + \vec{k} - \vec{p})\vec{r}} u^+(\vec{p}_1) \left(1 + \frac{i}{2E_1}\vec{\alpha}\nabla\right) \varphi_{p_1}^{(-)*}(\vec{r}) \\ \times \vec{\alpha}\vec{e}_s^{\,*} \left(1 - \frac{i}{2E_1}\vec{\alpha}\nabla\right) \varphi_p^{(+)}(\vec{r}) u(\vec{p}) \,, \tag{3.17}$$

i.e.,

$$\mathcal{M}_{fi} = u^+(\vec{p}_1)[\vec{\alpha}\vec{e}_s^{\,*} I_1 + (\vec{\alpha}\vec{e}_s^{\,*})(\vec{\alpha}\vec{I}_2) + (\vec{\alpha}\vec{I}_3)(\vec{\alpha}\vec{e}_s^{\,*})] u\vec{p}, \tag{3.18}$$

where, by analogy with [Olsen and Maximon (1959)], the following quantities are introduced

$$I_1 = \int e^{-i\vec{q}\vec{r}} \varphi_{p_1}^{(-)*}(\vec{r}) \varphi_p^{(+)}(\vec{r}) d^3 r \,,$$
$$\vec{I}_2 = -\frac{i}{2E} \int e^{-i\vec{q}\vec{r}} \varphi_{p_1}^{(-)*}(\vec{r}) \nabla_r \varphi_p^{(+)}(\vec{r}) d^3 r \,,$$
$$\vec{I}_3 = \frac{i}{2E_1} \int e^{-i\vec{q}\vec{r}} (\nabla_r \varphi_{p_1}^{(-)*}(\vec{r})) \varphi_p^{(+)}(\vec{r}) d^3 r \,, \tag{3.19}$$

$\vec{q} = \vec{p}_1 + \vec{k} - \vec{p}$ is the transmitted momentum.

For further consideration it should be noted that the integrals in (3.19) are related to each other [Olsen and Maximon (1959)].

Integration by parts in the equality for $\vec{I}_3$ gives:

$$\vec{I}_3 = -\frac{i}{2E_1} \int \varphi_{p_1}^{(-)*}(\vec{r}) \nabla_r [e^{-i\vec{q}\vec{r}} \varphi_p^{(+)}(\vec{r})] d^3 r \\ = -\frac{i}{2E_1} \int \varphi_{p_1}^{(-)*}(\vec{r}) e^{-i\vec{q}\vec{r}} \nabla_r \varphi_p^{(+)}(\vec{r}) d^3 r \\ - \frac{\vec{q}}{2E_1} \int e^{-iqr} \varphi_{p_1}^{(-)*}(\vec{r}) \varphi_p^{(+)}(\vec{r}) d^3 r \,. \tag{3.20}$$

Comparison of (3.20) and (3.19) gives

$$\vec{I}_3 = \frac{E}{E_1} \vec{I}_2 - \frac{\vec{q}}{2E_1} I_1 \,. \tag{3.21}$$

To further simplify the matrix element in (3.18), it is convenient to cast (3.18), using two-component spinor functions:

$$u(\vec{p}) = \sqrt{E+m}\begin{pmatrix} 1 \\ \sqrt{\frac{E-m}{E+m}}(\vec{\sigma}\vec{n}_p) \end{pmatrix} w;$$

$$u(\vec{p}_1) = \sqrt{E_{1+m}}\begin{pmatrix} 1 \\ \sqrt{\frac{E_1-m}{E_1+m}}(\vec{\sigma}\vec{n}_{p_1}) \end{pmatrix} w_1;\ \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \tag{3.22}$$

where $\vec{\sigma}$ are the Pauli matrices; $w(w_1)$ is the two-component spinor [Berestetsky *et al.* (1968)]; $\vec{n}_p$ is the unit vector in the $\vec{p}$ direction; $\vec{n}_{p_1}$ is the same for $\vec{p}_1$. Substitution of (3.22) into (3.18), gives quite an awkward expression for $\mathcal{M}_{ji}$, which, however, simplifies at E, $E_1 \gg m$. The calculations in this approximation are perfectly analogous to those performed by Olsen and Maximon in [Olsen and Maximon (1959)], making it possible to write $\mathcal{M}_{fi}$ as follows:

$$\mathcal{M}_{fi} = 2\sqrt{\frac{E}{E_1}} w_1^+ \left\{(E+E_1)(\vec{g}\vec{e}_s^{\,*}) + i\omega\vec{\sigma}[\vec{g}\times\vec{e}_s^{\,*}]\right\} w\,. \tag{3.23}$$

The vector

$$\vec{g} = \vec{g}_{\perp\vec{k}} + \vec{g}_{\parallel}\,, \tag{3.24}$$

where

$$\vec{g}_{\perp\vec{k}} = \vec{I}_{2\perp\vec{k}} + \frac{1}{2}\vec{n}_{\vec{p}\perp\vec{k}}I_1\,, \quad \vec{g}_{\parallel} = -\frac{m}{2E}\vec{n}_{\parallel}I_1\,,$$

the symbol $(\perp \vec{k})$ means the projection of the corresponding vector onto the plane perpendicular to the direction of the photon momentum $\vec{k}$; the symbol $\parallel$ is for the projection of the vector onto the $\vec{k}$ direction; $\vec{n}_{\parallel}$ is the unit vector in the $\vec{k}$ direction.

Upon introducing the polarization density matrices of the initial ρ and the final ρ_1 electrons, we obtain for the average square of the matrix element involved in the cross-section of (3.3),

$$\overline{|\mathcal{M}_{fi}|^2} = \mathrm{Tr}\,\rho\mathcal{M}^+\rho_1\mathcal{M}\,,$$

where

$$\mathcal{M} = 2\sqrt{\frac{E}{E_1}}\left\{(E+E_1)(\vec{g}\vec{e}_s^{\,*}) + i\omega\vec{\sigma}[\vec{g}\times\vec{e}_s^{\,*}]\right\};$$
$$\rho = \frac{1}{2}(1+\vec{\xi}\vec{\sigma})\,, \qquad \rho_1 = \frac{1}{2}(1+\vec{\xi}_1\vec{\sigma})\,, \tag{3.25}$$

$\vec{\xi}(\vec{\xi}_1)$ is the polarization vector of the particle in the initial (final) state; $0 \le \xi;\ \xi_1 \le 1$. After taking the trace appearing in (3.25), we get

$$\begin{aligned}
\overline{|\mathcal{M}_{fi}|^2} &= 4\frac{E}{E_1}\left\{\frac{1}{2}\omega^2|\vec{g}|^2 + 2EE_1(1+\vec{\xi}\vec{\xi}_1)|\vec{g}\vec{e}^*|^2\right.\\
&+\frac{1}{2}\omega^2\mathtt{Re}\left\{|\vec{g}|^2\vec{\xi}_1\vec{\xi} - 2(\vec{g}\vec{\xi})(\vec{g}^*\vec{\xi}_1)\right\} + \omega E_1\mathtt{Re}\left\{\left[|\vec{g}|^2(\vec{\xi}\vec{e})\right.\right.\\
&\left.\left.-2(\vec{g}\vec{e})(\vec{g}^*\vec{\xi})\right]\vec{\xi}_1\vec{e}^*\right\} - \omega E\,\mathtt{Re}\left\{\left[|\vec{g}|^2(\vec{\xi}_1\vec{e}) - 2(\vec{g}\vec{e})(\vec{g}^*\vec{\xi}_1)\right](\vec{\xi}\vec{e}^*)\right\}\\
&+\frac{1}{2}\omega|\vec{g}|^2(E\vec{\xi} + E_1\vec{\xi}_1)(i\vec{e}\times\vec{e}^*) + \frac{1}{2}\omega\,\mathtt{Re}\left\{|\vec{g}|^2\left(E_1\vec{\xi}\right.\right.\\
&\left.\left.+E\vec{\xi}_1\right)(i\vec{e}\times\vec{e}^*) - 2(g(E_1\vec{\xi} + E\vec{\xi}_1))(\vec{g}^*(i\vec{e}\times\vec{e}^*))\right\}\\
&+\frac{1}{2}\omega\left[\omega(1+\vec{\xi}\vec{\xi}_1)(i\vec{e}\times\vec{e}^*) + (E+E_1)(\vec{\xi}\times\vec{\xi}_1)\times(i\vec{e}\times\vec{e}^*)\right.\\
&\left.\left.+(E+E_1)(\vec{\xi}+\vec{\xi}_1) - 2\mathtt{Re}\left\{\vec{e}^*((E\vec{\xi}+E_1\vec{\xi}_1)\vec{e})\right\}\right](i\vec{g}\times\vec{g}^*)\right\}. \qquad (3.26)
\end{aligned}$$

Using (3.25) and (3.26), we get the required expression for the cross-section of the photon radiation in a crystal allowing for polarization of all the particles involved in the reaction.

$$d\sigma = e^2\delta(E_1+\omega-E)sp\,\rho\mathcal{M}^+\rho_1\mathcal{M}\frac{d^3p_1d^3k}{4(2\pi)^4pE_1\omega}. \qquad (3.27)$$

The relationships (3.26) and (3.27) solve in the general form the problem of finding $d\sigma$.

If we are not concerned about the polarization of the final particle, then with ξ_1 in (3.27) assumed to be zero, the entire expression (3.27) should to be multiplied by 2.

3.2 Photon Generation in Crystals under Channeling Conditions

We now turn to a more detailed treatment of the cross-section of (3.27). Let us take into account that in (3.19) the linear dimensions of the domain of integration only exceed the linear dimensions of the crystal by the magnitude of the vacuum coherence length

$$l_{\text{coh}} \sim \frac{1}{q_z} = \frac{2}{\omega}\frac{E(E-\omega)}{m^2}. \qquad (3.28)$$

(A thorough treatment of the properties of l_{coh} see in [Ter-Mikaelian (1969, 1972)]). For this reason, analyzing the radiation process in a crystal target with lateral dimensions much larger than its thickness, we can apply the expressions describing scattering of a plane wave by a crystal plate with infinite lateral dimensions, i.e., the functions considered in (). Substituting these functions into (3.19) with due account of the relation

$$\varphi_p^{(+)}(\vec{r}) = \varphi_{-p}^{(-)*}(\vec{r})$$

and integrating it with respect to the momentum $\vec{p}_1$, we get the below expression for the spectral–angular distribution of the number of photons emitted by a channeled particle $dN = \frac{1}{S}d\sigma$ (S is the area of the target surface):

$$\begin{aligned}
\frac{d^2N_s}{d\omega d\Omega} &= \frac{e^2\omega}{4\pi^2}\mathrm{Re}\sum_{nfj} Q_{nj}e^{i\tilde{\Omega}_{nj}L}\left[\frac{1-\exp(iq^*_{zjf}L)}{q^*_{zjf}}\right] \\
&\times\left[\frac{1-\exp(-iq_{znf}L)}{q_{znf}}\right]\Bigg\{\frac{\omega^2}{2E_1^2}\vec{g}_{nf}\vec{g}^*_{jf} + 2\frac{E}{E_1}(1+\vec{\xi}\vec{\xi}_1) \\
&\times(\vec{g}_{nf}\vec{e}^*_s)(\vec{g}^*_{jf}\vec{e}_s) + \frac{\omega^2}{2E_1^2}\mathrm{Re}[\vec{g}_{nf}\vec{g}^*_{jf}(\vec{\xi}\vec{\xi}_1) - 2(\vec{g}_{nf}\vec{\xi})(\vec{g}^*_{jf}\vec{\xi}_1)] \\
&+\frac{\omega}{E_1}\mathrm{Re}\left\{[\vec{g}_{nf}\vec{g}^*_{jf}(\vec{\xi}\vec{e}_s) - 2(\vec{g}_{nf}\vec{e}_s)(\vec{g}^*_{jf}\vec{\xi})](\vec{\xi}_1\vec{e}^*_s)\right\} \\
&-\frac{\omega E}{E_1^2}\mathrm{Re}\left\{[\vec{g}_{nf}\vec{g}^*_{jf}(\vec{\xi}_1\vec{e}_s) - 2(\vec{g}_{nf}\vec{e}_s)(\vec{g}^*_{jf}\vec{\xi}_1)](\vec{\xi}\vec{e}^*_s)\right\} \\
&+\frac{\omega}{2E_1^2}(\vec{g}_{nf}\vec{g}^*_{jf})(E\vec{\xi}+E_1\vec{\xi}_1)[i\vec{e}_s\times\vec{e}^*_s] + \frac{\omega}{2E_1^2}\mathrm{Re}\Big\{(\vec{g}_{nf}\vec{g}^*_{jf})(E_1\vec{\xi}+E\vec{\xi}_1)[i\vec{e}_s\times\vec{e}^*_s] \\
&-2(\vec{g}_{nf}(E_1\vec{\xi}+E\vec{\xi}_1))(\vec{g}^*_{jf}[i\vec{e}_s\times\vec{e}^*_s])\Big\} \\
&+\frac{\omega}{2E_1^2}\Big[\omega(1+\vec{\xi}\vec{\xi}_1)[i\vec{e}_s\times\vec{e}^*_s] + (E+E_1)\Big[[\vec{\xi}\times\vec{\xi}_1][i\vec{e}_s\times\vec{e}^*_s]\Big] \\
&+(E+E_1)(\vec{\xi}+\vec{\xi}_1) - 2\mathrm{Re}\left\{\vec{e}^*_s\left(\left(E\vec{\xi}+E_1\vec{\xi}_1\right)\vec{e}_s\right)\right\}\Big][i\vec{g}_{nf}\times\vec{g}^*_{jf}]\Bigg\},
\end{aligned} \tag{3.29}$$

where $q_{znf} = p_{zn} - p_{1zf} - k_z$ is the longitudinal momentum transmitted through radiation; $\tilde{\Omega}_{nj} = \varepsilon'_{n\kappa}(E) - \varepsilon'_{j\kappa}(E) = \frac{1}{\gamma}(\varepsilon_{n\kappa}(E) - \varepsilon_{j\kappa}(E))$; the argument E in the notation for the transverse energy of the initial state emphasizes that the particle in the initial state has the energy E.

In the general two–dimensional case (axial channeling), the following relations are valid

$$Q_{nj} = c_n(\vec{p}_\perp)c^*_j(\vec{p}_\perp)\,, \quad c_n(\vec{p}_\perp) = \sqrt{\frac{N_\perp}{S}}\int_S e^{i\vec{p}_\perp\vec{\rho}}\psi^*_{n\kappa}(\vec{\rho})d^2\rho\,, \tag{3.30}$$

where $N_\perp$ is the number of two–dimensional unit cells in a transverse plane of the crystal; s is the area of the unit cell. When a particle is channeled along the planes located periodically along the x-axis:

$$Q_{nj} = c_n(p_x)c_j^*(p_x); \quad c_n(p_x) = \sqrt{\frac{N_x}{a}} \int_0^a e^{ip_x x}\psi_{n\kappa}^*(x)dx\,, \tag{3.31}$$

where N_x is the number of the crystal periods along the x-axis; a is the lattice spacing along the x-axis;

$$\psi_{n\kappa}(x) = \frac{1}{\sqrt{N_x}} e^{i\kappa x} u_{n\kappa}(x)$$

is the Bloch function describing the transverse motion in zone n of a particle with the reduced quasi-momentum

$$\kappa = p_x - \frac{2\pi l}{a};$$

the integral number l is found from the condition

$$\left| p_x - \frac{2\pi l}{a} \right| < \frac{\pi}{a}.$$

In the two–dimensional case

$$\psi_{n\vec{\kappa}}(\vec{\rho}) = \frac{1}{N_\perp} e^{i\vec{\kappa}\vec{\rho}} u_{n\kappa}(\vec{\rho})$$

is the Bloch function with $\vec{\kappa} = \vec{p}_\perp - \vec{\tau}_\perp$; $\vec{\tau}_\perp$ is obtained from the condition of the reduction of $\vec{p}_\perp$ to the first Brillouin zone.

Vector $\vec{g}_{nf}$ in a two-dimensional (axial) case has the form

$$\begin{aligned}
\vec{g}_{nf} &= \vec{g}_{\perp nf} + \vec{g}_{\parallel nf} = \frac{1}{2E}\vec{W}_{nf} = \frac{1}{2E}(\vec{I}_{2nf} + \vec{p}_{z\perp\vec{k}} I_{1nf} - m\vec{n}_\parallel I_{1nf})\,,\\
\vec{I}_{2nf} &= -iN_\perp \int_s e^{-i\vec{k}_\perp\vec{\rho}} \psi_{f\vec{\kappa}_1}^*(\vec{\rho})\vec{\nabla}_\rho \psi_{n\vec{\kappa}}(\vec{\rho}) d^2\rho\,,\\
I_{1nf} &= N_\perp \int_s e^{-i\vec{k}_\perp\vec{\rho}} \psi_{f\vec{\kappa}}^*(\vec{\rho})\psi_{n\vec{\kappa}}(\vec{\rho}) d^2\rho;\ \vec{p}_{\vec{z}\perp\vec{k}} = p\vec{n}_{\vec{z}\perp\vec{k}},
\end{aligned} \tag{3.32}$$

where $\vec{n}_z$ is the unit vector along the z-axis direction; recall that the symbol $(\perp \vec{k}\,)$ stands for the projection of the corresponding vector onto the plane perpendicular to the direction of the photon momentum $\vec{k}$. In the one–dimensional (planar) case, vector

$$\begin{aligned}
\vec{g}_{nf} &= \frac{1}{2E}\vec{W}_{nf} = \frac{1}{2E}(I_{2nf}\vec{n}_x + (\vec{p} - \vec{p}_x)_{\perp k} I_{1nf} - m\vec{n}_\parallel I_{1nf})\,,\\
\vec{I}_{2nf} &= -iN_x \int_0^a e^{-ik_x x} \psi_{f\kappa_1}^*(x) \frac{\partial}{\partial x}\psi_{n\kappa}(x)dx\,,\\
I_{1nf} &= N_x \int_0^a e^{-ik_x x} \psi_{f\kappa_1}^*(x)\psi_{n\kappa}(x)dx,
\end{aligned} \tag{3.33}$$

where $\vec{p}_x = p_x \vec{n}_x$; $\vec{n}_x$ is the unit vector along the x-axis;

$$\kappa_1 = p_x - k_x - \frac{n_0}{a},$$

n_0 is found from the condition of the reduction of $p_x - k_x$ to the first Brillouin zone, i.e.,

$$|p_x - k_x - \frac{n_0}{a}| < \frac{\pi}{a},$$

$\vec{\kappa}_1 = \vec{p}_\perp - \vec{k}_\perp - \vec{\tau}_0$; $\vec{\tau}_0$ is found from the condition of the reduction of $\vec{p}_\perp - \vec{k}_\perp$ to the first Brillouin zone.

The formulas obtained above enable one to describe angular, spectral and polarization properties of radiation formed in a crystal in detail.

Let particles incident on a crystal be nonpolarized ($\xi = 0$), and the polarization of final particles be of no interest to us. As has already been mentioned, in this case it should be assumed that $\xi_1 = 0$ and the expression for the cross section should be multiplied by two. As a result, we obtain

$$\frac{d^2 N_s}{d\omega d\Omega} = \frac{e^2\omega}{2\pi^2} \mathrm{Re} \sum_{nfj} Q_{nj} e^{i\tilde{\Omega}_{nj} L} \left[\frac{1 - \exp(iq^*_{zjf} L)}{q^*_{zjf}}\right] \left[\frac{1 - \exp(iq_{znf} L)}{q^*_{znf}}\right] \tag{3.34}$$
$$\times \left\{ \frac{\omega^2}{2E_1^2} \vec{g}_{nf} \vec{g}^*_{jf} + 2\frac{E}{E_1} (\vec{g}_{nf} \vec{e}^*_s)(\vec{g}^*_{jf} \vec{e}_s) + \frac{\omega^2}{2E_1^2} [i\vec{e}_s \times \vec{e}^*_s][i\vec{g}_{nf} \times \vec{g}^*_{jf}] \right\}.$$

According to (3.34), the spectral angular distribution of photons oscillates with the change in the crystal thickness L at frequencies $\tilde{Q}_{nj}$ determined by the differences between the energies of the transverse motion levels which are populated when a particle enters the crystal. These oscillations of the radiation intensity are quite similar to those observed at radiation of atoms at the given angle under pulse-excitation into the superposition of states. If the characteristic frequencies $\tilde{\Omega}_{nj}$ and the crystal thickness L are such that $\tilde{\Omega}_{nj} L \gg 1$, the averaging of (3.34) over the thickness spread leads to the averaging of oscillations, and it should be assumed that in (3.34) $j = 0$ (integration of (3.34) with respect to $d\omega$ or $d\Omega$ also leads to vanishing of the oscillations). The characteristic oscillation frequencies in the transverse plane

$$\tilde{\Omega} \sim \frac{1}{T} = \frac{v_\perp}{d} \simeq \frac{\vartheta_L}{d}$$

where T is the oscillation period in the transverse plane; $v_\perp$ is the velocity of transverse motion; d is the channel width (cm); ϑ_L is the Lindhard angle; $v_\perp = \vartheta_L c$ (c is the velocity of light), at $c = 1$ $v_\perp = \vartheta_L$. Consequently, the inequality $\tilde{\Omega} L \gg 1$ can be cast as follows [Kagan and Kononets (1970)]

$$\frac{L\vartheta_L}{d} \gg 1 \tag{3.35}$$

At $\vartheta_L \simeq 10^{-4}$ for positrons with the energy 1 GeV and $d = 10^{-8}$ cm the inequality holds true for the thicknesses $L \gg 10^{-4}$ cm (in the absence of degeneracy of energy levels).

Thus, if $L \gg 1/\tilde{\Omega}_{nj}$, the sum in (3.34) should only contain the terms with $n = j$, which leads to the following relation

$$\frac{d^2N_s}{d\omega d\Omega} = \frac{e^2\omega}{2\pi^2}\sum_{nf} Q_{nn}\left|\frac{1-\exp(-iq_{znf}L)}{q_{znf}}\right|^2 \tag{3.36}$$

$$\times\left\{\frac{\omega^2}{2E_1^2}|\vec{g}_{nf}|^2 + 2\frac{E}{E_1}|\vec{g}_{nf}\vec{e}_s^{\,*}|^2 + \frac{\omega^2}{2E_1^2}[i\vec{e}_s\times\vec{e}_s^{\,*}][i\vec{g}_{nf}\times\vec{g}_{nf}^{\,*}]\right\}.$$

If we do not concern ourselves with the polarization of an emitted photon, (3.36) is to be summed over the polarization states:

$$\frac{d^2N}{d\omega d\Omega} = \frac{e^2\omega}{2\pi^2}\sum_{nf} Q_{nn}\left|\frac{1-\exp(-iq_{znf}L)}{q_{znf}}\right|^2$$

$$\times\left[\left(2\frac{E}{E_1}+\frac{\omega^2}{E_1^2}\right)|\vec{g}_{\perp nf}|^2 + \frac{\omega^2}{E_1^2}|\vec{q}_{\parallel nf}|^2\right]. \tag{3.37}$$

Recall that

$$\vec{g}_{\perp nf} = \frac{1}{2E}\vec{W}_{\perp nf} = \frac{1}{2E}(\vec{I}_{2nf} + \vec{p}_{z\perp k}I_{1nf}),$$

$$\vec{g}_{\parallel nf} = \frac{1}{2E}\vec{W}_{\parallel nf} = -\frac{m}{2E}\vec{n}_{\parallel}I_{1nf}. \tag{3.38}$$

The transferred momentum q_{znf} in the planar case can be written as follows

$$q_{znf} \simeq \frac{\omega}{2(E-\omega)}\left[\vartheta^2(E-\omega\cos^2\varphi) + \frac{m^2}{E} - 2\Omega_{nf} + 2\varepsilon'_{n\kappa}(E)\right] - \Omega_{nf}$$

$$\equiv \frac{\omega}{2(E-\omega)}\left[\vartheta^2(E-\omega\cos^2\varphi) + \frac{m^2}{E}\right] - (\varepsilon'_{n\kappa}(E) - \varepsilon'_{n\kappa_1}(E_1)),$$

$$\Omega_{nf} = \frac{m}{E}(\varepsilon_{n\kappa}(E) - \varepsilon_{n\kappa_1}(E_1)). \tag{3.39}$$

As far as we still analyze the process of photon radiation within the range of frequencies ω and crystal thicknesses, where the absorption and refraction of emitted quanta may be neglected, the expressions involved in (3.36) and (3.37) with high accuracy can be recast in the form

$$\left|\frac{1-\exp(-iq_{znf}L)}{q_{znf}}\right|^2 \simeq 2\pi L\delta(q_{znf}).$$

As a consequence,

$$\frac{d^2N_s}{d\omega d\Omega} = \frac{e^2\omega L}{\pi}\sum_{nf} Q_{nn}\left\{\frac{\omega^2}{2E_1^2}|\vec{g}_{nf}|^2 + 2\frac{E}{E_1}|\vec{g}_{nf}\vec{e}_s^{\,*}|^2\right.$$

$$\left. + \frac{\omega^2}{2E_1^2}[i\vec{e}_s\times\vec{e}_s^{\,*}][i\vec{g}_{nf}\times\vec{g}_{nf}^{\,*}]\right\}\delta(q_{znf}), \tag{3.40}$$

$$\frac{d^2N}{d\omega d\Omega} = \sum_s \frac{d^2N_s}{d\omega d\Omega} = \frac{e^2\omega L}{\pi}\sum_{nf} Q_{nn}\left\{\left(2\frac{E}{E_1} + \frac{\omega^2}{E_1^2}\right)|\vec{g}_{\perp nf}|^2 + \frac{\omega^2}{E_1^2}|\vec{g}_{\| nf}|^2\right\}\delta(q_{znf})\,. \tag{3.41}$$

If expression (3.41) only includes the sub-barrier transitions, then it coincides with that derived in [Zhevago (1978)]. Such a restriction, however, as we have pointed out repeatedly [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d); Baryshevsky *et al.* (1978)], does not fit the real experimental conditions, when at particle entering at a certain angle to the axis (plane), the above-barrier states (regions) are also necessarily populated.

In a most typical case, photons of frequency $\omega \ll E$ are emitted through channeling. If in this case the energies $\varepsilon_{n\kappa}$ and $\varepsilon_{f\kappa_1} \ll m$ (i.e., the transverse motion in the system with zero longitudinal velocity of a particle is nonrelativistic, which occurs for particles, whose energy is less than a few gigaelectronvolts), then expressions (3.40), (3.41) simplify considerably:

$$\frac{d^2N_s}{d\omega d\Omega} = \frac{2e^2\omega L}{\pi}\sum_{nf} Q_{nn}|\vec{g}_{\perp nf}\vec{e}_s^{\,*}|^2\delta(\omega(1-\beta\cos\vartheta) - \Omega_{nf})\,, \tag{3.42}$$

$$\frac{d^2N}{d\omega d\Omega} = \frac{2e^2\omega L}{\pi}\sum_{nf} Q_{nn}|\vec{g}_{\perp nf}|^2\delta(\omega(1-\beta\cos\vartheta) - \Omega_{nf})\,, \tag{3.43}$$

where $\beta = v_z/c$ and at $c = 1$, the value of $\beta = v_z$; v_z is the longitudinal particle velocity; the component $\vec{g}_{\|}$ in this approximation does not contribute to (3.42), (3.43).

It is worthy of mention that the quantity $\vec{W}_{\perp nf} = \vec{I}_{2nf} + \vec{p}_{z\perp k} I_{1nf}$ appearing in the expression for vector $\vec{g}_{\perp nf}$ can be represented in several equivalent forms. Using the notations agreed in [Baryshevsky *et al.* (1978)], we have the following expression for $\vec{W}_{\perp}$

$$\vec{W}_{\gamma\eta} = \vec{p}_n J_{\gamma\eta} - \vec{n}_x I_{\gamma\eta},\ \vec{p}_n = \vec{p} - \frac{2\pi n_0}{a}\vec{n}_x; \tag{3.44}$$

n_0 is obtained by reduction of vector $p_x - k_x$ to the first Brillouin zone, i.e., $|p_x - k_x - \frac{2\pi n_0}{a}| < \frac{\pi}{a}$;

$$J_{\gamma\eta} = \int_0^a e^{-i\frac{2\pi(l-n_0)}{a}x} u_{\gamma p_x}(x) u^*_{\eta p_{1x}}(x)dx;$$
$$I_{\gamma\eta} = \frac{1}{i}\int_0^a e^{-i\frac{2\pi(l-n_0)}{a}x} u_{\gamma p_x}(x)\frac{d}{dx}u^*_{\eta p_{1x}}(x)dx; \tag{3.45}$$

l is found from the condition $|p_x - \frac{2\pi l}{a}| < \frac{\pi}{a}$; Integration of the expression for $I_{\gamma\eta}$ by parts gives

$$I_{\gamma\eta} = -\frac{1}{i}\int_0^a u^*_{\eta p_{1x}}(x)\frac{d}{dx}(e^{-i\frac{2\pi(l-n_0)}{a}x}u_{\gamma p_x}(x))dx = \frac{2\pi(l-n_0)}{a}J_{\gamma\eta} - I'_{\gamma\eta}, \qquad (3.46)$$

where

$$I'_{\gamma\eta} = -i\int_0^a e^{-i\frac{2\pi(l-n_0)}{a}x}u^*_{\eta p_{1x}}(x)\frac{d}{dx}u_{\gamma p_x}(x)dx.$$

From the definitions of n_0 and l follows that $\frac{2\pi l}{a} = p_x - \kappa$, and $\frac{2\pi n_0}{a} = p_x - k_x - \kappa_1$. Hence, we can write:

$$J_{\gamma\eta} = \int_0^a e^{-i(k_x+\kappa_1-\kappa)x}u_{\gamma p_x}(x)u^*_{\eta p_{1x}}(x)dx;$$
$$I_{\gamma\eta} = (k_x+\kappa_1-\kappa)J_{\gamma\eta} - I'_{\gamma\eta};$$
$$I'_{\gamma\eta} = -i\int_0^a e^{-i(k_x+\kappa_1-\kappa)x}u^*_{\eta p_{1x}}(x)\frac{d}{dx}u_{\gamma p_x}(x)dx. \qquad (3.47)$$

Recall that the Bloch function is

$$\psi_{\gamma p_x}(x) \equiv \psi_{\gamma\kappa}(x) = \frac{1}{\sqrt{N_x}}e^{i\kappa x}u_{\gamma p_x}(x);\ u_{\gamma p_x}(x) \equiv u_{\gamma\kappa}(x);$$
$$\psi_{\eta p_{1x}}(x) \equiv \psi_{\eta\kappa_1}(x) = \frac{1}{\sqrt{N_x}}e^{i\kappa_1 x}u_{\eta p_{1x}}(x). \qquad (3.48)$$

Consequently,

$$u_{\gamma p_x} = \sqrt{N_x}e^{-i\kappa x}\psi_{\gamma p_x}(x);\ u_{\eta p_{1x}}(x) = \sqrt{N_x}e^{-i\kappa_1 x}\psi_{\eta p_{1x}}(x). \qquad (3.49)$$

Substituting (3.49) into (3.47), we obtain the following equality from (3.44):

$$\vec{W}_{\gamma\eta} = (\vec{p} - \vec{p}_x)J_{\gamma\eta} + I_{\gamma\eta}\vec{n}_x. \qquad (3.50)$$

As in (3.42) (see also (9) in [Baryshevsky *et al.* (1978)]) vector $\vec{W}$ is multiplied by the photon polarization vector $\vec{e}_s$, $(\vec{p} - \vec{p}_x)$ in (3.50) can be replaced by $(\vec{p} - \vec{p}_x)_{\perp k}$. [1]

[1] We obtained formula (3.42) in [Baryshevsky *et al.* (1978)] in a more general form (with the term $\frac{1-\exp(-iq_{znf})L}{q_{znf}}$ instead of δ-function). Two years after the work was published, the coincident formula was derived in [Bazylev *et al.* (1980, 1981)]. The authors of [Bazylev *et al.* (1980, 1981)] first did not notice that their relations coincide with those we had obtained before and declared our theory invalid. In [Baryshevsky (1980d); Baryshevsky *et al.* (1980e)] we proved that the criticism from the authors of [Bazylev *et al.* (1980, 1981)] is unfounded. Now compare (3.42) and the coincident

The presence of δ-functions in the derived expressions enables one to easily find spectral or angular distribution of emitted photons. It should be emphasized that the finite width of the bands for transverse motion leads to the fact that the radiation in question appears not only at transitions between different levels but also at the transitions within a given band. In the case of narrow bands the corresponding radiation for high-energy particles lies within the optical spectrum. For wide over-barrier bands these transitions cause radiation in the X-ray and shorter wavelength spectra. As follows from the presence of the δ-function in expressions (3.42), (3.43), the corresponding equation defining the photon frequency at the intraband transition has the form

$$(1-\beta\cos\vartheta)\omega - (\varepsilon'_{n\kappa} - \varepsilon'_{n\kappa_1}) = 0\,. \qquad (3.54)$$

At fixed frequency, this equation determines the radiation angle of a quantum. Note that in solving (3.54) in the case of over-barrier states it is vital to remember that $\varepsilon'_{n\kappa_1}$ depends on ω and ϑ.

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formula (9) in [Baryshevsky *et al.* (1978)] with formula (7) derived in [Bazylev *et al.* (1980, 1981)]. According to [Bazylev *et al.* (1980, 1981)] the formula for spectral-angular distribution of radiation at spontaneous transitions in the planar case has the form

$$\frac{d^2W}{d\omega d\Omega} = \frac{e^2\omega}{2\pi}\sum_f \Big\{ e_\sigma |j_{if}^{(x)}(k_x)|^2 \sin^2\varphi + e_\pi |j_{if}^{(z)}(k_x)\theta$$
$$-j_{if}^{(x)}(k_x)\cos\varphi|^2\Big\}\,\delta\left[\omega\Big(\frac{\theta^2+E^{-2}}{2} - \frac{\partial\varepsilon_f(E''_i)}{\partial E_i}\Big) - \tilde{\omega}_{if}\right]. \qquad (3.51)$$

The notations in (3.51) are the same as in [Bazylev *et al.* (1980, 1981)]. If a particle populates only one level, (3.42) could differ from (3.51) by the expression between the braces in (3.51), and by the $\frac{1}{E^2}|\vec{W}_{nf}\vec{e}_s|^2$. We will demonstrate that there is no difference. Consider π-polarization. In this case the polarization vector $\vec{e}_s = \vec{e}_\pi$ is in the plane formed by the particle and photon momenta. As a consequence, $\vec{e}_\pi\vec{n}_{z\perp k} = -\vartheta$, ϑ is the photon radiation angle; $\vec{e}_\pi\vec{n}_x \simeq \cos\varphi$. Then

$$\left|\frac{\vec{e}_\pi\vec{W}_{nf}}{E}\right|^2 = \left|J_{1nf}\vartheta - \frac{1}{E}I_{2nf}\cos\varphi\right|^2. \qquad (3.52)$$

It is clear from the definition of J_{1nf} and I_{2nf}, $j^{(x)}$ and $j^{(z)}$ that $j_{if}^{(x)} = \frac{1}{E}I^*_{2nf}$, $j_{if}^{(z)}(k_x) = J^*_{if}$ and, hence, the formulae for spectral-angular distribution of radiation coincide. Consider σ-polarization. Now $\vec{e}_s = \vec{e}_\sigma$ is perpendicular to the plane made up by the momenta of a photon and a particle. As a result, $\vec{e}_\sigma\vec{n}_{z\perp k} = 0$, $\vec{e}_\sigma\vec{n}_x \simeq \sin\varphi$ and

$$\left|\frac{\vec{e}_\sigma\vec{W}_{nf}}{E}\right|^2 = \frac{1}{E^2}|I_{2nf}|^2\sin^2\varphi, \qquad (3.53)$$

so the formulae coincide completely.

3.3 Spectral and Angular Distributions of Photons in the Dipole Approximation

Though simple at first sight, expressions (3.42), (3.43) are rather complicated. Matrix elements I_{2nf} and I_{1nf} defining vector $\vec{g}_{nf}$ are quite analogous to matrix elements used in the theory of atomic radiation (see, for example, [Berestetsky *et al.* (1968)]). Investigating the properties of radiation in the range where photon frequencies and exit angles are such that $k_\perp a \ll 1$, the exponentials in I_{2nf} and I_{1nf} may be expanded, and the reduced vectors κ and κ_1 in wave functions may be equated. At the same time, when solving (3.54), the distinction between κ and κ_1 should be taken into account especially for intrabad transitions. Under the condition $k_\perp a \ll 1$ $\varepsilon'_{n\kappa_1}$ can be expanded in terms of $k_\perp$. As a result, $\varepsilon'_{n\kappa} - \varepsilon'_{n\kappa_1} \simeq \frac{d\varepsilon'}{dk_\perp} k_\perp = vk_\perp$. Velocity v has the order of magnitude $\vartheta_L c$, i.e., $v \sim 10^6$ cm/s for $\vartheta_L \sim 10^{-4}$. From this $vk_\perp \sim 10^{12} - 10^{13}$ sec^{-1} for $k_\perp \sim 10^7$ cm^{-1}. The frequency $vk_\perp$ in this case is much smaller than the characteristic frequency of of interband transitions, so the corresponding radiation lies in a substantially softer spectra (in this case it lies in the optical region even for particles with energies of the order of 1 GeV). For this reason, when analyzing the radiation spectrum in the X-ray and shorter wavelength spectral regions, we shall not take into account intraband transitions, assuming that $\kappa = \kappa_1$ in the interband transition frequencies. As a result, equation (3.44) is easily solvable, and integration of (3.42), (3.44) with respect to the photon exit angles with the maximum collimation angle $\vartheta_k \ll m/E$, gives in the dipole approximation the following expressions for the spectrum [Baryshevsky *et al.* (1980a); Baryshevsky *et al.* (1980d,b)]:

$$\frac{dN_s}{d\omega} = e^2 L \sum_{nf} Q_{nn} |\vec{\rho}_{nf} \vec{e}_s^{\,*}|^2 \Omega_{nf}^2 \left[1 - \frac{\omega}{\Omega_{nf}}(1-\beta^2) + \frac{\omega^2}{2\Omega_{nf}^2}(1-\beta^2)^2\right] \theta\left(\frac{\vartheta_k^2}{2} - \alpha_{nf}(\omega)\right) \theta(\alpha_{nf}(\omega)), \tag{3.55}$$

$$\frac{dN}{d\omega} = e^2 L \sum_{nf} Q_{nn} |\vec{\rho}_{nf}|^2 \Omega_{nf}^2 \left[1 - \frac{\omega}{\Omega_{nf}}(1-\beta^2) + \frac{\omega^2}{2\Omega_{nf}^2}(1-\beta^2)^2\right] \theta\left(\frac{\vartheta_k^2}{2} - \alpha_{nf}(\omega)\right) \theta(\alpha_{nf}(\omega)), \tag{3.56}$$

where $\vec{\rho}_{nf} = N_\perp \int_s \psi_{n\kappa}(\vec{\rho}) \vec{\rho} \psi_{f\kappa}(\vec{\rho}) d^2\rho$; $\theta(z) = 1$ at $z > 0$ and $\theta(z) = 0$ at $z < 0$; $\alpha_{nf}(\omega) = (1-\beta)(\frac{\tilde{\omega}_{nf}}{\omega} - 1)$; $\tilde{\omega}_{nf} = \Omega_{nf}/(1-\beta)$ is the maximum radiation frequency at the $n \to f$ transition. If the collimation angle $\vartheta_k = \pi$,

$$\frac{dN}{d\omega} = e^2 L \sum_{nf} Q_{nn} |\vec{\rho}_{nf}|^2 \Omega_{nf}^2 \left\{ 1 - \frac{\omega}{\Omega_{nf}} (1-\beta^2) + \frac{\omega^2}{2\Omega_{nf}^2} (1-\beta^2)^2 \right\} \theta(2-\alpha_{nf}(\omega)) \theta(\alpha_{nf}(\omega)) \,. \tag{3.57}$$

In the particular case when only sub-barrier transitions remain in the sum over n, f, expression (3.57) turns into the one analyzed in [Zhevago (1978)].

Now consider the angular distribution. With this aim in view, integrate (3.42), (3.43) over the frequencies. Under real conditions, the detector registers the photons within a certain spectral interval $\omega_1 \le \omega \le \omega_2$. Integration within this interval gives

$$\begin{aligned} \frac{dN_s}{d\Omega} &= A_s \frac{(1-\beta\cos\vartheta)^2 - (1-\beta^2)\sin^2\vartheta\cos^2\varphi}{(1-\beta\cos\vartheta)^4} \\ &\times \theta[\cos\vartheta - b_{nf}(\omega_1)]\theta[b_{nf}(\omega_2) - \cos\vartheta] \,, \\ &\vartheta \le \frac{m}{E} \,, \quad b_{nf}(\omega) = \frac{1}{\beta}\left(1 - \frac{\Omega_{nf}}{\omega}\right) \,, \quad A_s = \frac{e^2 L}{2\pi} \sum_{nf} Q_{nn} |\vec{x}_{nf} \vec{e}_s^{\,*}|^2 \Omega_{nf}^3 \,, \end{aligned} \tag{3.58}$$

$$\begin{aligned} \frac{dN}{d\Omega} &= A \frac{(1-\beta\cos\vartheta)^2 - (1-\beta^2)\sin^2\vartheta\cos^2\varphi}{(1-\beta\cos\vartheta)^4} \\ &\times \theta[\cos\vartheta - b_{nf}(\omega_1)]\theta[b_{nf}(\omega_2) - \cos\vartheta] \,, \\ A &= \frac{e^2 L}{2\pi} \sum_{nf} Q_{nn} |\vec{x}_{nf}|^2 \Omega_{nf}^3 \,. \end{aligned} \tag{3.59}$$

From formulas (3.58), (3.59) follows the well-known result that the angular distribution of the radiation from a relativistic particle whose velocity and acceleration are mutually perpendicular is a universal function independent of the shape of the potential in which the particle moves [Landau and Lifshitz (1967)].

The number of γ-quanta ΔN_ω emitted by a channeled particle in the frequency interval $\Delta\omega$ in the dipole approximation can be estimated as follows:

$$\Delta N_\omega \simeq e^2 x_0^2 \Omega^2 L \Delta\omega,$$

where x_0 is the amplitude of particle oscillations in the channel. From this follows, for example, that the particle with the energy $E \sim 1$ GeV

($x_0 \sim 10^{-8}$ cm, $\Omega \sim 10^{16}$ s^{-1}) passing through a silicon plate of length $L \sim 10^{-2}$ cm in the spectral interval $\frac{\Delta\omega}{\omega} = 10^{-3}$ emits the number of quanta $\Delta N_\omega \sim 10^{-3} \div 10^{-4}$ in the vicinity of the maximum frequency, and $\Delta N_\omega \sim 10^{-7} \div 10^{-6}$ in the range of X-ray photons with the frequency of the order of tens of kiloelectron-volts (according to our estimations [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d, 1976a)]).

In the case of excitation of resonance nuclear levels the number of quanta formed in the interval of the order of the level width is important, which leads for example, for the number of quanta produced in ^{57}Fe Mossbauer target to the estimated value of $\Delta N_\omega \sim 10^{-14}$ quanta [Baryshevskii and Dubovskaya (1976a)]. The stated values follow from the formulae given in [Kumakhov (1977)], if taking into account that the estimate is given per unit length and the entire spectral interval.

It should be emphasized that for numerous applications in solid state physics and other fields, it is necessary to know the number of photons in a certain narrow frequency interval, rather than in the entire spectral interval. As a result, in narrow spectral intervals within the ranges of tens and hundreds of kiloelectronvolts the so-called parametric radiation often appears to be much more intense (see Section (4.7).

Chapter 4

The Influence of γ-Quanta Refraction and Diffraction on Angular and Spectral Characteristics of Radiation Produced by Particles in Crystals

4.1 Radiation in a Refractive Medium

Consider the theory of photon radiation in crystals when the effects caused by refraction and diffraction are of importance. The results obtained also describe radiation of diffracted electrons [Fedorov and Smirnov (1974); Fedorov *et al.* (1973); Fedorov (1980a); Baryshevsky (1980c,b)].

Refraction and diffraction are significant when the crystal thickness is $L > 1/k|n-1|$. As shown in Chapter (1.3), in this case spectral and angular distributions change drastically. In particular, the effects caused by diffraction lead to the appearance of radiation at large angles with the spectrum depending on the effects of anomalous transmission of γ-quanta through a crystal [Baryshevskii (1971)]. Moreover, diffraction gives rise to a new, quite a vigorous radiation mechanism, the so-called parametric mechanism for generating γ-quanta [Baryshevskii and Feranchuk (1971, 1973, 1976)] (see also [Garibyan and Yan Shi (1972); Avakyan *et al.* (1975); Afanas'ev and Aginyan (1978); Feranchuk (1979b)]).

Theoretical description of such phenomena requires (see Chapter (2.2) finding the transition matrix element M determined by the photon wave function being the exact solution of homogeneous Maxwell equations describing propagation of an electromagnetic wave in a medium. It should be emphasized that the photon wave function of the type $A^{(-)}$ satisfies Maxwell equations with the complex conjugate dielectric permittivity [Baryshevsky (1976)], and ignoring the asymptotic requirements may lead to the formation of misbehaving wave functions in an absorbing medium.

As before, consider photon emission in a plane-parallel crystal plate. If the photon exit angle of is not equal to the Wulff-Bragg angle, then in the X-ray and the frequency ranges with shorter wavelengths, where $|n-1| \ll 1$, the expression for A^{-}_{ks} has the form [Baryshevskii and Dubovskaya (1977d)]

$$\begin{aligned} A^{(-)}_{ks}(\vec{r}) &= \sqrt{4\pi}\left\{\vec{e}_s e^{i\vec{k}\vec{r}} e^{-ik_z n^* L}\theta(-z)\right. \\ &+\vec{e}_s e^{i\vec{k}\vec{r}} e^{-ik_z n^* L} e^{ik_z(n^*-1)z}\theta(z)\theta(L-z) \\ &\left.+\vec{e}_s e^{i\vec{k}\vec{r}} e^{-ik_z L}\theta(z-L)\right\}. \end{aligned} \quad (4.1)$$

According to (4.1) inside the plate with boundaries $0 \le z \le L$

$$A^{(-)}_{ks}(\vec{r}) = \sqrt{4\pi}\,\vec{e}_s e^{i\vec{k}^*_1\vec{r}} e^{-ik_z n^* L}, \quad (4.2)$$

where $\vec{k}_1 = (\vec{k}_\perp, k_z n)$. Comparison of (4.2) and the photon wave function (3.5) shows that taking into account the refractive effects in matrix elements is reduced to the substitution of vector $\vec{k}^*_1$ for vector $\vec{k}$. In other words, all general formulas written out in Chapter (2.2) preserve their form (for this purpose, we retained the complex conjugation symbol in q_{znf}). As a result, for example, at $\omega \gg E$ the spectral-angular distribution of photons has the form[1]

$$\begin{aligned} \frac{d^2 N_s}{d\omega d\Omega} &= \frac{e^2\omega}{\pi^2}\mathrm{Re}\sum_{nfj} Q_{nj} e^{i\tilde{\Omega}_{nj}L}\left[\frac{1-\exp(iq^*_{zjf}L)}{q^*_{zjf}}\right] \\ &\times\left[\frac{1-\exp(-iq_{znf}L)}{q_{znf}}\right](\vec{g}_{nf}\vec{e}^{\,*}_s)(\vec{g}^{\,*}_{jf}\vec{e}_s). \end{aligned} \quad (4.3)$$

When the crystal thickness L is much greater than the photon absorption depth l_{abs} in a crystal, (4.3) simplifies

$$\frac{d^2 N_s}{d\omega d\Omega} = \frac{e^2\omega}{\pi^2}\mathrm{Re}\sum_{nfj} Q_{nj} e^{i\tilde{\Omega}_{nj}L}\frac{1}{q^*_{zjf}q_{znf}}(\vec{g}_{nf}\vec{e}^{\,*}_s)(\vec{g}^{\,*}_{jf}\vec{e}_s). \quad (4.4)$$

Integration of expression (4.3) for the double-differential radiation spectrum over the angles with maximum opening ϑ_k equal the collimation angle of the photon beam that exits the crystal, we obtain the radiation spectrum

[1]Formula (4.3) is obtained if, when integrating matrix elements only the integrals over the path inside the crystal are retained, and the integrals over the path in a vacuum are discarded. The total radiation cross-section including vacuum terms is given in [Baryshevskii and Dubovskaya (1977d)]. The vacuum terms are important for a soft spectral range, when the vacuum coherent radiation length appears to be comparable with the plate thickness or the quantum absorption depth.

in the in the dipole approximation as

$$\frac{dN}{d\omega} = \frac{e^2}{4\pi}\sum_{nf} Q_{nn}|x_{nf}|^2 \left\{ \frac{[\Omega_{nf} - \omega(1-\beta^2 n'^2)]^2 + \Omega_{nf}^2}{\omega^2 n' n''} \right.$$
$$\times \left[(1+e^{-2\omega n'' L}) \left(\arctan\frac{\alpha_{nf}}{n''} + \arctan\frac{\frac{\vartheta_k^2}{2} - \alpha_{nf}}{n''} \right) \right.$$
$$-2\pi e^{-2\omega n'' L}\theta\left(\frac{\vartheta_k^2}{2} - \alpha_{nf}\right)\theta(\alpha_{nf}) + \frac{1}{2i}\left(\xi_{nf}^+ \right.$$
$$\left.\left. +\delta_{nf}^- - \xi_{nf}^-\delta_{nf}^+\right)\right] + (1+e^{-2\omega n'' L})\frac{3\vartheta_k^2}{2}$$
$$\times\theta\left(\frac{\vartheta_k^2}{2} - \alpha_{nf}\right)\theta(\alpha_{nf}) + \left[2(1-\beta^2 n'^2)\right.$$
$$\left. -2\frac{\Omega_{nf}}{\omega} + \frac{\Omega_{nf}^2}{\omega^2}\right]\left[(1+e^{-2\omega n'' L})\right.$$
$$\left.\left. \times \ln\frac{\alpha_{nf}^2 + \left(\frac{n''}{n'}\right)^2}{\left(\frac{\vartheta_k^2}{2} - \alpha_{nf}\right)^2 + \left(\frac{n''}{n'}\right)^2} - (\xi_{nf}^+ + \delta_{nf}^- - \delta_{nf}^+ + \xi_{nf}^-)\right]\right\}, \tag{4.5}$$

where

$$\alpha_{nf} = (1-\beta n')\left(\frac{\tilde{\omega}_{nf}}{\omega} - 1\right);\ \tilde{\omega}_{nf} = \frac{\Omega_{nf}}{1-\beta n'};$$

$$n' = \mathtt{Re}\, n(\omega);\ n'' = \mathtt{Im}\, n(\omega);$$

$$\xi_{nf}^{\pm} = Ei(-\omega n'' L \pm iL\omega\alpha_{nf})$$
$$-Ei\left[-\omega n'' L \pm iL\omega\left(\alpha_{nf} - \frac{\vartheta_k^2}{2}\right)\right];$$
$$\delta_{nf}^{\pm} = e^{-2\omega n'' L}\left\{Ei\left[\omega n'' L \pm iL\omega\left(\alpha_{nf} - \frac{\vartheta_k^2}{2}\right)\right]\right.$$
$$\left. -Ei(\omega n'' L \pm iL\omega\alpha_{nf})\right\} \tag{4.6}$$

($Ei(z)$ is the integral exponential function).

As the explicit form of the refractive index was not used in (4.5), this equation also holds true for crystals containing resonant nuclei. In this case it is essential to take account of the photon absorption in a crystal (the absorption length in such crystals can be of the order of $10^{-5} \div 10^{-4}$ cm).

Allowing for absorption is also necessary when considering radiation in a relatively soft X-ray spectrum (the absorption length l_{abs} of the photons with $\omega < 10$ keV in a Si crystal proves to be less than 10^{-2} cm (Figure 6).

Figure 6. Spectral distribution of photons in relative units ($f = \frac{dN\omega}{d\omega}A$). Photons are emitted by a particle (E = 1 GeV) in Si (110) crystals (crystal thicknesses in cm). Dashed curves - photon distribution without absorption; solid curves - photon distribution with account of absorption.

In the limiting case ($L \gg l_{abs}$ the expression (4.5) for the radiation spectrum simplifies and takes the form

$$\frac{dN}{d\omega} = \frac{e^2\omega}{2\pi}\sum_{nf} Q_{nn}|x_{nf}|^2 \left\{ \frac{l_{abs}}{\omega}\left[\Omega_{nf}^2 + (\Omega_{nf} \right.\right.$$
$$\left. -(1-\beta^2 n'^2)\omega)^2\right]\left(\arctan\frac{\alpha_{nf}}{n''} + \arctan\frac{\frac{\vartheta_k^2}{2} - \alpha_{nf}}{n''}\right)$$
$$+\frac{3}{4}\vartheta_k^2\theta\left(\frac{\vartheta_k^2}{2} - \alpha_{nf}\right)\theta(\alpha_{nf}) + \left[(1-\beta^2 n'^2)\right.$$
$$\left.\left. -\frac{\Omega_{nf}}{\omega} + \frac{\Omega_{nf}^2}{2\omega^2}\right]\ln\frac{\alpha_{nf}^2 + \left(\frac{n''}{n'}\right)^2}{\left(\frac{\vartheta_k^2}{2} - \alpha_{nf}\right)^2 + \left(\frac{n''}{n'}\right)^2}\right\}, \quad (4.7)$$

where $l_{abs} = \frac{1}{2\omega n''(\omega)}$.

For radiation at a small angle with respect to the direction of particle motion the last two terms in (4.7) are small, and the spectral intensity of radiation is practically proportional to the photon absorption length in a crystal:

$$\frac{dN}{d\omega} = \frac{e^2\omega}{2\pi}\sum_{nf} Q_{nn}|x_{nf}|^2 \frac{l_{abs}}{\omega}\left[\Omega_{nf}^2 + (\Omega_{nf}\right.$$
$$\left. -(1-\beta^2 n'^2)\omega)^2\right]\left(\arctan\frac{\alpha_{nf}}{n''} + \arctan\frac{\frac{\vartheta_k^2}{2} - \alpha_{nf}}{n''}\right). \quad (4.8)$$

The analysis shows that at an arbitrary ratio of the crystal thickness to the photon absorption depth, quite a simple formula may express the radiation spectrum in an absorbing crystal with high accuracy [Baryshevsky *et al.* (1980b)]:

$$\frac{dN}{d\omega} = l_{abs}(\omega)\left(1 - \exp\left(-\frac{L}{l_{abs}(\omega)}\right)\right)\frac{d\tilde{N}}{d\omega}, \quad (4.9)$$

where $\frac{d\tilde{N}}{d\omega}$ is the radiation spectrum in the absence of absorption, i.e., at $n'' = 0$.

4.2 Optical Radiation Produced by Channeled Particles

The effects associated with photon refraction in a medium in the optical spectrum, where the refractive index n (the dielectric permittivity $\varepsilon(\omega)$) can be appreciably different from unity, appear to be of particular importance. Formulae describing radiation of a moving oscillator in a refractive infinite medium were derived by Frank in [Frank (1942)]. For channeled (diffracting) particles the presence of boundaries in a crystal is essential. Classical theory generalizing [Frank (1942)] for the case of photon emission by an oscillating particle traversing a plate of finite thickness is given by the author together with I.M. Frank. In this case the formula describing spectral-angular characteristics of radiation of the oscillator moving along the z-axis and oscillating along the x-axis with the amplitude x_0 has the form

$$\frac{d^2N}{d\omega d\Omega} = \frac{e^2\omega^3 x_0^2}{4\pi^2\hbar c^3}\Big\{|G_1|^2\cos^2\varphi|\beta\varepsilon(\omega)-\sqrt{\varepsilon(\omega)-\sin^2\vartheta}|^2 + |G_2|^2\sin^2\varphi|1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta}|^2\Big\}|S(L)|^2\,, \tag{4.10}$$

where

$$S(L) = \frac{\exp\left\{i\left[\omega\left(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta}\right)-\omega_0\right]\frac{L}{v_z}\right\}-1}{i\left[\omega\left(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta}\right)-\omega_0\right]}\,,$$

ω_0 is the oscillation frequency of the oscillator in the laboratory system of coordinates; $\beta = v_z/c$; v_z is the velocity along the z-axis;

$$G_1 = g_1\left(\sqrt{\varepsilon(\omega)-\sin^2\vartheta}+\varepsilon(\omega)\cos\vartheta\right)\cos\vartheta;$$

$$G_2 = g_2\left(\sqrt{\varepsilon(\omega)-\sin^2\vartheta}+\cos\vartheta\right)\cos\vartheta\,,$$

$$g_1 = \left[\left(\sqrt{\varepsilon(\omega)-\sin^2\vartheta}+\varepsilon(\omega)\cos\vartheta\right)^2\exp\left(-i\frac{L\omega}{c}\sqrt{\varepsilon(\omega)-\sin^2\vartheta}\right) - \left(\sqrt{\varepsilon(\omega)-\sin^2\vartheta}-\varepsilon(\omega)\cos\vartheta\right)^2\exp\left(i\frac{L\omega}{c}\sqrt{\varepsilon(\omega)-\sin^2\vartheta}\right)\right]^{-1}\,,$$

$$g_2 = \left[\left(\sqrt{\varepsilon(\omega)-\sin^2\vartheta}+\cos\vartheta\right)^2\exp\left(-i\frac{L\omega}{c}\sqrt{\varepsilon(\omega)-\sin^2\vartheta}\right) - \left(\sqrt{\varepsilon(\omega)-\sin^2\vartheta}-\cos\vartheta\right)^2\exp\left(i\frac{L\omega}{c}\sqrt{\varepsilon(\omega)-\sin^2\vartheta}\right)\right]^{-1}\,.$$

To derive the expression

$$\frac{d^2N}{d\omega d\Omega}$$

describing the distribution of photons produced by a quantum emitter, suffice it to replace x_0 by a doubled matrix element of the transition from the emitter's coordinate: $x_0^2 \to 4|x_{nf}|^2$; and the oscillation frequency ω_0 by the transition frequency Ω_{nf}.

Without absorption

$$|S(L)|^2 = 4\frac{\sin^2[\omega(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta})-\omega_0]\frac{L}{2v_z}}{[\omega(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta})-\omega_0]^2}\,. \tag{4.11}$$

The contribution due to the waves reflected from the vacuum–plate entrance boundary should also be added to the intensity in (4.10). It is obtained by replacing $\beta \to -\beta$ and the sign "+" between the brackets of the multiplier appearing in $G_{1(2)}$ after $g_{1(2)}$ with "–". In the vicinity of the frequencies and angles for which $\sqrt{\varepsilon(\omega)-\sin^2\vartheta}$ vanishes, the interference terms may also gain in importance.

In view of the fact that the path L is finite, every angle ϑ has a corresponding frequency spectrum, covering the range $\Delta\omega$, which are close to the Doppler frequency. Photons with such frequency are emitted within the finite range of angles $\Delta\vartheta$ [Frank (1942)]. With increasing L the range of angles $\Delta\vartheta$ reduces. Thus, following [Frank (1942)], in our case we have the below equality for the range $\Delta\omega$ in the absence of absorption

$$\Delta\omega = \frac{\pm\pi}{|1-\beta n(\omega_\vartheta,\vartheta)\cos\vartheta| - \omega_\vartheta\frac{dn(\omega_\vartheta,\vartheta)}{d\omega_\vartheta}\cos\vartheta}\,\frac{2v}{L}\,,$$
$$\omega_\vartheta(1-\beta n(\omega_\vartheta,\,\vartheta)\cos\vartheta) = \omega_0\,, \tag{4.12}$$

where we introduce the refractive index

$$n(\omega,\vartheta) = \frac{\sqrt{\varepsilon-\sin^2\vartheta}}{|\cos\vartheta|} = \sqrt{1+\frac{\varepsilon(\omega)-1}{\cos^2\vartheta}}\,. \tag{4.13}$$

If we are not concerned with the width of the peak, then with high accuracy

$$\frac{\sin^2[\omega(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta})-\omega_0]\frac{L}{2v}}{[\omega(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta})-\omega_0]^2}$$
$$\simeq \frac{L}{2v}\pi\delta(\omega(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta})-\omega_0) \tag{4.14}$$

and the number of photons emitted by a linear oscillator is defined by formula

$$\frac{d^2N}{d\omega d\Omega} = \frac{e^2x_0^2\omega^3L}{2\pi\hbar c^3v}\left\{|G_1|^2\cos^2\varphi(\beta\varepsilon(\omega) - \sqrt{\varepsilon(\omega)-\sin^2\vartheta})^2\right.$$
$$\left.+|G_2|^2\sin^2\varphi(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta})^2\right\}$$
$$\times\delta(\omega(1-\beta\sqrt{\varepsilon(\omega)-\sin^2\vartheta})-\omega_0)\,. \qquad (4.15)$$

Find the angular distribution of photons with frequencies ω lying in the range $\omega_1 \le \omega \le \omega_2$:

$$\frac{dN}{d\Omega} = \frac{e^2x_0^2L}{2\pi\hbar c^3v}\sum_\alpha \omega_{\vartheta_\alpha}^3\mathcal{M}(\omega_{\vartheta_\alpha},\,\vartheta,\varphi)\eta_\alpha$$
$$\times\left|1-\beta n(\omega_{\vartheta_\alpha},\vartheta)\cos\vartheta - \beta\omega_{\vartheta_\alpha}\frac{dn(\omega_{\vartheta_\alpha},\,\vartheta)}{d\omega_{\vartheta_\alpha}}\cos\vartheta\right|^{-1}, \qquad (4.16)$$

where $\mathcal{M}$ denotes the curly bracket, appearing in (4.15), taken at the frequency value $\omega = \omega_{\vartheta_\alpha}$; the sign η_α reminds that (4.16) is nonzero in the range of polar angles ϑ, which is determined by the direction of the photon escape with the maximum ω_2 and minimum ω_1 frequencies;

$$\omega_{\vartheta_\alpha}(1-\beta n(\omega_{\vartheta_\alpha},\vartheta)\cos\vartheta) = \omega_0\,.$$

Now consider spectral distribution. Integration of expression (4.15) with respect to the angle φ is reduced to replacing $\cos^2\varphi$ and $\sin^2\varphi$ by π. As a consequence,

$$\frac{dN}{d\omega} = \frac{e^2x_0^2\omega^3L}{2\hbar c^3v}\int\limits_{\vartheta_{\min}}^{\vartheta_{\max}}\left\{|G_1|^2(\beta\varepsilon-\sqrt{\varepsilon-\sin^2\vartheta})^2+|G_2|^2\right.$$
$$\left.\times(1-\beta\sqrt{\varepsilon-\sin^2\vartheta})^2\right\}\delta(\omega(1-\beta\sqrt{\varepsilon-\sin^2\vartheta})-\omega_0)\sin\vartheta d\vartheta\,, \quad (4.17)$$

where $\vartheta_{\min}$, $\vartheta_{\max}$ are the minimum and maximum angles defining the boundaries of the angular range within which the radiation is detected.

To determine $dN/d\omega$, it is necessary to find the roots of equation

$$\omega(1-\beta\sqrt{\varepsilon-\sin^2\vartheta})-\omega_0 = 0\,. \qquad (4.18)$$

From (4.18) follows that

$$\cos\vartheta_{1,2} = \pm\sqrt{\left(\frac{\omega-\omega_0}{\beta\omega}\right)^2-(\varepsilon-1)}\,. \qquad (4.19)$$

If the radiation propagating at an acute angle relative to the particle velocity is registered, the contribution to (4.17) comes from only one positive root of (4.19). As a result,

$$\frac{dN}{d\omega} = \frac{e^2\omega^2 x_0^2 L}{2cv^3}\left\{|G_1|^2\left(\beta\varepsilon - \frac{1}{\beta} + \frac{\omega_0}{\omega}\right)^2 + |G_2|^2\left(\frac{\omega_0}{\omega}\right)^2\right\}\left|1 - \frac{\omega_0}{\omega}\right|\frac{\xi_\vartheta}{\cos\vartheta_1}\,. \tag{4.20}$$

Here the functions G_1 and G_2, which include the quantities $\cos\vartheta$ and $\sqrt{\varepsilon - \sin^2\vartheta}$ are expressed in terms of frequency according to (4.18), (4.19), e.g., $\sqrt{\varepsilon - \sin^2\vartheta} = \frac{1}{\beta}(1-\frac{\omega_0}{\omega})$. The symbol ξ_ϑ reminds that (4.20) is nonzero within the frequency range determined by the frequency values of the photons escaping at the angle $\vartheta_{\max}$ and $\vartheta_{\min}$.

Note that to describe the phenomena occurring under the anomalous Doppler effect, it takes only to replace ω_0 by $-\omega_0$ (in the quantum case $\Omega_{nf} > 0$ under the normal Doppler effect, and $\Omega_{nf} < 0$ under the anomalous one) in all the above formulas.

The relations derived simplify appreciably if mirror-reflected waves can be neglected, i.e., for example, in the case when $n(\omega,\,\vartheta)$ slightly differs from unity. Under such conditions with good accuracy

$$|G_1|^2 = \frac{\cos^2\vartheta}{(\varepsilon\cos\vartheta + \sqrt{\varepsilon - \sin^2\vartheta})^2}\,; \quad |G_2|^2 = \frac{\cos^2\vartheta}{(\cos\vartheta + \sqrt{\varepsilon - \sin^2\vartheta})}\,.$$

Generalization of formulae (4.10) to the two-dimensional case (axial channeling) was given by the author together with I.Ya. Dubovskaya. Spectral-angular distribution of radiation has the form:

for σ -polarization ($\hbar = c = 1$)

$$\frac{d^2N_\sigma}{d\omega d\Omega} = \frac{e^2\omega}{4\pi^2}\left\{\frac{|D_\sigma|^2}{q_z^2}\left(\frac{\vec{\beta}[\vec{n}\times\vec{n}_z]}{\sin\vartheta}\right)^2 + \frac{2}{q_z}\frac{\vec{\beta}[\vec{n}\times\vec{n}_z]}{\sin\vartheta}\right.$$
$$\times \mathtt{Im}\sum_{nf} Q_{nf}\Omega_{nf}D_\sigma^*\left[\left(\frac{1}{q_z}+\frac{A_\sigma^r}{\tilde{q}_z}\right)e^{i(p_{zn}-p_{1zf})L}\right.$$
$$\left.+q_{znf}^{-1}B_\sigma(e^{iq_{znf}L}-1)+B_\sigma^r\tilde{q}_{znf}^{-1}(e^{i\tilde{q}_{znf}L}-1)\right]$$
$$\times(\rho_{xnf}\sin\varphi-\rho_{ynf}\cos\varphi)+\mathtt{Re}\sum_{nfj}Q_{nj}\Omega_{nf}\Omega_{jf}$$
$$\times[B_\sigma^*q_{zjf}^{1*}(e^{-iq_{zjf}^*L}-1)+B_\sigma^{*r}\tilde{q}_{zjf}^{-1*}(e^{-i\tilde{q}_{zjf}^*L}-1)]$$
$$\times\left[B_\sigma q_{znf}^{-1}(e^{iq_{znf}L}-1)+B_\sigma^r\tilde{q}_{znf}^{-1}(e^{i\tilde{q}_{znf}L}-1)\right.$$
$$\left.-2\left(\frac{1}{q_z}+\frac{A_\sigma^r}{\tilde{q}_z}\right)e^{i(p_{zn}-p_{1zf})L}\right](\rho_{xnf}\sin\varphi-\rho_{ynf}\cos\varphi)$$
$$\times(\rho_{xjf}^*\sin\varphi-\rho_{yjf}^*\cos\varphi)-(1-\sin2\varphi)$$
$$\left.\times\mathtt{Re}\sum_{nj}Q_{nj}e^{i(p_{zn}-p_{zj})L}F_{nj}\right\}, \tag{4.21}$$

where

$$q_z = p_z - p_{1z} - k_z \simeq (1-\beta_z\cos\vartheta)\omega;$$

$$\tilde{q}_z = p_z - p_{1z} + k_z \simeq (1+\beta_z\cos\vartheta)\omega;$$

$$q_{znf} = p_{zn} - p_{1zf} - k_{1z} \simeq \omega(1-\beta_z\sqrt{\varepsilon(\omega)-\sin^2\vartheta}) - \Omega_{nf};$$

$$k_{1z} = \omega\sqrt{\varepsilon - \sin^2\vartheta};\ \tilde{q}_{znf} = p_{zn} - p_{1zf} + k_{1z}$$

$$\simeq \omega(1+\beta_z\sqrt{\varepsilon(\omega)-\sin^2\vartheta}) - \Omega_{nf};$$

$$A_\sigma^r = (1-\varepsilon(\omega))(e^{-i\omega a_0 L} - e^{i\omega a_0 L})g_2;$$

$$a_0 = \sqrt{\varepsilon(\omega)-\sin^2\vartheta};\ B_\sigma = 2G_2;$$

$$B_\sigma^r = 2\cos\vartheta(cos\vartheta - \sqrt{\varepsilon(\omega)-\sin^2\vartheta})g_2;$$

$$D_\sigma = 4a_0\cos\vartheta g_2;\ Q_{nf} = c_n(\vec{p}_\perp)c_f^*(\vec{p}_{1\perp});$$

$\vec{n} = \frac{\vec{k}}{\omega}$: $\vec{n}_z$ is the unit vector along the z-axis.

$$F_{n\vec{j}} = \frac{1}{E^2} N_\perp \int \varphi_{n\vec{\kappa}}(\vec{\rho}) \Delta_{\vec{\rho}} \varphi^*_{j\vec{\kappa}}(\vec{\rho}) d^2\rho;$$

for π-polarization

$$\frac{d^2 N_\sigma}{d\omega d\Omega} = \frac{e^2\omega}{4\pi^2} \left\{ \frac{|D_\pi|^2 |\beta_z - (\vec{\beta}\vec{n})\cos\vartheta|^2}{q_z^2 \sin^2\vartheta} + \beta_z^2 \sin^2\vartheta \right.$$
$$\times \left| \frac{1}{q_z} + \frac{A^r_\pi}{\tilde{q}_z} \right|^2 + 2\mathtt{Im} \sum_{nf} Q_{nf} \frac{D^*_\pi(\beta_z - (\vec{\beta}\vec{n})\cos\vartheta)}{q_z \sin\vartheta}$$
$$\times \Big[(\beta_z \omega \sin^2\vartheta - \Omega_{nf} a_0) B_\pi q^{-1}_{znf} (e^{iq_{znf}L} - 1)$$
$$+ (\beta_z \omega \sin^2\vartheta + \Omega_{nf} a_0) B^r_\pi \tilde{q}^{-1}_{znf} (e^{i\tilde{q}_{znf}L} - 1)$$
$$+ (\beta_z \omega \sin^2\vartheta + \Omega_{nf} \cos\vartheta) q_z^{-1} e^{i(p_{zn} - p_{1zf})L}$$
$$+ (\beta_z \omega \sin^2\vartheta + \Omega_{nf} \cos\vartheta) A^r_\pi \tilde{q}_z^{-1} e^{i(p_{zn} - p_{1zf})L} \Big]$$
$$\times (\rho_{xnf} \cos\varphi + \rho_{ynf} \sin\varphi) + \mathtt{Re} \sum_{nfj} Q_{nj} \left[\left(\beta_z \omega \sin^2\vartheta \right. \right.$$
$$\left. - \Omega_{jf} a_0^* \right) B^*_\pi q^{-1*}_{zjf} (e^{-iq^*_{zjf}L} - 1) + \left(\beta_z \omega \sin^2\vartheta \right.$$
$$\left. \left. + \Omega_{jf} a_0^* \right) B^{r*}_\pi \tilde{q}^{-1*}_{zjf} (e^{-i\tilde{q}^*_{zjf}L} - 1) \right] \Big[(\beta_z \omega \sin^2\vartheta - \Omega_{nf} a_0) B_\pi q^{-1}_{znf}$$
$$\times (e^{iq_{znf}L} - 1) + (\beta_z \omega \sin^2\vartheta + \Omega_{nf} a_0) B^r_\pi \tilde{q}^{-1}_{znf} (e^{i\tilde{q}_{znf}L} - 1)$$
$$-2(\beta_z \omega \sin^2\vartheta - \Omega_{nf} \cos\vartheta) q_z^{-1} e^{i(p_{zi} - p_{1zf})L}$$
$$-2(\beta_z \omega \sin^2\vartheta + \Omega_{nf} \cos\vartheta) A^r_\pi \tilde{q}_z^{-1} e^{i(p_{zn} - p_{1zf})L} \Big]$$
$$\times (\rho_{xnf} \cos\varphi + \rho_{ynf} \sin\varphi)(\rho^*_{xjf} \cos\varphi + \rho^*_{yjf} \sin\varphi)$$
$$-2\beta_z \sin\vartheta \cos\vartheta \mathtt{Im} \left[\left(\frac{1}{q_z} + \frac{A^r_\pi}{\tilde{q}_z} \right) \left(\frac{1}{q_z} - \frac{A^{r*}_\pi}{\tilde{q}_z} \right) \sum_{nj} Q_{nj} \right.$$
$$\left. \times e^{-i\tilde{\Omega}_{nj}L} \tilde{\Omega}_{nj} (\rho_{xnj} \cos\varphi + p_{ynj} \sin\varphi) \right] - \cos^2\vartheta (1 + \sin 2\varphi)$$
$$\left. \times \left| \frac{1}{q_z} - \frac{A^r_\pi}{\tilde{q}_z} \right|^2 \mathtt{Re} \sum_{nj} Q_{nj} e^{-i\tilde{\Omega}_{nj}L} F_{nj} \right\}; \quad (4.22)$$

$$A^r_\pi = (\varepsilon^2(\omega)\cos^2\vartheta - a_0^2)(e^{-i\omega a_0 L} - e^{i\omega a_0 L}) g_1;$$

$$B_\pi = 2\cos\vartheta(\varepsilon(\omega)\cos\vartheta + a_0) g_1 = 2G_1;$$

$$B^r_\pi = 2\cos\vartheta(a_0 - \varepsilon\cos\vartheta) g_1; \;\; D_\pi = 4a_0 \cos\vartheta \varepsilon g_1.$$

4.3 Angular Distribution of Radiation Produced by Particles in a Crystal under Refraction

Let us give a more detailed treatment of $dN/d\Omega$ in the case of radiation in a crystal whose thickness exceeds the photon absorption length in a medium, i.e., assume that the condition $\omega \mathtt{Im}\, n(\omega) L \gg 1$ is satisfied. In this case in order to integrate the cross-section (4.4) with respect to frequencies, we shall make use of the fact that the function $1/|q_{znf}|^2$ has a sharp maximum in the vicinity of the point ω_v, where $\mathtt{Re}\, q_{znf} = 0$. At the same time other terms appearing in (4.4) change smoothly with the change in the photon frequency. Therefore the function before $|q_{znf}|^{-2}$ may be factored outside the integral sign at the maximum point $\omega = \omega_\vartheta$.

Expand $\mathtt{Re}\, q_z$ into a series in the vicinity of the point $\omega = \omega_\vartheta$:

$$\mathtt{Re}\, q_z \simeq \frac{d(\mathtt{Re}\, q_z)}{d\omega}(\omega - \omega_\vartheta) + \frac{1}{2}\frac{d^2(\mathtt{Re}\, q_z)}{d\omega^2}(\omega - \omega_\vartheta)^2 \tag{4.23}$$

and expand the limits of integration to the infinite interval. As a result, the angular distribution of photons emitted by a channeled particle is written as follows:

$$\begin{aligned}\frac{dN}{d\Omega} &= \sum_{nfa} Q_{nn}|x_{nf}|^2 \frac{e^2 l_{abs}(\omega_\vartheta^{(a)})\Omega_{nf}^3}{2\pi} \\ &\times \frac{[1-\beta n'(\omega_\vartheta^{(a)})\cos\vartheta]^2 - [1-\beta^2 n'^2(\omega_\vartheta^{(a)})]\sin^2\vartheta\cos^2\varphi}{[1-\beta n'(\omega_\vartheta^{(a)})\cos\vartheta]^4} \\ &\times \Delta_{nf}(\omega_\vartheta^{(a)})\eta_a(\omega_1,\omega_2)\,,\end{aligned} \tag{4.24}$$

where $l_{\rm abs}(\omega_\vartheta^{(a)})$ is the absorption length of the photon with the frequency $\omega_\vartheta^{(a)}$.

Summation over (a) indicates summation over all possible solutions of equation $\mathtt{Re}\, q_z(\omega) = 0$ in the spectral range of the detector $[\omega_1, \omega_2]$. The term Δ_{nf} is due to the dispersion of the medium, and it has the form

$$\begin{aligned}\Delta_{nf}(\omega_\vartheta^{(a)}) &= \frac{1-\beta n(\omega_\vartheta^{(a)})\cos\vartheta}{\left|1-\beta n(\omega_\vartheta^{(a)})\cos\vartheta - \omega_\vartheta^{(a)}\beta\cos\vartheta\left(\frac{\partial n(\omega)}{\partial\omega}\right)_{\omega=\omega_\vartheta^{(a)}}\right|} \\ &= \left|1 - \frac{(\omega_\vartheta^{(a)})^2\beta\cos\vartheta}{\Omega_{nf}}\left(\frac{\partial n(\omega)}{\partial\omega}\right)_{\omega=\omega_\vartheta^{(a)}}\right|^{-1}.\end{aligned} \tag{4.25}$$

The angular distribution of photons for which $\omega(n'-1)L \gg 1$, $l_{\rm abs} < L$ is obtained by replacing $l_{\rm abs}$ in (4.24) with the crystal thickness. Due to a particular relationship between the observed frequency and the photon emission angle, the shape of angular distribution depends significantly on the frequency value within the detection range (ω_1, ω_2). The function $\eta_a(\omega_1, \omega_2)$ takes account of this circumstance. For example, if the frequencies ω_1 and ω_2 lie in the X-ray spectrum, where

$$n'(\omega) = 1 - \frac{\omega_L^2}{2\omega^2} \, ,$$

the function $\eta_a(\omega_1, \omega_2)$ may be represented as

$$\eta_a(\omega_1, \omega_2) = \begin{cases} \theta(b_{nf}(\omega_a) - \cos\vartheta)\theta(\cos\vartheta - \cos\vartheta_m) & \text{at} \quad \omega_1 < \omega_0 < \omega_2 \, , \\ \theta(b_{nf}(\omega_1) - \cos\vartheta)\theta(\cos\vartheta - b_{nf}(\omega_2)) & \text{at} \quad \omega_1, \quad \omega_2 > \omega_0 \, , \\ \theta(b_{nf}(\omega_2) - \cos\vartheta)\theta(\cos\vartheta - b_{nf}(\omega_1)) & \text{at} \quad \omega_0 > \omega_1, \ \omega_2 \, , \end{cases} \tag{4.26}$$

here $\omega_0 = \omega_L^2/\Omega_{nf}$. The multiplier

$$\Delta_{nf}(\omega_\vartheta^{(a)}) \simeq \left| 1 - \frac{\omega_L^2}{\omega_\vartheta^{(a)} \Omega_{nf}} \right|^{-1} \, ,$$

$$b_{nf}(\omega) = \frac{1}{\beta} \left(1 - \frac{\Omega_{nf}}{\omega} \right) \, , \tag{4.27}$$

where $\omega_\vartheta^{(a)}$ is defined by the formula

$$\omega_\vartheta^{(a)} = \frac{\Omega_{nf} \pm \sqrt{\Omega_{nf}^2 - 2\omega_L^2(1 - \beta\cos\vartheta)}}{2(1 - \beta\cos\vartheta)} \, . \tag{4.28}$$

According to (4.27), for the radiation angles ϑ, at which

$$\omega_\vartheta^{(a)} \gg \frac{\omega_L^2}{\Omega_{nf}} \, ,$$

the multiplier $\Delta_{nf}(\omega_\vartheta^{(a)})$ may be assumed equal to unity and consequently, the effect of the frequency dispersion of the medium on the angular distribution of quanta may be neglected.

To define the frequency range of the spectrum, where Δ_{nf} being different from unity is of importance, rewrite (4.25) as follows

$$\Delta_{nf} = \left(1 - \frac{\beta\cos\vartheta}{v_{\mathrm{ph}}(\omega_\vartheta^{(a)})} \right) \left(1 - \frac{\beta\cos\vartheta}{W(\omega_\vartheta^{(a)})} \right)^{-1} \, , \tag{4.29}$$

where $v_{\mathrm{ph}}(\omega_\vartheta^{(a)})$ and $W(\omega_\vartheta^{(a)})$ are the phase and group velocities of light in the medium at the frequency $\omega = \omega_\vartheta^{(a)}$.

According to (4.29), the multiplier Δ_{nf} becomes essential for photons with the frequencies, at which the group and phase velocities in a medium are different. This occurs, for example, in the vicinity of the resonances.

The expression for angular distribution (4.24) simplifies if the condition

$$\delta = \frac{2\omega_L^2}{\Omega_{nf}^2}(1 - \beta\cos\vartheta) \ll 1$$

is fulfilled (e.g. for an electron with the energy $E \geq 100\,\mathrm{MeV}$ at $\vartheta = 0$ in a silicon crystal $\delta < 10^{-5}$). Then the root of (4.28) may be decomposed, which gives

$$\omega_\vartheta^{(1)} \simeq \frac{\Omega_{nf}}{1 - \beta\cos\vartheta} \tag{4.30}$$

for the upper frequency radiation mode and

$$\omega_\vartheta^{(2)} \simeq \frac{\omega_L^2}{2\Omega_{nf}}\left(1 + \frac{\omega_L^2}{\Omega_{nf}^2}\left(\frac{1}{\gamma^2} + \vartheta^2\right)\right) \tag{4.31}$$

for the lower one. In view of (4.31), at the lower mode, the observed photon frequency is practically independent of the radiation angle ϑ. As a result, for $\omega_1 < \omega_0 < \omega_2$, the expression for angular distribution takes the form

$$\begin{aligned}\frac{dN}{d\Omega} &= \frac{e^2L}{2\pi}\sum_{nf} Q_{nn}|x_{nf}|^2 \left\{\Omega_{nf}^3\Phi(\vartheta,\,\varphi)\theta\left[\cos\vartheta - b(\omega_0)\right]\theta\left[b(\omega_2) - \cos\vartheta\right]\right.\\ &+\frac{\omega_L^6\beta^2}{8\Omega_{nf}^3}\sin^4\vartheta\cos^2\varphi - \frac{\omega_L^4\beta}{2\Omega_{nf}}\sin^2\vartheta\cos\vartheta\cos^2\varphi\\ &\left.+\Omega_{nf}\omega_L^2(1 - \sin^2\vartheta\cos^2\varphi)\right\}\,,\end{aligned} \tag{4.32}$$

where $\omega_0 = \omega_L^2/\Omega_{nf}$. If $\omega_{1,2} > \omega_0$, the angular distribution of radiation is described by (3.58).

At a certain energy E_{cr} of a channeled particle (or at a fixed particle energy for a limiting radiation angle $\vartheta_{\max}$), being such that the condition

$$\Omega_{nf}^2 - 2\omega_L^2(1 - \beta\cos\vartheta) = 0 \tag{4.33}$$

is fulfilled, the difference between the frequencies $\omega_\vartheta^{(1)}$ and $\omega_\vartheta^{(2)}$ disappears, and at an angle $\vartheta_{\max}$ a γ-quantum with the frequency $\omega_0 = \omega_L^2/\Omega_{nf}$ is emitted (at the angles $\Omega > \Omega_{\max}$, photon emission by channeled particles is impossible).

Under the conditions of one frequency observation the photon group velocity $W(\omega_0)$ is equal to the projection of the particle velocity along the direction of γ-quantum emission $v\cos\vartheta$ [Frank (1942)]. In this case expression (4.24) is not applicable for describing angular distribution, as the first derivative in the expansion of (4.23) to which we confined ourselves when calculating (4.24) vanishes. Therefore, the quadratic expansion terms in (4.23) should be taken into account when integrating the differential cross-section in (4.4) over the frequencies:

$$\mathtt{Re}q_z \simeq -\frac{\beta\cos\vartheta}{2}\left(2\frac{dn(\omega)}{d\omega}+\omega\frac{d^2n(\omega)}{d\omega^2}\right)_{\omega=\omega_\vartheta}(\omega-\omega_\vartheta)^2\,. \tag{4.34}$$

As a result, when the frequency $\omega_0 = \omega_L^2/\Omega_{nf}$ is within the range (ω_1, ω_2), which is the domain of integration of the detector, the number of γ-quanta emitted by a channeled particle over the angular range $\Delta\vartheta$ near the angle $\vartheta = \vartheta_{max}$ is given by the following expression:

$$\frac{dN}{d\varphi} \simeq \frac{e^2\omega_L^3}{\sqrt{2}}\sum_{nf} Q_{nn}|x_{nf}|^2\sqrt{\Omega_{nf}}\,l_{\text{abs}}^{3/2}(\omega_0) \tag{4.35}$$

$$\times\left\{1-\cos\varphi\left(1-\frac{\omega_L^4}{\gamma^4\Omega_{nf}^4}\right)\right\}\theta_{\max}\Delta\theta\,,\quad \theta_{\max}\simeq\left(\frac{\Omega_{nf}^2}{\omega_L^2}-\frac{1}{\gamma^2}\right)^{1/2},$$

where for a lower mode $\Delta\theta \sim \theta_{\max}$
and for an upper mode

$$\theta_{\max}\Delta\theta \sim \frac{\Omega_{nf}}{\omega_L}\sqrt{n''}\,.$$

The characteristic feature of angular distribution near the critical confluence point of the two frequencies is the following dependence of $dN/d\varphi$ on the absorption length: $l_{\text{abs}}^{3/2}$ (compare [Zhevago (1978)]).

4.4 Influence of Diffraction on the Process of Photon Emission in Crystals

Diffraction of produced photons in a crystal gives rise to a new phenomenon: emission of γ-quanta at large angles with respect to the direction of the fast particle motion, and formation of a characteristic diffraction pattern. Two fundamentally different mechanisms contribute to the latter [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d)]: one caused by the deceleration of electrons in a single crystal, being most pronounced in the process of photon emission through radiative transitions between the bands (levels) of transverse energy; the other, occurring even for a particle moving at a constant velocity, is due to scattering of pseudo-photons associated with a particle by atoms and crystal nuclei (the so-called parametric radiation [Baryshevskii (1971); Baryshevskii and Feranchuk (1971, 1973, 1976); Garibyan and Yan Shi (1972); Avakyan *et al.* (1975); Afanas'ev and Aginyan (1978); Feranchuk (1979b)]). The number of photons in the diffraction peak appears to be quite large, which enables obtaining information about the crystal structure directly from the analysis of the frequency and angular photon spectra. At the particle energy exceeding several tens of megaelectron volts the spectral density of radiation caused by a parametric mechanism in the frequency range up to several hundreds of kiloelectron volts, proves to be one or two orders of magnitude higher than density of radiation emerging at radiative transitions between the levels of the particle transverse energy [Baryshevsky and Feranchuk (1980b)].

As mentioned above, to determine the radiation intensity, one should first find the photon wave function $A_{ks}^{(-)}$ under diffraction conditions. If the photon wave length is comparable with a lattice spacing, $A_{ks}^{(-)}$ may be found using the two-wave approximation of the dynamical theory of diffraction. If it is much less than the lattice spacing, the theory developed for the case of electron channeling is applicable (see Section (1.1, 1.2) [Baryshevsky (1979f,e)].

In the two-wave approximation of the dynamical theory of diffraction (see, for example, [Pinsker (1974)]) the wave function $A_{ks}^{(-)}$ may be represented in the general form as follows:

$$A_{ks}^{(-)}(\vec{r}) = \vec{e}_s\Phi(z)e^{i\vec{k}\vec{r}} + \vec{e}_{1s}\Phi_1(z)e^{k_1 r}, \tag{4.36}$$

where $\vec{e}_s$ and $\vec{e}_{1s}$ are the polarization vectors of the direct and diffracted waves satisfying the the transversality condition: $(\vec{e}_s\vec{k}) = (\vec{e}_{1s}\vec{k}_1) = 0$; $\vec{k}_1 = \vec{k} + 2\pi\vec{\tau}$; $s = 1, 2$; $\vec{e}_1 \parallel \vec{e}_{11} \parallel [\vec{k}, 2\pi\vec{\tau}]$; $e_2 \parallel [\vec{k}[\vec{k}, 2\pi\tau]]$; $\vec{e}_{12} \parallel [\vec{k}_1[\vec{k}, 2\pi\vec{\tau}]]$; $2\pi\vec{\tau}$ is the reciprocal lattice vector characterizing the family of planes, where the photon diffraction occurs. Note here that in the general case of diffraction in polarized and magnetically ordered crystals equations (4.36) turns out to be more complicated. Methods of constructing solutions describing such a diffraction see in [Baryshevsky (1976); Belyakov (1975)].

The photon wave functions corresponding to various cases of the Laue and Bragg diffraction only differ by the shape of the amplitudes $\Phi(z)$ and $\Phi_1(z)$:

a. The Bragg case ($k_z > 0,\ k_z + 2\pi\tau_z < 0$):

$$\begin{aligned}A_{ks}^{(-)}(\vec{r}) &= \vec{e}_s[\gamma_{1s}^{0*} + \gamma_{2s}^{0*}]e^{i\vec{k}\vec{r}}e^{-ik_zL}\theta(-z)\\ &+ \left\{\vec{e}_s e^{i\vec{k}\vec{r}}e^{-ik_zL}\left[\gamma_{1s}^{0*}e^{i\frac{\omega}{\gamma_0}\varepsilon_{1s}^{*}z} + \gamma_{2s}^{0*}e^{i\frac{\omega}{\gamma_0}\varepsilon_{2s}^{*}z}\right]\right.\\ &\left.+\vec{e}_{1s}\beta_1\gamma_s^{\tau*}e^{i\vec{k}_1\vec{r}}e^{-ik_zL}\left[e^{i\frac{\omega}{\gamma_0}\varepsilon_{1s}^{*}z} - e^{i\frac{\omega}{\gamma_0}\varepsilon_{2s}^{*}z}\right]\right\}\\ &\times\theta(z)\theta(L-z) + \left\{e_s e^{i\vec{k}\vec{r}}e^{-ik_zL} + \vec{e}_{1s}\beta_1\gamma_s^{\tau*}\right.\\ &\left.\times\left[e^{i\frac{\omega}{\gamma_0}\varepsilon_{1s}^{*}L} + e^{i\frac{\omega}{\gamma_0}\varepsilon_{2s}^{*}L}\right]e^{i\vec{k}_1\vec{r}}e^{-ik_zL}\right\}\theta(z-L),\end{aligned} \tag{4.37}$$

where

$$\gamma_0 = \cos\theta;\ \theta = \frac{\vec{k}\vec{n}_z}{\omega};\ \vec{k}_1 = \vec{k} + 2\pi\vec{\tau};$$

$$\gamma_{1,2s}^{0} = \pm\frac{2\varepsilon_{2,1s} - g_{00}}{\Delta_s^{*}};\ \gamma_s^{\tau} = \frac{g_{10}^{s}}{\Delta_s^{*}};$$

$$\Delta_s = (2\varepsilon_{2s}^{*} - g_{00}^{*})e^{i\frac{\omega}{\gamma_0}\varepsilon_{1s}^{*}L} - (2\varepsilon_{1s}^{*} - g_{00}^{*})e^{i\frac{\omega}{\gamma}\varepsilon_{2s}^{*}L};$$

$$\varepsilon_{1,2s} = \frac{1}{4}\left\{g_{00} + \beta_1 g_{11} - \beta_1\alpha \pm\sqrt{(g_{00} + \beta_1 g_{11} - \alpha\beta_1)^2 - 4\beta_1(\alpha g_{00} - g_{00}g_{11} + g_{10}^{s}g_{01}^{s})}\right\};$$

$$\beta_1 = \frac{k_z}{k_z + 2\pi\tau_z} = \frac{k_z}{k_{1z}};\ \alpha = \frac{2(\vec{k}2\pi\vec{\tau}) + (2\pi\vec{\tau})^2}{\omega^2};$$

the quantities g_{ik}^{s} are determined by the expansion of the dielectric permittivity of a crystal into a series in terms of reciprocal lattice vectors. The crystal dielectric permittivity is a periodic function of the position of nuclei and atoms;

b. the Bragg case ($k_z < 0,\ k_z + 2\pi\tau_z > 0$):

$$\begin{aligned}A_{ks}^{(-)}(\vec{r}) &= \left\{\vec{e}_s e^{i\vec{k}\vec{r}} - \vec{e}_{1s}\beta_1\gamma_s^{\tau*}e^{i\vec{k}_1\vec{r}}\right.\\ &\left.\times\left[e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{2s}^{*}L} - e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{1s}^{*}L}\right]\right\}\theta(-z)\\ &+ \left\{\vec{e}_s e^{i\vec{k}\vec{r}}\left[\gamma_{1s}^{0*}e^{-i\frac{\omega}{|\gamma_0|}(\varepsilon_{2s}^{*}L + \varepsilon_{1s}^{*}z)} + \gamma_{2s}^{0*}e^{-i\frac{\omega}{|\gamma_0|}(\varepsilon_{1s}^{*}L + \varepsilon_{2s}^{*}z)}\right]\right.\\ &\left.-\vec{e}_{1s}\beta_1\gamma_s^{\tau*}e^{i\vec{k}_1 r}\left[e^{-i\frac{\omega}{|\gamma_0|}(\varepsilon_{2s}^{*}L + \varepsilon_{1s}^{*}z)} - e^{-i\frac{\omega}{|\gamma_0|}(\varepsilon_{1s}^{*}L + \varepsilon_{2s}^{*}z)}\right]\right\}\\ &\times\theta(z)\theta(L-z) + \vec{e}_s e^{i\vec{k}\vec{r}}\left[\gamma_{1s}^{0*}e^{-i\frac{\omega}{|\gamma_0|}(\varepsilon_{2s}^{*} + \varepsilon_{1s}^{*})L}\right.\\ &\left.+\gamma_{2s}^{0*}e^{-i\frac{\omega}{|\gamma_0|}(\varepsilon_{1s}^{*} + \varepsilon_{2s}^{*})L}\right]e^{-ik_zL}\theta(z-L);\end{aligned} \tag{4.38}$$

c. the Laue case ($k_z > 0$, $k_z + 2\pi\tau_z > 0$):

$$A_{ks}^{(-)}(\vec{r}) = \left\{\vec{e}_s e^{i\vec{k}\vec{r}}\left[-\xi_{1s}^{0*} e^{-i\frac{\omega}{\gamma_0}\varepsilon_{1s}^* L} - \xi_{2s}^{0*} e^{-i\frac{\omega}{\gamma_0}\varepsilon_{2s}^* L}\right]\right.$$
$$\left. + \vec{e}_{1s} e^{i\vec{k}_1\vec{r}}\beta_1\left[\xi_{1s}^{\tau*} e^{-i\frac{\omega}{\gamma_0}\varepsilon_{1s}^* L} + \xi_{2s}^{\tau*} e^{-i\frac{\omega}{\gamma_0}\varepsilon_{2s}^* L}\right]\right\}\theta(-z)$$
$$+ \left\{\vec{e}_s e^{i\vec{k}\vec{r}}\left[-\xi_{1s}^{0*} e^{-i\frac{\omega}{\gamma_0}\varepsilon_{1s}^*(L-z)} - \xi_{2s}^{0*} e^{-i\frac{\omega}{\gamma_0}\varepsilon_{2s}^*(L-z)}\right]\right.$$
$$\left. + \vec{e}_{1s}\beta_1 e^{i\vec{k}_1\vec{r}}\left[\xi_{1s}^{\tau*} e^{-i\frac{\omega}{\gamma_0}\varepsilon_{1s}^*(L-z)} + \xi_{2s}^{\tau*} e^{-i\frac{\omega}{\gamma_0}\varepsilon_{2s}^*(L-z)}\right]\right\}$$
$$\times\theta(z)\theta(L-z) + \vec{e}_s e^{i\vec{k}\vec{r}} e^{-ik_z L}\theta(z-L), \qquad (4.39)$$

where

$$\xi_{1,2s}^0 = \mp\frac{2\varepsilon_{2,1s} - g_{00}}{2(\varepsilon_{2s} - \varepsilon_{1s})};\ \xi_{1,2s}^{\tau} = \mp\frac{g_{01}^s}{2(\varepsilon_{2s} - \varepsilon_{1s})};$$

d. the Laue case ($k_z < 0$, $k_z + 2\pi\tau_z < 0$):

$$A_{ks}^{(-)}(\vec{r}) = \vec{e}_s e^{i\vec{k}\vec{r}}\theta(-z) + \left\{\vec{e}_s e^{i\vec{k}\vec{r}}\left[-\xi_{1s}^{0*} e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{1s}^* z}\right.\right.$$
$$\left. - \xi_{2s}^{0*} e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{2s}^* z}\right] + \vec{e}_{1s}\beta_1 e^{i\vec{k}_1\vec{r}}\left[\xi_{1s}^{\tau*} e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{1s}^* z}\right.$$
$$\left.\left. - \xi_{2s}^{\tau*} e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{2s}^* z}\right]\right\}\theta(z)\theta(L-z)$$
$$+ \left\{\vec{e}_s^{i\vec{k}\vec{r}}\left[-\xi_{1s}^{0*} e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{1s}^* L} - \xi_{2s}^{0*} e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{2s}^* L}\right]\right.$$
$$+ \vec{e}_{1s}\beta_1 e^{i\vec{k}_1\vec{r}} e^{i2\pi\tau_z L}\left[\xi_{1s}^{\tau*} e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{1s}^* L}\right.$$
$$\left.\left. + \xi_{1s}^{\tau*} e^{-i\frac{\omega}{|\gamma_0|}\varepsilon_{2s}^* L}\right]\right\}\theta(z-L). \qquad (4.40)$$

4.5 Spectral-Angular Distribution in the Bragg and Laue Cases

Consider the influence of diffraction on spectral-angular distribution of photons emitted by a particle passing through a crystal. The particle enters the crystal at a certain small angle with respect to the z-axis directed perpendicular to the crystal surface.

Let, for example, a photon be diffracted by a family of crystallographic planes described by the reciprocal lattice vector $2\pi\vec{\tau}$, which is directed antiparallel to the z-axis, i.e., $2\pi\tau_x = 2\pi\tau_y = 0$, $2\pi\tau_z < 0$. In this case under diffraction conditions $k_z + 2\pi\tau_z < 0$; according the Wulff-Bragg condition, the anomalies in the photon spectrum should be expected at the frequencies $\omega = \frac{\pi n}{a}$ (a is the lattice spacing along the z-axis; $n = 1, 2...$).

In the case in question (*the Bragg case a.*) the photon wave function has the form (4.37). Its substitution into the general expression for the radiation cross-section enables us to find the explicit form of the cross-section (see [Baryshevsky *et al.* (1978)], formula (8)) (the photon is emitted at a small angle with respect to the particle momentum.):

$$\frac{d^2N_s}{d\omega d\Omega} = \frac{e^2\omega}{\pi^2}\sum_{nf} Q_{nn}|\vec{g}_{nf}\vec{e}_s^{\,*}|^2 \times \left|\sum_{\mu=1,2}\gamma_{\mu s}^{(0)} l_{nf}^{\mu s}(1-e^{-iL/l_{nf}^{\mu s}})\right|^2, \tag{4.41}$$

where $l_{nf}^{\mu s} = (q_{znf}^{\mu s})^{-1}$ is the coherent length; $q_{znf}^{\mu s} = p_{zn} - p_{1zf} - k_z - \omega\varepsilon_{\mu s}$. The cross-section will take on its maximum value for all the frequencies satisfying the inequality

$$\mathtt{Re}\, q_{znf}^{\mu s} \equiv \frac{\omega}{2}\left(\frac{m^2}{E^2}+\theta^2\right) - (\varepsilon'_{n\kappa} - \varepsilon'_{f\kappa_1}) - \omega\mathtt{Re}\varepsilon_{\mu s} \leq \omega\mathtt{Im}\,\varepsilon_{\mu s}. \tag{4.42}$$

Using (4.42), the spectrum can be written as

$$\omega_{nf}^{\mu s} = \frac{(\varepsilon'_{n\kappa} - \varepsilon'_{f\kappa_1}) + |\varepsilon|\delta_{\mu s}}{\frac{1}{2}\left(\frac{m^2}{E^2}+\vartheta^2\right) - \mathtt{Re}\,\varepsilon_{\mu s}}, \tag{4.43}$$

where $\delta_{\mu s} = \omega_{nf}^{\mu s}\mathtt{Im}\,\varepsilon_{\mu s}$; $|\varepsilon| \leq 1$.

Consider *the Bragg case b.*. Now the emitted photons can fly into the left half-plane from the crystal target. The diffraction pattern obtained coincides with that produced by a polychromatic beam of photons incident along the z-axis with the opening angle $\vartheta \sim m/E$.

The Laue case d. The analysis shows that the radiation intensity is sharply suppressed, as none of the coherent lengths can become large.

The Laue case c. Spectral-angular distribution for the number of photons escaping (outcoming) at a large angle with respect to the direction of particle motion has the form [Baryshevsky *et al.* (1980a); Baryshevsky *et al.* (1980c)]

$$\frac{d^2N_s}{d\omega d\Omega} = \frac{e^2\beta_1^2\omega}{\pi^2}\sum_{nf} Q_{nn}|\vec{e}_{1s}^{\,*}\vec{g}_{\vec{n}f}|^2 \times \left|\sum_{\mu=1,2}\xi_{\mu s}^{\tau}\frac{1-e^{-iq_{znf}^{\mu s}L}}{q_{znf}^{\mu s}}\right|^2. \tag{4.44}$$

The quantity (4.44) attains the maximum value at the minimum $q^{\mu s}_{znf}$. As $q^{\mu s}_{znf}$ is the complex value the minimum value is limited by the imaginary part. The inequality

$$\begin{aligned}\mathrm{Re}\, q^{\mu s}_{znf} &= p_{zn} - p_{1zf} - (k_z + 2\pi\tau_z) \\ &-\frac{\omega}{\gamma_1}\mathrm{Re}\,\varepsilon_{\mu s} \le \frac{\omega}{\gamma_1}\mathrm{Im}\,\varepsilon_{\mu s}\end{aligned} \tag{4.45}$$

leads to the relation between ω and the emission angle of the photons, and, thus determining the photon spectrum. It should be emphasized that, due to the effect of anomalous transmission, the imaginary part of q_{znf} in the case of the Laue diffraction may become anomalously small, which results in an appreciable increase in the radiation intensity of γ-quanta as compared to the case of the absence of diffraction.

Pay attention to the fact that upon introducing the notation $k_{1z} = k_z + 2\pi\tau_z$, inequality (4.45) takes the form analogous to that of the longitudinal momentum at the emission of a photon with the wave vector $\vec{k}_1$.

It is common knowledge that at photon emission by fast particles, the photon emission angle is small. Hence, the angle that vector $\vec{k}_1$ makes with the direction of particle motion is small too. From this follows that a large emission angle is exhibited by a photon whose wave vector $\vec{k}$ is such that together with vector $2\pi\vec{\tau}$ it sums up into vector $\vec{k}_1$, which makes a small angle with the direction of the particle momentum. As a result, the analysis of kinematics is perfectly analogous to the case of emission at a small angle ϑ.

Expression (4.44) can be simplified considerably at $\mathrm{Re}\,\varepsilon_{\mu s} \gg \mathrm{Im}\,\varepsilon_{\mu s}$ and the crystal thickness small as compared to the absorption depth, or much greater than absorption depth for a γ-quantum. Using in the former case the relation $\left|\frac{1-e^{-iqL}}{q}\right|^2 \simeq 2\pi L\delta(q)$, we may integrate (4.44) with respect to, for example, frequencies and obtain the photon angular distribution. As diffraction is most pronounced within the range of photon wave lengths $\lambda \sim 10^{-8} - 10^{-9}$ cm, and the angle of $\vec{k} + 2\pi\vec{\tau}$ with $\vec{p}$ is small, then $(\vec{k} + 2\pi\vec{\tau})_x a \ll 1$, and in $\vec{I}_{nf}$ and I_{1nf} we may expand the exponents into a series [Baryshevskii and Dubovskaya (1977d); Baryshevsky *et al.* (1978)]. Confining ourselves to the first nonzero terms, we can obtain, for example for planar channeling, the following expression for the angular distribution of γ-quanta emitted at a large angle to the polarization plane $\vec{e}_{11}$ perpendicular to the diffraction plane [Baryshevsky *et al.* (1980a);

Baryshevsky *et al.* (1980c)]:

$$\frac{dN_{11}^{\tau}}{d\Omega} = \frac{e^2 L}{2\pi} \sum_{nf} Q_{nn} |x_{nf}|^2 \sum_{\mu} \frac{(\omega_{nf}^{\mu 1})^2}{\Omega_{nf}} \beta_1^2 |\xi_{\mu 1}^{\tau}(\omega_{nf}^{\mu 1})|^2$$
$$\times \left[1 - \frac{(\omega_{nf}^{\mu 1})^2}{\gamma_1 \Omega_{nf}} \mathtt{Re} \left(\frac{\partial \varepsilon \mu_1}{\partial \omega}\right)_{\omega = \omega_{nf}^{\mu 1}}\right]^{-1}$$
$$\times \left[\beta_1 \omega_{nf}^{\mu 1} \sin^2\theta \cos\varphi \frac{\tau_y \cos\varphi - \tau_x \sin\varphi}{|\tau_\perp|}\right.$$
$$\left. + \Omega_{nf} \frac{\tau_z \sin\theta \sin\varphi - \tau_y \cos\theta}{|\tau_\perp|}\right]^2, \qquad (4.46)$$

where $\omega_{nf}^{\mu 1} = \Omega_{nf}(1 - \beta\cos\theta - \gamma_1^{-1} \mathtt{Re}\, \varepsilon_{\mu 1}(\omega_{nf}^{\mu 1}))^{-1}$; θ is the angle of $\vec{k} + 2\pi\vec{\tau}$ with the z-axis.

The angular distribution of γ-quanta emitted with the same polarization at a small angle with respect to the particle momentum is described by the same expression (4.46), where $\xi^{\tau} \to \xi^0$, $\gamma_1 \to \gamma_0$, $\beta_1 = 1$, $\theta = \vartheta$ is the photon emission angle. If the polarization of γ-quanta is $\vec{e}_2$, i.e., it lies in the diffraction plane, then their angular distribution is obtained by additional replacement of $\tau_y \to \tau_x$, $\tau_x \to -\tau_y$ in the augend of (4.46), and $tau_y \cos\theta \to \tau_x$ in the addend.

Angular distribution of γ-quanta emitted at a large angle with the polarization $\vec{e}_{12}$ lying in the diffraction plane differs from (4.46) by lengthy terms of the order of unity and has the form

$$\frac{dN_{12}^{\tau}}{d\Omega} = \frac{e^2 L \beta_1^2}{2\pi} \sum_{nf} Q_{nn} |x_{nf}|^2 \sum_{\mu} \frac{(\omega_{nf}^{\mu 2})^2}{\Omega_{nf}} |\xi_{\mu 2}^{\tau}(\omega_{nf}^{\mu 2})|^2$$
$$\times \left[1 - \frac{(\omega_{nf}^{\mu 2})^2}{\gamma_1 \Omega_{nf}} \mathtt{Re} \left(\frac{\partial \varepsilon \mu_2}{\partial \omega}\right)_{\omega = \omega_{nf}^{\mu 2}}\right]^{-1}$$
$$\left\{\frac{\beta_1 \sin\theta \cos\varphi [\cos\theta (\vec{n}_1 \vec{\tau}) - \tau_z] \omega_{nf}^{\mu 2}}{|\tau_\perp|} + \Omega_{nf} \frac{[\sin^2\theta \cos\varphi (\vec{n}_1 \vec{\tau}) - \tau_x]}{|\tau_\perp|}\right\}^2 \qquad (4.47)$$

where $\vec{n}_1 = \frac{\vec{k} + 2\pi\vec{\tau}}{|\vec{k} + 2\pi\vec{\tau}|}$.

The derived expressions for angular distribution of radiation simplify considerably, if the particle energy is such, that $1 - \beta \gg \frac{1}{\gamma_1} \mathtt{Re} \varepsilon_{\mu s}$. In this case we may assume that the frequency corresponding to the radiation maximum is very likely to be independent of dielectric properties of a crystal, being determined only by the radiation angle and the frequency of the corresponding transition, i.e.,

$$\omega_{nf}^{\mu s} \simeq \omega_{nf} = \frac{\Omega_{nf}}{1-\beta\cos\vartheta}. \tag{4.48}$$

As a result, for example, angular distributions under diffraction conditions in the Laue case will be recast as follows

1. For radiation at a large angle with respect to the direction of the particle motion

$$\frac{dN_s^\tau}{d\Omega} = \frac{e^2L}{2\pi}\beta_1^2\sum_{nf} Q_{nn}|x_{nf}|^2\Omega_{nf}^3 R_s^\tau(\theta,\varphi)B_s^\tau(\omega_{nf}) \tag{4.49}$$

(recall that s means photon polarization which may be of two types: σ-polarization $\vec{e}_\sigma^{\,\tau} \parallel [\vec{k}, 2\pi\vec{\tau}]$ and π-polarization $\vec{e}_\pi^{\,\tau} \parallel [\vec{k}_1[\vec{k}, 2\pi\vec{\tau}]]$), where

$$R_\sigma^\tau(\theta,\varphi) = \left[\frac{\beta\sin^2\theta\cos\varphi(\tau_y\cos\varphi-\tau_x\sin\varphi)}{(1-\beta\cos\vartheta)^2\sqrt{\tau^2-(\vec{n}_1\vec{\tau})^2}} + \frac{(\tau_z\sin\theta\sin\varphi-\tau_y\cos\theta)}{(1-\beta\cos\theta)\sqrt{\tau^2-(\vec{n}_1\vec{\tau})^2}}\right]^2;$$

$$R_\sigma^\tau(\theta,\varphi) = \left[\frac{\beta\sin\theta\cos\varphi[\cos\theta(\vec{n}_1\vec{\tau})-\tau_z]}{(1-\beta\cos\theta)^2\sqrt{\tau^2-(\vec{n}_1\vec{\tau})^2}} + \frac{[\sin\theta\cos\varphi(\vec{n}_1\vec{\tau})-\tau_x]}{(1-\beta\cos\theta)^2\sqrt{\tau^2-(\vec{n}_1\vec{\tau})^2}}\right]^2;$$

$$\vec{n}_1 = \frac{\vec{k}+2\pi\vec{\tau}}{|\vec{k}+2\pi\vec{\tau}|};\ \ B_s^\tau(\omega_{nf}) \equiv \sum_\mu |\xi_{\mu s}^\tau(\omega_{nf})|^2; \tag{4.50}$$

θ is the angle of vector $\vec{k}+2\pi\vec{\tau}$ with the z-axis, φ is the polar angle in the xy plane.

2. For radiation along the direction of particle motion

$$\frac{dN_s^0}{d\Omega} = \frac{e^2L}{2\pi}\sum_{nf} Q_{nn}|x_{nf}|^2\Omega_{nf}^3 R_s^0(\theta,\varphi)B_s^0(\omega_{nf}) \tag{4.51}$$

where

$$R_\sigma^0(\vartheta,\varphi) = \left[\frac{\beta\sin^2\vartheta\cos\varphi(\tau_y\cos\varphi-\tau_x\sin\varphi)}{(1-\beta\cos\vartheta)^2\sqrt{\tau^2-(\vec{n}\vec{\tau})^2}} + \frac{\tau_z\sin\vartheta\sin\varphi-\tau_y\cos\vartheta}{(1-\beta\cos\vartheta)\sqrt{\tau^2-(\vec{n}\vec{\tau})^2}}\right]^2;$$

$$R_\sigma^0(\vartheta,\varphi) = \left[\frac{\sin^2\vartheta\cos\varphi(\tau_x\cos\varphi + \tau_y\sin\varphi)}{(1-\beta\cos\vartheta)^2\sqrt{\tau^2-(\vec{n}\vec{\tau})^2}}\right.$$

$$\left. + \frac{\tau_z\sin\vartheta\cos\varphi - \tau_x}{(1-\beta\cos\vartheta)\sqrt{\tau^2-(\vec{n}\vec{\tau})^2}}\right]^2;$$

$$B_s^0(\omega_{nf}) \equiv \sum_\mu |\xi_{\mu s}^0(\omega_{nf})|^2;\ \vec{n} = \frac{\vec{k}}{|\vec{k}|}$$

(ϑ is the angle of vector $\vec{k}$ with the z-axis; φ is the polar angle in the xy plane).

Approximate integral expressions for the number of γ-quanta emitted at a large angle with respect to the direction of particle motion within the diffraction peak may be found, using the fact that the frequency of the photon produced and the position of the diffraction peak are determined, on the one hand, by the Bragg condition, and, on the other hand, by the laws of conservation in emission for a corresponding transition Ω_{nf}. As a result, we obtain the following expressions for the number of γ-quanta emitted within the diffraction peak at a large angle with respect to the direction of particle motion:

a. for π-polarization

$$\Delta N_\pi^\tau \approx \frac{\pi e^2 L\beta_1^2\tau^4|g_{00}'|}{8|\tau_z|^3}\sum_{nf} Q_{nn}|x_{nf}|^2\Omega_{nf}^2$$

$$\times\left\{\frac{\tau_x^2}{\tau_\perp^2} + \frac{\pi\tau^2|\tau_z|\Omega_{nf}}{\tau_\perp^2\tilde{\omega}_{nf}^2}\left(1-\frac{\pi\tau^2}{|\tau_z|\tilde{\omega}_{nf}}\right)\right\},\ \tilde{\omega}_{nf} = 2\Omega_{nf}\gamma^2; \tag{4.52}$$

b. for σ-polarization

$$\Delta N_\sigma^\tau \approx \frac{\pi e^2\beta_1^2 L\tau^4|g_{00}'|}{8|\tau_z|^3}\sum_{nf} Q_{nn}|x_{nf}|^2\Omega_{nf}^2$$

$$\times\left\{1-\frac{\pi\tau^2(\tau_x^2-\tau_y^2)}{|\tau_z|\tau_\perp^2\tilde{\omega}_{nf}}\right\}. \tag{4.53}$$

To estimate the number of quanta emitted within the diffraction peak, note that in the order of magnitude expression (4.49) can be represented as the product of the spectrum of photon emission by a channeled particle without regard to diffraction into the function $|\xi|^2$ characterizing reflection of photons by a crystal in the presence of diffraction. The value of the stated function is close to unity under the fulfillment of the Bragg conditions in the range of angles $\delta\vartheta \sim \varepsilon_{\mu s}$, close to the Bragg ones, i.e., in the range $\delta\vartheta \sim 10^{-6}$ rad, vanishing rapidly at great deviation from the diffraction condition. Hence, the number of quanta emitted within the diffraction peak is of the same order of magnitude as that emitted without diffraction in the range of angles $\delta\vartheta \sim 10^{-6}$ rad near the intensity maximum. As follows from the estimations [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d); Baryshevsky *et al.* (1978, 1980a, 1979); Baryshevsky *et al.* (1980c)] (see Section (3.3), depending on the energy and the type of matter, $10^{-8}E/m$ quanta (it is assumed that $m/E > \delta\vartheta$) will be emitted in the range $\delta\vartheta \sim 10^{-6}$ rad over the crystal thickness $L \sim 10^{-2}$ cm. From this follows that at the current of 10^{-6} A and the energy of, for example, 50 MeV one should expect emission of about 10^7 quanta/sec, which appreciably exceeds the intensity of conventional X-ray sources for the same angular and spectral ranges.

The above formulae also hold true in the case when the crystal thickness is much greater than the absorption depth of quanta, if in expressions (4.46)-(4.51) the thickness L is understood as the quantum absorption depth

$$L(\omega_{nf}^{\mu s}) = \gamma_{1(0)}[2\omega_{nf}^{\mu s}\mathtt{Im}\varepsilon_{\mu s}(\omega_{nf}^{\mu s})]^{-1},$$

where the subscripts 1(0) refer to radiation at a large (small) angle with respect to the direction of particle motion, respectively.

Thus, radiation produced through radiative transitions between the levels of transverse motion of a channeled particle, form behind a crystal a diffraction pattern which can be decoded by means of the methods applied in X-ray structural analysis.

4.6 Radiation Spectrum in the Quasi-classical Approximation

Due to a large relativistic mass, transverse motion of ultra-relativistic channeled particles is quasi-classical for a vast majority of levels. This makes it possible to apply particle wave functions in the quasi-classical approximation to calculate the matrix elements x_{nf} appearing in (4.46)-(4.51) [Feranchuk (1979a); Baryshevsky *et al.* (1980a); Baryshevsky *et al.* (1980c,d,b)]. The sum over a appearing in (4.46)-(4.50) is split into two sums relating to: 1. over-barrier states and 2. sub-barrier states [Baryshevsky and Dubovskaya (1976a); Baryshevskii and Dubovskaya (1977d); Baryshevsky *et al.* (1980a)].

The Bloch functions of sub-barrier states which are not located near the the barrier top may be taken in the tight binding approximation with quasiclassical functions in a well. For example, for planar channeling we may write

$$\psi_{n\kappa}(x) = c_n \frac{1}{\sqrt{p_n(x)}} \cos\left(\int_{x_n}^{x} p_n(x')dx' - \frac{\pi}{4}\right) \tag{4.54}$$

with the quantization condition $\int_{x_n}^{a-x_n} p_n(x')dx' = \pi(n+\frac{1}{2})$, where $p_n(x) = \{2E[\varepsilon_n' - V(x)]\}^{1/2}$; $V(x) = V(x+a)$ is the one–dimensional periodic in x potential of crystal planes; x_n is the turning point in the well for n level; $c_n^2 = 4E(T_{\text{sub}}^n)^{-1}$; T_{sub}^n is the period of particle motion in the well:

$$T_{\text{sub}}^n = 2E\int_{x_n}^{a-x_n} p_n^{-1}(x')dx'.$$

The Bloch functions of the over-barrier states in this approximation may be written as follows

$$\psi_{n\kappa}(x) = \tilde{c}_n \frac{1}{\sqrt{p_n(x)}} e^{i\int_0^x p_n(x')dx'} \tag{4.55}$$

with the quantization condition $\int_0^a p_n(x')dx' = \kappa a + 2\pi n$, where $\tilde{c}_n = E(N_x T_{\text{over}}^n)^{-1}$; T_{over}^n is the period of the over-barrier motion:

$$T_{\text{over}}^n = E\int_0^a p_n^{-1}(x')dx'.$$

Using the stated wave functions, one may obtain the following expressions for the occupation coefficients:

$$Q_{nn} = 2\pi(aT_{\text{sub}}^n|V'(x_0)|)^{-1} \quad \text{for sub-barrier states}, \tag{4.56}$$

$$Q_{nn} = 2\pi(aT_{\text{over}}^n|V'(x_0)|)^{-1} \quad \text{for over-barrier states}. \tag{4.57}$$

The point x_0 is found from the condition $\varepsilon_n' = \frac{p_x^2}{2E} + V(x_0)$.

As a result we have, for example, in the case when $\frac{n-f}{n} \ll 1$: for sub-barrier transitions

$$x_{nf} = \frac{2E}{T_{\text{sub}}^n}\int_{x_n}^{a-x_n} xp_n^{-1}(x)\cos\left(\Omega_{nf}^{\text{sub}}\int_{x_n}^{x} p_n^{-1}(x')Edx'\right)dx,$$
$$\Omega_{nf}^{\text{sub}} = \frac{2\pi(n-f)}{T_{\text{sub}}^n}, \tag{4.58}$$

for over-barrier transitions

$$x_{nf} = \frac{E}{T^n_{\text{sub}}} \int_0^a p_n^{-1}(x) x \exp\left\{ i\Omega^{\text{over}}_{nf} \int_0^x p_n^{-1}(x') E dx' \right\} dx,$$
$$\Omega^{\text{over}}_{nf} = \frac{2\pi(n-f)}{T^n_{\text{over}}}. \tag{4.59}$$

Using the expressions derived for x_{nf}, it is possible to demonstrate directly that formulas of the type (3.43) calculated in the quasiclassical approximation for the transitions

$$\frac{n-f}{n} \ll 1,$$

coincide with similar expressions obtained by means of classical electrodynamics calculations.

Consider in more detail the spectrum of forward radiation in (3.56) in the case when refraction and absorption can be neglected. From (3.56) follows that in the dipole approximation the spectral distribution of radiation intensity in the absence of absorption and refraction may be written as follows

$$\frac{dw_\omega}{d\omega} = Le^2\omega \sum_{nf} Q_{nn} |x_{nf}|^2 \tag{4.60}$$
$$\times \Omega^2_{nf} \left[1 - \frac{\omega}{\Omega_{nf}}(1-\beta^2) + \frac{\omega^2}{2\Omega^2_{nf}}(1-\beta^2)^2 \right] \theta\left[\frac{\vartheta_k^2}{2} - \alpha_{nf}(\omega) \right] \theta[\alpha_{nf}(\omega)].$$

If $\vartheta_k = \pi$, then in view of (3.57) we have

$$\frac{dw_\omega}{d\omega} = Le^2\omega \sum_{nf} Q_{nn} |x_{nf}|^2 \tag{4.61}$$
$$\times \Omega^2_{nf} \left\{ 1 - \frac{\omega}{\Omega_{nf}}(1-\beta^2) + \frac{\omega^2}{2\Omega^2_{nf}}(1-\beta^2)^2 \right\} \theta[2 - \alpha_{nf}(\omega)] \theta[\alpha_{nf}(\omega)].$$

Note that in the particular case $Q_{nn} = \delta_{n\bar{n}}$, where $\bar{n}$ belongs to the states lying inside the well, expression (4.62) converts into the expression discussed in [Kumakhov (1977); Zhevago (1978)].

On the other hand, according to [[Landau and Lifshitz (1967)], formula to problem 2 on p. 278] in the case when the particle deviation angle in the field is small in comparison with the radiation angle, we have

$$\frac{dw_\omega}{d\omega} = \frac{e^2\omega}{2\pi} \int\limits_{\frac{\omega}{2}(1-\beta^2)}^{\infty} \frac{|w_{\omega'}|^2}{\omega'^2} \left[1 - \frac{\omega}{\omega'}(1-\beta^2) + \frac{\omega^2}{2\omega'^2}(1-\beta^2)^2 \right] d\omega', \tag{4.62}$$

where $w_{\omega'}$ is the Fourier transform of the particle acceleration, which, due to the periodic character of motion in a transverse plane, is the set of harmonics multiple of $2\pi/T$, where T is the period of classical motion for the given initial conditions. Substitution of $w_{\omega'}$ for the periodic motion in (4.62) gives formula (4.62) at $Q_{nn} = \delta_{nn'}$. Averaging of (4.62) over various initial conditions of motion, which in (4.61) corresponds to summation over n with the weights Q_{nn} leads to the complete coincidence of these formulas. The theory of radiation of channeled particles based on (4.62) is given in [Akhiezer *et al.* (1979)].

Now consider another extreme case, when at particle motion in the field produced by crystallographic axes (planes), the particle deviation angle (which is of the order of the Lindhard angle $\vartheta_\pi = \sqrt{\frac{2V}{E}}$) is much larger than the characteristic angle of the photon emission $\vartheta_\gamma \sim \frac{m}{E}$. Coherent radiation length $l = \frac{2}{\omega}\left(1 - \frac{\omega}{E}\right)\gamma^2$ is small as compared to the spatial period of particle oscillation in a channel. In view of [Landau and Lifshitz (1967)], radiation in the given direction occurs mainly from that part of the classical trajectory of the particle, where the particle velocity is almost parallel to this direction. Along this part, the field acting on the particle may be considered constant, and the part of the trajectory contributing to radiation may be considered a circle. This enables application of the theory of photon emission in uniform circular motion for analyzing the problem. As a result, in view of the problem 1 §77 in [Landau and Lifshitz (1967)], the spectral distribution of radiation intensity has the form

$$\frac{dw_\omega}{d\omega} = -\frac{2e^2\omega}{\sqrt{\pi}\gamma^2}\int\limits_{-\infty}^{+\infty}\left[\frac{\Phi'(u)}{u} + \frac{1}{2}\int\limits_{u}^{\infty}\Phi(u')du'\right]dt\,, \tag{4.63}$$

where $\Phi(u)$ is the Airy function of argument

$$u = \left[\frac{m\omega}{e\mathcal{E}(\vec{r}(t))\gamma^2}\right]^{2/3},$$

$\mathcal{E}(\vec{r}(t))$ in our case is the magnitude of the electric field strength at the particle location point.

Next consider planar channeling $\mathcal{E}(\vec{r}(t)) = \mathcal{E}(x(t))$. Change the variables

$$dt = \frac{dx}{v(x, x_0)}\,,$$

where

$$v(x, x_0) = \sqrt{\frac{2}{E}(E_\perp(x_0) - V(x))}$$

is the velocity in the transverse plane of the particle entering the channel at point x_0;

$$E_\perp(x_0) = E\frac{\vartheta^2}{2} + V(x_0)$$

where ϑ is the particle angle of incidence with respect to the chosen family of crystallographic planes; $E = m\gamma$ is the energy of the particle entering the crystal.

Take into account that the particle motion in a periodic potential is periodic. The time of particle motion from the left turning point to the right one (see Figure (1.2))

$$\tau(x_0) = \int\limits_{x_1(x_0)}^{a-x_1(x_0)} \frac{dx}{v(x,\, x_0)}\,,$$

$x_1(x_0)$ is determined from the equation $E_\perp(x_0) = V(x_1(x_0))$.

If $E_\perp(x_0)$ is greater than the maximum value of V, then $x_1(x_0) = 0$. Hence, the entire integral over t may be represented as a sum of $L/\tau(x_0)$ identical integrals, i.e., (4.63) can be written as follows:

$$\frac{dw_\omega(\vartheta, x_0)}{d\omega} = -\frac{2e^2\omega}{\sqrt{\pi}\gamma^2}\frac{L}{\tau(x_0)} \times \int\limits_{x_1(x_0)}^{a-x_1(x_0)} \left[\frac{\Phi'(u)}{u} + \frac{1}{2}\int_u^\infty \Phi(u')du'\right]\frac{dx}{v(x,\, x_0)}\,. \tag{4.64}$$

Upon averaging (4.64) over the points of entrance and initial angular distribution of the incident particle, we obtain

$$\frac{dw_\omega}{d\omega} = \frac{1}{a}\int f(\vartheta)d(\vartheta)\int_0^a dx_0 \frac{dw_\omega(\vartheta, x_0)}{d\omega}\,. \tag{4.65}$$

Equation (4.63) is derived using the methods of classical electrodynamics, so it is valid for describing the spectrum of soft photons with the energy $\omega \ll E$ (but one should bear in mind that the coherence length l should be less than the characteristic spatial period of the trajectory). To analyze the spectrum in a short–wave range $\omega \sim E$, make use of the fact that, as shown by Nikishov and Ritus [Nikishov (1979); Ritus (1970)], with due account of the quantum recoil effects the spectral distribution of radiation produced by a particle moving along a circular trajectory has the form:

$$\frac{dI}{d\omega} = -\frac{e^2m^2}{\sqrt{\pi}E}\frac{\eta}{1+\eta}\left\{\int\limits_\xi^\infty \Phi(\xi')d\xi' + \frac{2}{\xi}\left(1 + \frac{\eta^2}{2(1+\eta)}\right)\Phi'(\xi)\right\}\,, \tag{4.66}$$

where $\eta = \omega/E - \omega$; $\xi = (\eta/\chi)^{2/3}$; $\chi = eH\gamma/m^2$, H is the strength of the external magnetic field.

In a similar manner as has been done above, replacing the strength of the external magnetic field by the strength of the electric field which acts on a particle moving at a certain small angle with the crystallographic axis (plane) and integrating (4.66) over the flight time, we obtain the following expression for the spectral distribution of radiation energy:

$$\frac{dw}{d\omega} = -\frac{e^2m^2}{\sqrt{\pi}E}\frac{\eta}{1+\eta}\int\limits_{-\infty}^{+\infty}\left\{\int\limits_{\xi}^{\infty}\Phi(\xi')d\xi' + \frac{2}{\xi}\left(1+\frac{\eta^2}{2(1+\eta)}\right)\Phi'(\eta)\right\}dt\,. \tag{4.67}$$

From this we obtain for planar channeling

$$\frac{dw(\vartheta, x_0)}{d\omega} = -\frac{e^2m^2}{\sqrt{\pi}E}\frac{\eta}{1+\eta}\frac{L}{\tau(x_0)} \tag{4.68}$$

$$\times\int\limits_{x_1(x_0)}^{a-x_1(x_0)}\left\{\int\limits_{\xi}^{\infty}\Phi(\xi')d\xi' + \frac{2}{\xi}\left(1+\frac{\eta^2}{2(1+\eta)}\right)\Phi'(\eta)\right\}\frac{dx}{v(x_1x_0)}\,.$$

Averaged spectral distribution is given by (4.65).

4.7 Parametric Radiation

As mentioned above, the contribution to radiation intensity under diffraction conditions comes from radiation through transition between the levels along with radiation which is due to scattering of pseudo-photons associated with a particle by crystal atoms and nuclei (parametric radiation). This mechanism manifests itself in its purest form in particle motion in a crystal beyond the channeling regime. Recall that parametric radiation is the photon production in the transmission of a uniformly moving charged particle through a periodically inhomogeneous medium.

Parametric optical radiation in a one-dimensional medium with dielectric permittivity of one-dimensional periodicity was first studied by Fainberg and Khizhnyak [Feinberg and Khizhnyak (1957)]. The phenomenon of photon production when a particle passes through a medium with space-periodic dielectric permittivity was reviewed by Ter-Mikaelyan [Ter-Mikaelian (1969, 1972)]. In [Baryshevskii (1971)] attention was focused on the fact that the effect of anomalous transmission can drastically change the spectral properties of radiation produced by a particle in a thick crystal. Classical theory of parametric radiation in a thick crystal, when the effects caused by anomalous transmission are of importance was developed by Feranchuk and the author [Baryshevskii and Feranchuk (1971, 1973, 1976)], Garibyan and Yan Shi [Garibyan and Yan Shi (1972); Avakyan *et al.* (1975)]. Thorough analysis carried out in [Baryshevskii and Feranchuk (1971, 1973, 1976); Feranchuk (1979b)] made it possible to not only find general expressions for spectral-angular distributions of emitted photons but also to obtain explicit expressions for the number of quanta emitted by a particle within the diffraction peak as well as to analyze the process of radiation in crystals containing Mossbauer nuclei. Formulae for the number of quanta produced by a particle analogous to those in [Baryshevskii and Feranchuk (1971, 1973, 1976)] were later derived in [Afanas'ev and Aginyan (1978)].

To obtain the formulae describing parametric radiation in its pure form sufficient it to assume that the angle of a particle entrance into the crystal is much larger than the Lindhard angle. In this case the particle wave functions in a crystal are plane waves. As a result, we have the following expression for the differential number of quanta emitted by a particle forward into the narrow cone along the direction of its velocity $\vec{v}$ in the Laue case (c.): From this we obtain for planar channeling

$$dN_s^{(0)} = \frac{e^2}{\pi^2}(\vec{e}_s\vec{v})^2 \left| \sum_{\mu=1,2} \frac{2\varepsilon_{\mu s} - g_{00}}{2(\varepsilon_{2s} - \varepsilon_{1s})} (l_0^{(0)} - l_{\mu s}^{(0)}) \right.$$
$$\left. \times (e^{-iL/l_{\mu s}^{(0)}} - 1) \right|^2 \vartheta_0 d\vartheta_0 d\varphi_0 \omega d\omega, \tag{4.69}$$

where ϑ_0 and φ_0 are the polar and azimuthal angles of the photon.

Moreover, there appears radiation concentrated in the narrow cone with the axis along the direction $\omega_B^\tau\vec{v} + 2\pi\vec{\tau}$, $\omega_B^\tau = \frac{\pi\tau^2}{|(\vec{\tau}\vec{v})|}$. [2] The differential number of quanta emitted in the direction of diffraction is given by

$$dN_s^{(\tau)} = \frac{e^2}{\pi^2}(\vec{e}_{1s}\vec{v})^2 \left| \sum_{\mu=1,2} \frac{(-1)^\mu g_\tau^{(s)}}{2(\varepsilon_{2s} - \varepsilon_{1s})} (l_0^{(\tau)} - l_{\mu s}^{(\tau)}) \right.$$
$$\left. \times (e^{-iL/L_{\mu s}^{\tau}} - 1) \right|^2 \vartheta_\tau d\vartheta_\tau d\varphi_\tau \omega d\omega, \tag{4.70}$$

where $\cos\vartheta_\tau = (\vec{k}, \omega_B^\tau\vec{v} + 2\pi\vec{\tau})/\omega^2$. Coherent radiation lengths $l_{\mu s}^{(\nu)}$ are

[2] Note that here and below, unlike [Baryshevskii and Feranchuk (1971, 1973, 1976); Baryshevsky and Feranchuk (1980b)], for the sake of uniformity of symbols, we use the notation $2\pi\tau$ to denote the reciprocal lattice vector. In [Baryshevskii and Feranchuk (1971, 1973, 1976); Feranchuk (1979b)] the reciprocal lattice vector is denoted by $\vec{\tau}$.

determined as follows:

$$l_{\mu s}^{(\nu)} = \frac{1}{q_{z\mu s}^{(\nu)}} = \frac{2}{\omega}\left(\frac{m^2}{E^2} + \vartheta_\nu^2 - 2\varepsilon_{\mu s}\right)^{-1};$$

$$l_{\mu s}^{(\nu)} = \frac{2}{\omega}\left(\frac{m^2}{E^2} + \vartheta_\nu^2\right)^{-1}. \tag{4.71}$$

From the analysis of (4.69) and (4.70) follows that the radiation cross-section is maximum when the real part of the longitudinal momentum transmitted to the medium vanishes. From the requirement $\mathtt{Re}q_{z\mu s}^{(\nu)} = 0$ we find the dispersion equation defining the condition for emergence of parametric radiation in a crystal [Baryshevsky and Feranchuk (1974)].

$$\cos\vartheta_\nu = \frac{1}{v} - \mathtt{Re}2\varepsilon_{1,2s}. \tag{4.72}$$

Equation (4.72) differs from the equation defining the condition for emergence of Vavilov-Cherenkov radiation in a homogeneous medium by the dielectric permittivity $\varepsilon(\omega)$, substituted for the corresponding expression for a crystal $1 + 2\varepsilon_{1,2s}$.

In two limiting cases of thin ($\omega L|g_{00}| \ll 1$) and thick ($\omega L\mathtt{Im}g_{00} \gg 1$) crystals, it is possible to obtain analytical expressions for the total number of quanta produced by one particle, which are valid at $\ln\frac{E}{m} \gg 1$:

a. $\omega_B L|g_{00}| \ll 1$, with the results for the Laue and Bragg cases coinciding:

$$N_s^\tau = e^2\frac{(2\pi\vec{\tau})^2|(2\pi\vec{\tau}_\perp)^2 - (2\pi\tau_z)^2|}{8|2\pi\tau_z|^3}L|g_{10}^s(\omega_B)|^2\ln\frac{E}{m}, \tag{4.73}$$

where $|2\pi\tau_z| \gg \omega_B\frac{m}{E}$; $\frac{m^2}{E^2} \geq |g_{00}|$;

b. $\omega_B L\mathtt{Im}g_{00} \gg 1$. In the Laue case .

$$N_s^\tau = e^2\frac{|(2\pi\vec{\tau}_\perp)^2 - (2\pi\tau_z)^2|}{8(2\pi\vec{\tau}_z)^2}\frac{|g_{10}^s(\omega_B^\tau)|^2}{|g_{00}''(\omega_B^\tau)}$$
$$\times\left|\ln\left\{\left(\frac{m^2}{E^2} + \delta_s - g_{00}'\right)^2 + |g_{10}^s|^2 - \delta_s^2\right\}\right|, \tag{4.74}$$

where $g_{00} = g_{00}' + ig_{00}''$; $\delta_s = \frac{\mathtt{Re}\sqrt{g_{10}^s g_{01}^s}\mathtt{Im}\sqrt{g_{10}^s g_{01}^s}}{g_{00}''}$; g_{00}' is the real part of g_{00}; g_{00}'' is the imaginary part of g_{00}. And the angular divergence of quanta $\Delta\vartheta = \sqrt{\frac{m^2}{E^2} + g_{00}'}$, the order of magnitude of the frequency spread near $\omega = \omega_B^\tau$ is defined by the formula

$$\frac{\Delta\omega}{\omega_B^\tau} \simeq \sqrt{\frac{m^2}{E^2} + g_{00}'}. \tag{4.75}$$

In the Bragg case when the below condition is satisfied

$$\frac{m^2}{E^2} - 2g_{10}^{s\prime} - 2\sqrt{|\beta_1|\mathtt{Re}(g_{10}^s g_{01}^s)} \gg g_{00}''. \tag{4.76}$$

the intensity is defined by formula (4.74). If the condition (4.76) is violated,

$$N_s^\tau \simeq e^2 \frac{|(2\pi\vec{\tau}_\perp)^2 - (2\pi\tau_z)^2|}{8(2\pi\tau_z)^2}$$

$$\times \frac{|g_{10}^s(\omega_B)|^2 \ln|\frac{m^2}{E^2} + g_{00}'|}{[\sqrt{|\beta_1|\mathtt{Re}(g_{10}^s g_{01}^s)}(g_{00}'' + \sqrt{|\beta_1| g_{10}'' g_{01}''})]^{1/2}}. \tag{4.77}$$

Numerical analysis showed that the values of N_s^τ found from formulae (4.74)-(4.75) coincide with the results of calculations by the exact formulae with the accuracy of $5-10\%$

Now go over to considering the frequency spectrum of parametric radiation concentrated along the direction of particle motion. Assume a crystal to be quite thick ($\omega L \mathtt{Im} g_{00} \gg 1$). Expression (4.69) can be represented in the form

$$dN_s^{(0)} = dN_s^n + d\tilde{N}_s,$$

where

$$dN_s^n = \frac{4e^2}{\pi^2} p_s^2 \frac{|g_{00}|^2}{(\gamma^{-2}+\vartheta^2)^2|\gamma^{-2}+\vartheta^2-g_{00}|^2} \vartheta^3 d\vartheta d\varphi \frac{d\omega}{\omega} \tag{4.78}$$

$$\begin{aligned} d\tilde{N}_s = {} & \frac{4e^2}{\pi^2} p_s^2 \\ & \times \frac{(2\varepsilon_{2s} - g_{00})(2\varepsilon_{1s} - g_{00})}{(\gamma^{-2}+\vartheta^2)^2|\gamma^{-2}+\vartheta^2-g_{00}|^2|\gamma^{-2}+\vartheta^2-2\varepsilon_{1s}|^2|\gamma^{-2}+\vartheta^2-2\varepsilon_{2s}|^2} \\ & \times \mathtt{Re}\left[4\varepsilon_{2s}\varepsilon_{1s}(\gamma^{-2}+\vartheta^2-g_{00})\right. \\ & + 2g_{00}(\gamma^{-2}+\vartheta^2)(\varepsilon_{2s}-\varepsilon_{1s}) - g_{00}(\gamma^{-2}+\vartheta^2) \\ & \left.\times(2\gamma^{-2}+2\vartheta^2+g_{00})\right]\vartheta^3 d\vartheta d\varphi \frac{d\omega}{\omega}, \end{aligned} \tag{4.79}$$

and $p_1 = \sin\varphi$; $p_2 = \cos\varphi; \gamma = E/m$. Formula (4.78) coincides with the expression for the cross-section of transient radiation in a homogeneous medium with dielectric permittivity $\varepsilon(\omega) = 1 + 2g_{00}$. The addend is associated with parametric radiation, it contains information about the crystal structure.

Analysis of expression (4.79) shows that $d\tilde{N}_s$ has a pronounced resonance character: when the conditions (4.72) hold, its value exceeds dN_s^n by a factor of $(g_0'/g_0'')^2$ (i.e., by a factor of $10^4 \div 10^5$). The width of the peak formed by parametric radiation is very small (see (4.75)), so the contribution to the integral intensity of forward radiation due to parametric effect is insignificant as compared to the intensity of transient radiation.

As seen from a through analysis carried out by Feranchuk [Feranchuk (1979b)], the study of the energy spectrum of the forward-emitted photons enables one to simultaneously measure a larger number of structure amplitudes, which may appreciably reduce the duration of physical experiments in X-ray diffraction analysis. Below we follow the same line of reasoning as in [Feranchuk (1979b)].

The most direct method to measure $d\tilde{N}_s$ is to use X-ray detectors with high angular and frequency resolution. But good reliability of the parametric effect study against transient radiation is possible when the relative angular and energy resolution of a detector is not poorer than $10^{-2}\%$. Though the investigation of radiation spectrum with such a resolution is feasible, using another single crystal with known parameters as a detector, such an experiment seems to be tedious, and above all, it leads to a considerable loss of radiation intensity.

Another opportunity is to use detectors, which enable detecting X-ray radiation with given (preset) polarization. In this case suffice it to register photons polarized perpendicular to the radiation plane, i.e., the plane formed by vectors $\vec{k}$ and $\vec{v}$. The radiation registered by such a detector will be completely associated with the parametric effect. Nevertheless, this method also exhibits the shortcomings mentioned above.

Therefore we only give a more detailed analysis of one experimental method which seems to provide the simplest way of measuring $d\tilde{N}_s$ making the most out of the advantages of the parametric Vavilov-Cherenkov effect: high intensity and the possibility of simultaneous study of numerous structure amplitudes.

Thus, suppose that a detector registers the total radiation propagating in the cone with the apex angle $\Delta\vartheta = \sqrt{\gamma^{-2} + g_{00}'}$ along the direction of particle motion and has a relative energy resolution $\Delta\omega/\omega = \omega \sim 3 - 5\%$, typical of semiconductor detectors. Assume also that the electron beam does not get into the detector after leaving the crystal. For this purpose one may use a holed detector, or change the beam direction after the crystal by means of a magnetic field.

The number of photons with the frequency ω_0 registered by the detector per unit time, which are formed in the transmission of a beam of monochromatic electrons with the energy E and current J through a thick perfect single crystal, is defined by the expression derived from (4.78), (4.79) by integration with respect to the exit angles and frequency and summation over polarizations of quanta:

$$n_0(\omega_0) = J\frac{4e^2}{\pi}\left[\frac{2\gamma^{-2} - g'_{00}(\omega_0)}{|g'_{00}(\omega_0)|}\right.$$
$$\left.\times \ln\frac{\gamma^{-2} - g'_{00}(\omega_0)}{\gamma^{-2}} - 2\right]\frac{\Delta\omega}{\omega}, \tag{4.80}$$

if $\omega_0 < \omega_B^\tau - \Delta\omega$ or $\omega_0 > \omega_B^\tau + \Delta\omega$, and

$$n(\omega_0) = n(\omega_0) + n_\tau(\omega_0) = n_0(\omega_0) + Je^2\frac{|\tau_\perp^2 - \tau_z^2|}{\tau^2}$$
$$\times\frac{|g_\tau(\omega_0)|^2}{g''_{00}(\omega_0)}|\ln B|f\left(\frac{\Delta\omega}{\omega_B^\tau}\right), \tag{4.81}$$

if $\omega_B^\tau - \Delta\omega < \omega_0 < \omega_B^\tau + \Delta\omega$. Here

$$\omega_B^\tau = \frac{(2\pi\tau)^2}{2|2\pi\tau_z|};\ B = \ln[(\gamma^{-2} + \delta - g'_{00})^2 + |g_\tau|^2 - \delta^2];$$

$$\delta = g'_\tau\frac{g''_\tau(\omega_0)}{g''_{00}(\omega_0)};$$

$$f(x) = \begin{cases} 1 & x > \sqrt{\gamma^{-2} + g'_{00}}, \\ x/\sqrt{\gamma^{-2} + g'_{00}} & x < \sqrt{\gamma^{-2} + g'_{00}}; \end{cases}$$

$\Delta\omega$ is the energy resolution of the detector, we shall assume to be appreciably less than the distance between the nearest resonance frequencies, i.e.,

$$\Delta\omega \ll \min_{\tau_1\tau_2}[\omega_B^{\tau_1} - \omega_B^{\tau_2}] \simeq \pi\tau_{\min}.$$

Complete information about the crystal structure is contained in the quantities n_τ, which, according to (4.81) are determined by structure amplitudes. The relative value of n_τ as compared to the background counting rate n_0 associated with the transient radiation depends on $\Delta\omega$:

$$\xi \equiv \frac{n_\tau}{n_0} \approx \frac{|\tau_\perp^2 - \tau_z^2}{\tau^2|}\frac{|g_\tau|^2}{g''_{00}}\frac{\omega_B^\tau}{\Delta\omega};$$
$$\frac{\Delta\omega}{\omega_B^\tau} \geq \sqrt{\gamma^{-2} + g'_{00}}. \tag{4.82}$$

From (4.82) follows that at a relative resolution of the detector $\Delta\omega/\omega \sim 0.03$ the quantity ξ for real crystals varies within the limits from 0.01 to 0.1.

The method enabling one to select a weak signal with the intensity n_s against the noise of intensity $n_n \gg n_s$ ia applicable to measure n_τ. This method is widely used in the problems dealing with the measurement of weak luminous fluxes [Komarov and Pisarevsky (1965)]. It is based on splitting of the total measurement time t into two equal parts t_1 and t_2, with all the photons associated with both mainstream and noise flows being registered during time t_1. During the time period t_2 only noise pulses are taken into account. Then the difference of the number of photons N_1, gathered in time t_1 and the number of photons registered by the detector in time t_2 determines the signal intensity:

$$n_s = 2(N_2 - N_1)/t \pm \Delta n_s, \tag{4.83}$$

the relative accuracy $\Delta n/n_s$ is obviously dependent on the measurement time t. The time necessary to attain the the given accuracy β can be easily found

$$t_\beta = 2(2n_b + n_s)/n_s^2\beta^2. \tag{4.84}$$

In the problem in question this method may be used as follows. Suppose that a multichannel analyzer with the channel width $\Delta\omega$ corresponding to the energy resolution of the detector is used to study the pulses from the x-ray detector. Let during time $t/2$ the pulses be summed up in each analyzer channel, which appear at the detector output when an X-ray quantum with the energy corresponding to the given channel gets into the detector. Then the crystal should be turned through the angle ψ satisfying the condition

$$2\pi\tau_{\min}\sin\psi \gg \Delta\omega \tag{4.85}$$

about the direction of the velocity of the electrons, with $\tau_{\min}$ being the smallest of the vectors $\vec{\tau}$.

If the condition (4.85) is fulfilled, the photon frequency ω_1, which was close to the resonance one for a certain reciprocal lattice vector $\vec{\tau}(\omega_1 \approx \omega_B^\tau)$, after the crystal rotation will appreciably differ from it, so that the intensity of quanta with the frequency ω_1 will only be determined by the quantity n_0. If now in each channel of the analyzer we subtract the number of quanta registered by the detector during time $t/2$ after the crystal rotation, then in the analyzer channels corresponding to the resonance frequencies in the first time period, the number of pulses N_τ will be defined by formula

$$N_\tau = n_\tau T/2 \pm \Delta N, \tag{4.86}$$

while in the rest of the channels the number of pulses is equal in magnitude to ΔN - the number of pulses due to statistic fluctuations of photons, and

$$\Delta N \simeq \sqrt{(n_{\vec{\tau}} + n_0)t/2}. \tag{4.87}$$

Using (4.86), we may find n_τ along with the structure amplitude $F(\vec{\tau})$ with the absolute error determined by the quantity $2\Delta N/t$. The time necessary to measure $F(\tau)$ with the specified relative accuracy β is found, using (4.84), if assume that $n_s = n_\tau$, $n_b = n_0$:

$$t_\beta = \frac{1}{\beta^2 J |\ln B| e^2} \frac{g_{00}''^2}{|g_\tau|^4} \frac{\Delta\omega}{\omega_B^\tau}. \tag{4.88}$$

To estimate t_β choose the electron current $J = 10^{-6}$ A, $E = 50$ MeV, $\Delta\omega/\omega_B^\tau = 0.03$, $|g_\tau| = 10^{-6}$, $g_{00}'' = 10^{-8}$ are the typical values for real crystals. Then, to measure $F(\tau)$ with the relative accuracy 0.01, we need the time $t_\beta \simeq 10^{-2}$ s.

Mention also a simpler way of selecting transient radiation suitable for investigating crystals containing atoms with the small number of electrons, when the frequencies ω_B^τ of photon emitted in parametric effect are greater than characteristic atomic frequencies. In this case n_0 has a universal dependence on the frequency ω. That is why it is not necessary to rotate a crystal to determine n_τ: suffice it to subtract $N_k = N_0\omega_0^2/\omega_k^2$, where N_0 is the number of pulses in the channel corresponding to the frequency ω_0 (ω_0 satisfies $\omega_0 < \pi\tau_{\min}$) from the total number of photons registered in the channel of the analyzer corresponding to the frequency ω_k. It may be demonstrated that in this case the time of accumulation is also determined by (4.88).

Chapter 5

Classical Theory of Radiation Formation by Particles in a Medium

5.1 Particle Radiation in a Medium in the Presence of Scattering and Energy Losses

Classical theory of production of electromagnetic radiation by particles passing through a single crystal without regard to refraction, absorption and diffraction was developed by M.A. Kumakhov [Kumakhov (1976, 1977)], M.I. Podgoretsky [Podgoretsky (1977a,b)], A.I. Akhiezer, V.F. Boldyshev and N.F. Shulga [Akhiezer *et al.* (1979)], D.A. Alferov, Yu. A. Bashmakov, E. G. Bessonov [Bessonov (1978); Alferov *et al.* (1977a,b)], V.N. Baier, V.M. Katkov, V.M. Strakhovenko [Baier *et al.* (1979)].

Presented below is the classical theory of photon formation by particles in a medium with due account of the effects caused by refraction, absorption and diffraction, which also enables one directly to allow for possible multiple scattering of particles [Baryshevsky (1976); Baryshevskii (1974); Baryshevskii *et al.* (1977, 1976); Baryshevsky and Grubich (1979c)].

So, let a charge move in a medium (e.g., in a crystal) in an arbitrary manner. The spectral density of radiation energy per unit solid angle $W_{\vec{n}\omega}$($\vec{n} = \vec{k}/k$; the differential number of quanta $dN_{\vec{n}\omega} = W\vec{n}\omega/\hbar\omega$) as well as the polarization characteristics of radiation may be easily obtained if the field $\vec{E}(\vec{r}, \omega)$ produced by a charge at large distances from the crystal is known. For instance,

$$W_{\vec{n}\omega} = \frac{cr^2}{4\pi^2}\overline{\left|\vec{E}(\vec{r},\omega)\right|^2}, \tag{5.1}$$

where c is the speed of light; the vinculum means averaging over all possible states of the system under consideration. To find the field $\vec{E}(\vec{r}, \omega)$, one should solve Maxwell's equations which for an arbitrary medium have the form

$$\left[-\mathrm{rot}\,\mathrm{rot}\,\vec{E}(\vec{r},\,\omega)+\frac{\omega^2}{c^2}\vec{E}(\vec{r},\,\omega)\right]_i+\frac{4\pi i\omega}{c^2}\hat{\sigma}_{ij}E_j=-\frac{4\pi i\omega}{c^2}j_{0i}(\vec{r},\,\omega)\,, \quad (5.2)$$

where $\hat{\sigma}_{ij}$ is the conductivity tensor of matter; $j_{0i}(\vec{r},\,\omega)$ is the Fourier transform of the i-th component of the current induced by a moving charge. In the quantum mechanical case by "$j_{0i}(\vec{r},\,\omega)$" one should understand the unaveraged over the crystal states current of transition from one quantum mechanical state to another.

The transverse solution of (5.2) can be found, using the Green function G of this equation satisfying the relation of the form

$$G=G+G_0\frac{i\omega}{c^2}\hat{\sigma}G, \quad (5.3)$$

where G_0 is the transverse Green function of equation (5.2) at $\hat{\sigma}=0$ (its explicit form see, for example, in [Morse and Feshbach (1953)]). Using G, it is easy to find the field we are concerned with:

$$\vec{E}(\vec{r},\omega)=\int G_{il}(\vec{r},\vec{r}\,',\omega)\frac{i\omega}{c^2}j_{0l}(\vec{r}\,')d^3\vec{r}\,'\,. \quad (5.4)$$

According to [Baryshevsky (1976)] at $r\to\infty$ the Green function is expressed via the solution of homogeneous Maxwell's equations $E_i^{(-)}(\vec{r},\omega)$ containing a converging spherical wave at infinity:

$$\lim_{r\to\infty}G_{il}(\vec{r},\vec{r}\,',\omega)=\frac{e^{ikr}}{r}\sum_s e_i^s E_{\vec{k}l}^{s(-)^*}(\vec{r}\,',\omega)\,, \quad (5.5)$$

$$\left[-\mathrm{rot}\,\mathrm{rot}\,\vec{E}^{(-)}(\vec{r},\,\omega)+\frac{\omega^2}{c^2}\vec{E}^{(-)}(\vec{r},\,\omega)\right]_i-\frac{4\pi i\omega}{c^2}\hat{\sigma}_{ij}^*E_j^{(-)}=0\,, \quad (5.6)$$

where $\vec{e}^{\,s}$ is the transverse unit vector of polarization; $s=1,\,2$.

If the wave is incident onto the object of finite dimensions, then, at $r\to\infty$,

$$\vec{E}_{\vec{k}l}^{(-)}(\vec{r},\omega)=\vec{e}^{\,s}e^{i\vec{k}\vec{r}}+\quad\mathrm{const}\quad\frac{e^{-ikr}}{r}\,. \quad (5.7)$$

Using (5.4) and (5.5), we find

$$E_i(\vec{r},\omega)=\frac{e^{-ikr}}{r}\frac{i\omega}{c^2}\sum_s e_i^s\int\vec{E}_{\vec{k}}^{(-)s^*}(\vec{r}\,',\omega)\vec{\jmath}(\vec{r}\,',\omega)d^3r'\,. \quad (5.8)$$

In view of (5.1) and (5.8), the spectral density of radiation is

$$W_{\vec{n}\omega}=\sum_s W_{s\vec{n}\omega}=\frac{\omega^2}{4\pi^2c^3}\sum_s\left|\int\vec{E}_{\vec{k}}^{s(-)^*}(\vec{r},\omega)\vec{\jmath}(\vec{r},\omega)d^3r\right|^2\,, \quad (5.9)$$

where $W_{s\vec{n}\omega}$ is the spectral density of radiation per unit solid angle for photons, characterized by the polarization vector $\vec{e}^{\,s}$. To explicitly find $W_{\vec{n}\omega}$, it is necessary to know the field $\vec{E}_{\vec{k}}^{s(-)}$ and the current $\vec{\jmath}$. With known solution $\vec{E}_{\vec{k}}^{s(+)}$ of the homogenous Maxwell equations describing the process of photon scattering by the target, field $\vec{E}_{\vec{k}}^{s(-)}$ can be found using the below relation:

$$\vec{E}_{\vec{k}}^{s(-)^*} = \vec{E}_{-\vec{k}}^{s(+)} . \tag{5.10}$$

Introduce the following explicit expression for the Fourier transform of the current into (5.9):

$$\begin{aligned} \vec{\jmath}(\vec{r},\omega) &= \int e^{i\omega t}\vec{\jmath}(\vec{r},t)dt\,, \\ \vec{\jmath}(\vec{r},t) &= e\vec{v}(t)\delta(\vec{r}-\vec{r}(t))\,. \end{aligned} \tag{5.11}$$

Substitution of (5.11) into (5.9) gives

$$\begin{aligned} W_{s\vec{n}\omega} = \frac{e^2\omega^2}{4\pi^2c^3}\int_{t_1}^{t_2}\int (\vec{E}_{\vec{k}}^{(-)s^*}(\vec{r}(t))\vec{v}(t))e^{i\omega t} \\ \times(\vec{E}_{\vec{k}}^{(-)s}(\vec{r}(t'))\vec{v}(t'))e^{-i\omega t'}dtdt', \end{aligned} \tag{5.12}$$

where t_1 and t_2 are the starting and finishing moments of the charge motion, respectively.

In (5.2) perform averaging over the possible particle trajectories in a medium. Such averaging is usually performed with the combined probability density $w(\vec{r},\vec{v},t;\vec{r}\,',\vec{v}\,',t')$ of finding the coordinate $\vec{r}$ and the velocity $\vec{v}$ of a particle at moment t, the coordinate $\vec{r}\,'$ and the velocity $\vec{v}\,'$ at moment t'. However, when investigating the effects of the energy losses, it is more convenient to perform averaging with a similar function, which depends on variables $\vec{r}$ and $\vec{p}$, where $\vec{p}$ is the particle momentum. As a result ($c = 1$),

$$\begin{aligned} W_{s\omega} = \frac{e^2\omega^2}{4\pi^2}\int\int_{t_1}^{t_2}\int\left(\vec{E}_{\vec{k}}^{(-)s^*}(\vec{r})\frac{\vec{p}}{E(p)}\right)\left(\vec{E}_{\vec{k}}^{(-)s}(\vec{r}\,')\frac{\vec{p}\,'}{E'(p')}\right) \\ \times w(\vec{r},\vec{p},t\,,\,\vec{r}\,',\vec{p}\,',t')e^{i\omega(t-t')}d^3rd^3r'd^3pd^3p'dtdt'\,. \end{aligned} \tag{5.13}$$

Choose the coordinate system so that the $x\,y$ plane coincides with the matter–vacuum boundary. Direct the z-axis from the medium to vacuum. Suppose that a particle with momentum $\vec{p}_0$ directed along the z-axis starts moving at time $(-T)$ at point (0, 0, $-z_0$) inside the medium. Let it cross the matter–vacuum boundary at moment $t = 0$.

In the case of high energies of γ-quanta we are concerned with, we may neglect mirror reflected waves in the expressions for the fields $\vec{E}^{(-)s}_{\vec{k}}$. As a result,

$$\vec{E}^{(-)s}_{\vec{k}} = \begin{cases} \vec{e}^{\,s} e^{i\vec{k}\vec{r}} & \text{at } z > 0\,, \\ \vec{e}^{\,s} e^{i\vec{k}'^{*}\vec{r}} & \text{at } z < 0\,, \end{cases} \tag{5.14}$$

where $\vec{k}'$ is the photon wave vector in the medium with the components $\vec{k}'_\perp = \omega\vec{n}_\perp$, $k'_z = \omega\sqrt{\varepsilon}\vec{n}_z$.

Using (5.14) and going from variables $(p,\ \theta,\ \varphi)$ to variables $(E,\ \vec{\theta})$, where E is the energy and $\vec{\theta} = \theta_x\vec{\imath} + \theta_y\vec{\jmath}$ is the transverse angular vector, one can obtain the following expression for the intensity distribution $W_{\|\vec{n}\omega}$ of photons polarized in the plane of exit from matter:

$$\begin{aligned} W_{\|\vec{n}\omega} = \frac{e^2\omega^2\vartheta^2}{4\pi^2} & \left\{ \int_0^\infty dt \int_0^\infty dt' \int d\xi d\xi' F(\theta)F(\theta') \right. \\ & \times \exp[-i\vec{k}(\vec{r}-\vec{r}\,') + i\omega(t-t')] \\ & \times w_1(\vec{r},\vec{\theta},\ E,\ t+\tau) w_2(\vec{r},\vec{\theta},\ E,\ t|\vec{r}\,',\vec{\theta}',E',t') \\ & + 2\mathtt{Re} \int_{-T}^0 dt \int_0^\infty dt' \int d\xi d\xi' F(\theta)F(\theta') \\ & \times \exp[-i(\vec{k}'\vec{r} - \omega t) + i(\vec{k}\vec{r}\,' - \omega t')] \\ & \times w_1(\vec{r},\vec{\theta},E,t+T) w_2(\vec{r},\vec{\theta},\ E,\ t|\vec{r}\,',\vec{\theta}',E',t') \\ & + 2\mathtt{Re} \int_{-T}^0 dt \int_0^{-t} d\tau \int d\xi d\xi' F(\theta)F(\theta') \\ & \times \exp[-i\omega\tau + i\vec{k}'^{*}(\vec{r}\,' - \vec{r})] \exp(\omega z \mathtt{Im}\varepsilon) \\ & \left. \times w_1(\vec{r},\vec{\theta},\ E,\ t+T) w_2(\vec{r},\vec{\theta},\ E,\ 0|\vec{r}\,',\vec{\theta}',E',\tau') \right\}\,, \end{aligned} \tag{5.15}$$

where ξ is the set of coordinates $(\vec{r},\ \vec{\theta},\ E)$; $w_1(\vec{r},\vec{\theta},\ E,\ t)$ is the probability of finding the particle coordinates $(\vec{r},\ \vec{\theta},\ E)$ at time t; $w_2(\vec{r},\vec{\theta},\ E,\ t|\vec{r}\,',\vec{\theta}',E',\ t')$ is the conditional probability of finding the particle coordinates $(\vec{r}\,',\vec{\theta}',E')$ at time t' if at moment t the particle coordinates were $(\vec{r},\vec{\theta},E)$; $F(\theta) = 1 - (\theta_x\cos\vartheta_x + \theta_y\cos\vartheta_y)\vartheta^{-2}$; $\vartheta_x,\ \vartheta_y,\ \vartheta_z \equiv \vartheta$ are the direction angles of vector $\vec{k}(\vec{n})$.

The expression for spectral–angular distribution of the intensity of photons whose polarization vector is perpendicular to the exit plane is derived from (5.15) by substitution of ϑ^{-2} for ϑ^2 and $B(\theta) = \theta_x \cos\vartheta_y - \theta_y \cos\vartheta_x$ for $F(\theta)$.

Pay attention to the fact that some integrals in (5.15) contain the probability densities w_2 which depend on the instants of time corresponding to particle motion both in the medium and outside it. However, it is more convenient to deal with the densities which depend on the instants of time referring to particle motion in the medium or outside it alone. With this aim in view, make use of the following general property of the distribution functions:

$$w_2(\xi, t|\xi', t') = \int w_2(\xi, t|\xi'', t'') w_2(\xi'', t''|\xi', t') d\xi'' . \tag{5.16}$$

Substituting (5.16) into (5.15) and choosing the instant of time corresponding to the moment of particle exit from matter, i.e., $t'' = 0$, we obtain the expression for $W_{s\vec{n}\omega}$ which only depends on the distribution functions describing the particle motion in the medium or outside it.

The probabilities w_1 and w_2 satisfy the kinetic equation which, for example, in a chaotic medium has the form

$$\frac{\partial w}{\partial t} + \frac{\vec{p}}{E}\frac{\partial w}{\partial \vec{r}} = \left(\frac{\partial w}{\partial t}\right)_{col} . \tag{5.17}$$

(The case of a crystal is discussed in Chapter (9.5).

In our case the change of the collisional term $\left(\frac{\partial w}{\partial t}\right)_{col}$ in time is due to scattering and radiation processes, and it may be described by the equations of the form:

$$\left(\frac{\partial w^{(n)}}{\partial t}\right)_{\text{col}} = -\sum_{n'} g_{n'n} w^{(n)} + \sum_{n'} g_{nn'} w^{(n')} , \tag{5.18}$$

where $g_{nn'}$ is the probability of the system transition from state n (in our case of the electron in state n) to state n' per unit time. The probabilities $g_{nn'}$ may be found by conventional rules [Berestetsky *et al.* (1968)].

As a result, for example, in a chaotic medium, taking account of the change in $w^{(n)}$, which is only due to multiple scattering and bremsstrahlung, gives

$$\begin{aligned}\left(\frac{\partial w(\vec{p}, t)}{\partial t}\right)_{\text{col}} &= -N\sigma_{\text{tot}} w(\vec{p}, t) \\ &+N \int \frac{d^3p'}{(2\pi)^2}\delta(E_p - E_{p'})\frac{|M_{\vec{p}'\vec{p}}|^2}{4E_p^2} w(\vec{p}, t) \\ &+N \int \frac{d^3p' d^3k}{(2\pi)^5}\delta(E_p - E_{p'} - k)\frac{|M_{\vec{p}'\vec{k},\vec{p}}|^2}{8E_p E_{p'} k} w(\vec{p}', t) ,\end{aligned} \tag{5.19}$$

where N is the number of scatterers (nuclei) per unit volume; the amplitude $M_{\vec{p}'\vec{p}}$ describes electron scattering in the nuclear Coulomb field, the amplitude $M_{\vec{p}'\vec{k},\vec{p}}$ describes the emission of γ-quanta by the electron in the nuclear field; $\sigma_{\rm tot}$ is the total cross section of all the processes.

Give a more detailed treatment of the case when the emission of γ-quanta may be described by the Bethe-Heitler expression at complete screening of the nuclear field [Berestetsky *et al.* (1968); Heitler (1984)].[1]

Turning from the probability describing the electron distribution in the momenta to the probabilities describing particle distribution in energies and scattering angles and taking the appropriate transformations of (5.19), we obtain the following equation

$$\left(\frac{\partial w(\vec{\theta}, E, t)}{\partial t}\right)_{col} = q(E)\Delta_\theta w(\vec{\theta}, E, t) + K(E)w(\vec{\theta}, E, t), \tag{5.20}$$

where $\Delta_\theta = \frac{\partial^2}{\partial\theta_x^2} + \frac{\partial^2}{\partial\theta_y^2}$; $q(E) = \delta E^{-2}$; $\delta = \frac{1}{4}E_s^2$; $K(E)$ is the integral operator of the form [2]

$$K(E)w(\vec{\theta}, E, t) = \int_E^\infty \frac{u^2 + E^2 - \frac{2}{3}uE}{u^2(u-E)} w(\vec{\theta}, u, t)du - \int_0^E \frac{u^2 + E^2 - \frac{2}{3}uE}{E^2(E-u)} w(\vec{\theta}, E, t)du. \tag{5.21}$$

The initial conditions for the distribution functions w_1 and w_2 have the form

$$w_1(t = -T) = \delta(\vec{r} - \vec{r}_0)\delta(\theta)\delta(E - E_0)\,,$$
$$w_2(t = t') = \delta(\vec{r} - \vec{r}\,')\delta(\vec{\theta} - \vec{\theta}')\delta(E - E')\,,$$

where E_0 is the initial energy of the particle;

$$\frac{\vec{p}}{E} = \vec{v}\,, \vec{v} \quad \text{has the components} \quad \theta_x,\, \theta_y,\, 1 - \frac{\theta_x^2 + \theta_y^2}{2}\,.$$

Using kinetic equation (5.20), it is possible to find the time–dependence of the mean–square angle $\langle\theta^2(E,\, t)\rangle$ of multiple scattering of the electron of

[1]The influence of multiple scattering (the Landau-Pomeranchuk effect)and the effect of the medium polarization on the bremsstrahlung cross-section may be taken into account, using the method described by Ter-Mikaelian in [Ter-Mikaelian (1969, 1972)].

[2]It is interesting to note that the equation obtained can be derived from well known equations of the shower theory [Belenky (1948)] with the terms referring to the pair formation being dropped.

energy E within the interval dE. For this purpose, we shall multiply (5.20) by $\theta^2 = \theta_x^2 + \theta_y^2$ and integrate it over $\vec{r}$ and $\vec{\theta}$. As a result, we have

$$\frac{\partial\langle\theta^2(E,t)\rangle}{\partial t} = 4q(E)w(E,t) + K(E)\langle\theta^2(E,t)\rangle\,, \tag{5.22}$$

where $w(E,t) = \int w(\vec{r},\vec{\theta},E,t)d^3rd^2\theta$ is the probability of finding an electron with the energy E at time t if at $t = 0$ its energy is E_0.

Solving (5.22) using the Mellin transform, find

$$\langle\theta^2(E,t)\rangle = 4\delta\int\limits_0^t dt'\int\limits_E^{E_0}\frac{dE'}{E'}w(E_0|E',\,t')w(E'|E,t'-t)\,. \tag{5.23}$$

For particular calculation of (5.23), make use of the approximate expression, derived by Bethe and Heitler [Heitler (1984)]:

$$w(E_0,E,t) = \frac{1}{E_0}\frac{\left(\ln\frac{E_0}{E}\right)^{\frac{t}{\ln 2}-1}}{\Gamma\left(\frac{t}{\ln 2}\right)}\,, \tag{5.24}$$

where t is measured in radiation units; $\Gamma\left(\frac{t}{\ln 2}\right)$ is the gamma function. Substitution of (5.24) into (5.23) gives

$$\langle\theta^2(E,t)\rangle = \frac{4q(E)\left(\ln\frac{E_0}{E}\right)^{\frac{t}{\ln 2}-1}}{E_0\Gamma\left(\frac{t}{\ln 2}\right)}\int\limits_0^t\Phi\left(\frac{\tau}{\ln 2},\frac{t}{\ln 2},2\ln\frac{E_0}{E}\right)d\tau\,, \tag{5.25}$$

where Φ is the degenerate hypergeometric function. Expression (5.25) differs considerably from a simple exponential dependence, obtained through substitution of the equality $\langle E\rangle = E_0e^{-\vec{t}/L}$ into $\langle\theta^2(E,t)\rangle = 4qt$.

Like in the case without losses, further analysis is convenient to perform, studying the equations for the functions of the type given below, which appear in (5.15)

$$u_0(\vec{\theta},E,t+T) = \int d^3rw_1(\vec{r},\vec{\theta},\,E,\,t+T)e^{\omega z\,\mathtt{Im}\varepsilon} \tag{5.26}$$

$$u_2(\vec{\theta},\,\vec{\theta}',\,E,\,E',\,\vec{\tau}) = \int d^3\rho w_2(\vec{\rho},\,\vec{\theta},\,\vec{\theta}',\,E,\,E',\vec{\tau})e^{-i(\omega\vec{\tau}-\vec{k}'^*\vec{\rho})}\,. \tag{5.27}$$

In view of (5.26) and (5.27), multiplication of (5.20) by the corresponding multipliers gives gives the following equation for u in a chaotic medium

$$\frac{\partial u}{\partial t} + A(\theta,E)u = q(E)\Delta_\theta u + K(E)u\,, \tag{5.28}$$

where in the case of u_0

$$A(\theta,E)u \equiv A_0(\theta) = -\omega\beta\left(1-\frac{1}{2}\theta^2\right)\mathtt{Im}\varepsilon\,, \tag{5.29}$$

in the case of u_2

$$A(\theta, E) \equiv A_2(\theta) \tag{5.30}$$
$$= i\omega \left[1 - \beta(\theta_x \cos\vartheta_x + \theta_y \cos\vartheta_y) - \beta\left(1 - \frac{1}{2}\theta^2\right)\left(1 - \frac{1}{2}\theta^2 + \frac{1}{2}\delta\varepsilon^*\right)\right] ,$$

where $\beta = v/c$; $c = 1$.

Thus, to find the radiation spectrum one should solve equation (5.28). Similar equations can be analyzed, using the methods developed in the cascade theory [Belenky (1948)], though the formal solution obtain thus obtained is sophisticated in form.

Let us give a more detailed treatment of the case of radiation in an amorphous medium when the energy losses can be neglected (the plate thickness is much smaller than the radiation length). The problem of photon radiation in the X-ray and optical regions by particles passing through the matter-vacuum boundary was discussed in many publications. However, angular, spectral and polarization properties of the radiation produced in the presence of multiple scattering were analyzed regardless photon absorption in the medium, and for this reason they are not suitable for the study of, e.g., generation of resonance photons (optical, X-ray, or Mössbauer ones). Below the results obtained in [Baryshevskii *et al.* (1977, 1976)] are presented.

5.2 Spectral-Angular Distribution in the Absence of the Energy Loss

First, consider the problem of the relation between bremsstrahlung, transition and Cherenkov radiations in the range of high energy γ-quanta.

It is worth mentioning that the analysis of the role of transition radiation in the X-ray spectral range used the expression of the form below for dielectric permittivity of the the medium:

$$\varepsilon = 1 - \frac{\omega_L^2}{\omega^2} , \tag{5.31}$$

where $\omega_L^2 = 4\pi e^2 zN/m$; ω_L is the Langmuir frequency; ω is the frequency of the emitted quantum.

Equation (5.31) is only valid for the energy ranges where the Compton scattering is the major mechanisms of scattering of γ-quanta. In the range of high energies of γ-quanta we are concerned with, the significant contribution to ε also comes from the pair production processes, and $\mathtt{Re}(\varepsilon-1) < \mathtt{Im}\,\varepsilon$.

1. To clarify the relationship between the transition radiation and bremsstrahlung in the high energy regions, it is necessary to find the intensity $W_{n\omega}$ of radiation emerging in a vacuum in the direction of motion of a particle passing through a layer of matter. Assume for simplicity that the layer thickness is much larger than the absorption length of γ-quanta. In this case in a similar manner as when solving the problem of optical radiation of a particle entering the matter [Pafomov (1969)], one may obtain the expression for $W_{\vec{n}\omega}$ coinciding with the that derived by Pafomov [see [Pafomov (1969)], formulae (27.44)-(27.49)], upon substituting in the latter $\beta \to -\beta$ and $\varepsilon \to \varepsilon^*$ in all the functions except $\eta = [4\omega\beta q \mathtt{Im}\sqrt{\varepsilon - \sin^2\vartheta}]^{1/2}$. Taking account of the fact that at high energies the angular distribution of radiation is concentrated within a very narrow angle relative to the direction of particle motion, and the dielectric permittivity of matter is close to unity, allows us to simplify formulas (27.44)-(27.49) in [Pafomov (1969)]. They take the simplest form if the following condition ifs fulfilled

$$\langle\theta^2\rangle \ll \frac{\omega}{c}(\mathtt{Im}\varepsilon)^2), \tag{5.32}$$

where $\langle\theta^2\rangle$ is the mean-square angle of multiple scattering per unit length. The condition (5.32) may be recast as follows:

$$\langle\theta^2\rangle L_c \ll \vartheta_c^2, \tag{5.33}$$

where $L_c = \frac{c}{\omega \mathtt{Im}\varepsilon}$ is the absorption depth of γ-quanta; $\vartheta_c^2 = \frac{c}{\omega L_c}$ is the squared effective angle of quantum emission.

According to (5.33) the formulas in question simplify, if the mean-square angle of multiple scattering of an electron over the absorption length of γ-quantum is smaller than the squared effective angle of radiation. Then the expression for angular and spectral distributions of the radiation intensity $W_{\vec{n}\omega}$ takes the form

$$\begin{aligned} W_{\vec{n}\omega} &= \frac{e^2\vartheta^2}{4\pi^2 c}\frac{|\delta\varepsilon|^2|1-\beta^2-\beta-\frac{1}{2}\delta\varepsilon+\frac{1}{2}\vartheta^2|^2}{(1-\beta^2+\vartheta^2)^2|1-\beta-\frac{1}{2}\delta\varepsilon+\frac{1}{2}\vartheta^2|^2} \\ &+\frac{e^2\langle\theta^2\rangle}{\pi^2\omega}\frac{\vartheta^2}{(1-\beta^2+\vartheta^2)}\mathtt{Im}\frac{1-\beta-\frac{1}{2}\delta\varepsilon}{(1-\beta-\frac{1}{2}\delta\varepsilon+\frac{1}{2}\vartheta^2)^4} \\ &+\frac{e^2\langle\theta^2\rangle}{4\pi^2\omega\mathtt{Im}\varepsilon|1-\beta-\frac{1}{2}\delta\varepsilon+\frac{1}{2}\vartheta^2|}, \end{aligned} \tag{5.34}$$

where ϑ is the radiation angle; $\delta\varepsilon = \varepsilon - 1$. The first term in (5.34) describes transition radiation, the second and third ones are non-zero only allowing for the particle scattering in a medium and describe the interference of the transition radiation and bremsstrahlung, and bremsstrahlung itself.

Using the condition (5.32), one may notice that the second and third terms are smaller than the first term, i.e., the intensity of transition radiation is greater than that of bremsstrahlung. To make sure, use the following common expression for $\langle\theta^2\rangle_1$ [Ter-Mikaelian (1969)]

$$\langle\theta^2\rangle_1 = 4E_s^2/L_\mathrm{r}E^2\,, \tag{5.35}$$

where L_r is the radiation length; $E_s = 21\,\mathrm{MeV}$; E is the electron energy. As in the energy range under consideration $L_c \simeq L_\mathrm{r}$, the condition (5.32) may be recast as

$$E \gg E_s\sqrt{\frac{\omega}{c}L_\mathrm{r}}\,. \tag{5.36}$$

Fulfilment of (5.36) entails satisfying the condition

$$1-\beta \simeq \frac{(m_0c^2)^2}{2E^2} \ll \frac{E_s^2}{E^2} \ll \frac{c}{\omega L_c} = \mathtt{Im}\varepsilon\,. \tag{5.37}$$

Integration of (5.34) with respect to the angles using (5.37), gives the following expression for the first term describing the intensity of transition radiation:

$$W_\mathrm{tr} \simeq \frac{2e^2}{\pi c}\ln\frac{E\sqrt{|\delta\varepsilon|}}{m_0c^2}\,, \tag{5.38}$$

for the second and third terms, we obtain the estimate coinciding with that for the bremsstrahlung [Pafomov (1969)]:

$$W_\mathrm{br} \simeq \frac{e^2\langle\theta^2\rangle}{\pi\omega}\frac{1}{(\mathtt{Im}\varepsilon)^2}$$

or, taking into account (5.32),

$$W_\mathrm{br} \ll \frac{e^2}{\pi c}\,. \tag{5.39}$$

Thus, in the energy range, where the condition (5.32) holds, the intensity of bremsstrahlung is much less than that of transition radiation of γ-quanta. If a less stringent condition $\langle\theta^2\rangle \sim \omega/c(\mathtt{Im}\varepsilon)^2$ is fulfilled, the intensities W_tr and W_br (and their interference) become comparable in magnitude, so the separate consideration of transition radiation and bremsstrahlung is also impossible for high–energy γ-quanta. In view of (5.36) such a situation arises at the electron energy

$$E \sim E_s\sqrt{\frac{\omega}{c}L}\,,$$

i.e., at $E \sim 10^{14}$ eV for γ-quanta in the range of several gigaelectronvolts, and substances for which $L \sim 1$ cm (for example, for copper $L = 1.29$ cm, for lead $L = 0.46$ cm).

Interestingly enough, in the ranges of high–energy γ-quanta the contribution of the pair production processes to the real part of dielectric permittivity is positive, in contrast to the negative contribution from the Compton scattering (see (5.31). According to [Toptygin (1964)], allowing for pair production

$$\mathtt{Re}\,\varepsilon = 1 + \frac{4\pi N c^2}{\omega^2}\left[-\frac{ze^2}{mc^2} + \frac{7}{18}z(z+1)\alpha^3 a\right], \tag{5.40}$$

where a is the shielding radius; α is the fine structure constant, z is the atomic number of the nucleus.

From (5.40) follows that under standard conditions the contribution of pair production processes to $\mathtt{Re}\varepsilon$ is by the order of magnitude less than that of the Compton scattering even for heavy substances [Toptygin (1964)]. As a consequence, $\mathtt{Re}\,\varepsilon < 1$, and the Vavilov–Cherenkov effect is impossible. Nevertheless, according to (5.40), to increase the contribution of pair production processes to $\mathtt{Re}\,\varepsilon$ is possible by increasing the shielding radius which is attainable in plasma. Thus, in hydrogen plasma the contribution of pairs to $\mathtt{Re}\,\varepsilon$ becomes greater than the Compton contribution for the shielding radii $a > 2\cdot 10^{-6}$ cm. As a result, $\mathtt{Re}\,\varepsilon > 1$, and Cherenkov radiation is possible even in the high energy range in the isotropic homogeneous medium.

3. Formula (5.34) also enables analyzing transition and Cherenkov radiations of resonance γ-quanta and the effect that multiple scattering of electrons in matter produce on the stated processes.

Let us first dwell upon the role of bremsstrahlung. To avoid the influence of multiple scattering on transition radiation, the condition (5.32) should to be fulfilled. Using (5.35) recast (5.32) as follows

$$E \gg 2E_s\sqrt{\frac{\omega}{c}\frac{L_c^2(\omega)}{L}}, \tag{5.41}$$

where

$$L_c(\omega) = \frac{c}{\omega\,\mathtt{Im}\,\varepsilon(\omega)}$$

is the absorption depth of a quantum of frequency ω. From (5.41) follows that, for example, for ^{119}Sn ($\hbar\omega = 24$ keV, $L_c(\omega)$ in the resonance is $2.8\cdot 10^{-4}$ cm) multiple scattering may be neglected at the electron energies $E \gg 400$ MeV. Assuming that the condition (5.41) is fulfilled, upon integrating (5.34) with respect to the angles, we obtain the following expression for spectral intensity of transition (and, if possible, Cherenkov) radiation [Baryshevskii and Ngo Dan Nyan (1974)]:

$$\begin{aligned} W_\omega &= \frac{e^2}{2\pi c}\left[1-\frac{4(1-\beta)\mathtt{Re}\,\delta\varepsilon}{|\delta\varepsilon|^2}\right]\ln\frac{[2(1-\beta)-\mathtt{Re}\,\delta\varepsilon]^2+(\mathtt{Im}\,\delta\varepsilon)^2}{4(1-\beta)^2} \\ &+\frac{e^2}{\pi c}\left[\frac{\mathtt{Re}\,\delta\varepsilon}{\mathtt{Im}\,\delta\varepsilon}+\frac{2(1-\beta)}{\mathtt{Im}\,\delta\varepsilon}\frac{(\mathtt{Im}\,\delta\varepsilon)^2-(\mathtt{Re}\,\delta\varepsilon)^2}{|\delta\varepsilon|^2}\right] \\ &\times\left[\frac{\pi}{2}-\arctan\frac{2(1-\beta)-\mathtt{Re}\,\delta\varepsilon}{\mathtt{Im}\,\delta\varepsilon}\right]-\frac{e^2}{\pi c}\,. \end{aligned} \tag{5.42}$$

Using the values of $\delta\varepsilon$ given in [Perelshtein and Podgoretsky (1970)], one may obtain the below estimation of the number of γ-quanta in the center of the Mössbauer line in the energy region $\Delta\omega$ of the order of the level width Γ, which are emitted by an electron with the energy of the order of 1 GeV:

$$N_\gamma = \frac{W_\omega}{\hbar\omega}\frac{\Gamma}{\hbar} \simeq 10^{-12}\,. \tag{5.43}$$

The estimate (5.43) was obtained in [Samsonov (1978)] by numerical solution. This estimate is also valid for the case when the difference $2(1-\beta)-\mathtt{Re}\delta\varepsilon$ may vanish, i.e., in the presence of the Cherenkov radiation mechanism. Note that for the electron energy, at which $1-\beta \ll |\delta\varepsilon|$, the radiation intensity W_ω is determined by formula (5.38), demonstrating weak dependence on the type (form) of ε, unlike the case of a thin target. Thus, to detect in the emission spectrum the anomalies associated with the resonance level, it is necessary that the electron energy should not be very high (for $\delta\varepsilon \sim 10^{-5}$ the term in (5.42) similar to that in (5.38) exceeds tenfold the terms depending on $\delta\varepsilon$ if the electron energy $E \sim 100$ GeV).

Now turn to quantitative analysis of the radiation spectra in the absence of the energy losses.

Thus, assume that in (5.28) $K(E) = 0$, i.e., neglect the bremsstrahlung loss. Following the similar lines as given by Pafomov [Pafomov (1969)], one may find explicit solutions of equations (5.28) for u. Substitution of thus derived expressions for u into (5.15) and integration with respect to the angles of electron scattering, give the following expressions for the radiation intensity $W_{s\vec{n}\omega}$

$$W_{\perp\vec{n}\omega} = \frac{e^2\omega^2}{2\pi^2 c}\left\{\int_0^\infty dt\int_0^\infty dt'\frac{T}{p_0^2}\exp\left[i\omega(t-t')(1-\beta+\frac{1}{2}\vartheta^2)\right.\right.$$
$$\left.+\frac{\vartheta^2\eta_0^4 T(t-t')^2}{4p_0 q}\right] + 2\mathtt{Re}\int_0^T dt\int_0^\infty dt'\frac{T-t}{H^2\cosh^2\eta t}$$
$$\times\exp\left[-i\omega t'(1-\beta+\frac{1}{2}\vartheta^2) - i\omega t(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon)\right.$$
$$\left.+\frac{\eta\vartheta^2}{4q(1-\vartheta^2+\delta\varepsilon)}\left(\eta t-\frac{\tanh\eta t}{H}\right)+S\right]$$
$$+2\mathtt{Re}\int_0^T dt\int_0^t d\tau\frac{\eta_1}{p^2\cosh^2\eta_2\tau\sinh\eta_1(T-t)}$$
$$\times\exp\left[-i\omega\vec{\tau}(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon^*)-\frac{\eta_1^2}{2q}+\frac{\eta_2\vartheta^2}{4q(1-\vartheta^2+\delta\varepsilon^*)}\right.$$
$$\left.\left.\times\left(\eta_2\tau-\frac{\eta_1\coth\eta_1(T-t)\tanh\eta_2\tau}{p}\right)\right]\right\}, \tag{5.44}$$

$$W_{\parallel\vec{n}\omega} = \frac{e^2\omega^2\vartheta^2}{4\pi^2 c}\left\{\int_0^\infty dt\int_0^\infty dt'\frac{1}{p_0^2}\left(\frac{1}{p_0^2}+2qT\vartheta^{-2}\right)\right.$$
$$\times\exp\left[i\omega(t-t')(1-\beta+\frac{1}{2}\vartheta^2)+\frac{\vartheta^2\eta_0^4 T(t-t')^2}{4p_0 q}\right]$$
$$+2\mathtt{Re}\int_0^T dt\int_0^\infty dt'\frac{1}{H\cosh\eta t}\left[1+R+\frac{2q(T-t)\vartheta^{-2}}{H\cosh\eta t}\right]$$
$$\times\exp\left[-i\omega t'(1-\beta+\frac{1}{2}\vartheta^2)-i\omega t(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon)\right.$$
$$\left.+\frac{\eta\vartheta^2}{4q(1-\vartheta^2+\delta\varepsilon)}\left(\eta t-\frac{\tanh\eta t}{H}\right)+S\right]$$
$$+2\mathtt{Re}\int_0^T dt\int_0^t d\tau\frac{\eta_1^2\cosh\eta_1(T-t)}{p^2\cosh^2\eta_2\tau\sinh^2\eta_1(T-t)}$$
$$\times\left[\frac{\eta_1\coth\eta_1(T-t)}{p}+\frac{2q\vartheta^{-2}\tanh\eta_1(T-t)}{\eta_1}\right]$$
$$\times\exp\left[-i\omega\tau(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon^*)\right. \tag{5.45}$$
$$\left.\left.-\frac{\eta_1^2}{2q}+\frac{\eta_2\vartheta^2}{4q(1-\vartheta^2+\delta\varepsilon^*)}\left(\eta_2\tau-\frac{\eta_1\coth\eta_1(T-t)\tanh\eta_2\tau}{p}\right)\right]\right\},$$

where

$$\delta\varepsilon=\varepsilon-1;\quad \eta_0=\sqrt{2i\omega\beta q(1-\frac{1}{2}\vartheta^2)},$$

$$\eta = \sqrt{2i\omega\beta q(1 - \frac{1}{2}\vartheta^2 + \frac{1}{2}\delta\varepsilon)}\,, \quad \eta_1 = \sqrt{2\omega\beta q \mathtt{Im}\varepsilon}\,,$$

$$\eta_2 = \sqrt{2i\omega\beta q(1 - \frac{1}{2}\vartheta^2 + \frac{1}{2}\delta\varepsilon^*)}\,,$$

$$p_0 = 1 - \eta_0^2 T(t - t')\,,\ p = [\eta_2 \tanh \eta_2 \tau + \eta_1 \coth \eta_1 (T - t)]\,,$$

$$H = \left[1 + \eta(T-t)\tanh \eta t + \frac{\eta_0^2 t'}{\eta}(\eta(T-t) + \tanh \eta t)\right]\,,$$

$$S = \frac{\vartheta^2 \eta (T-t) \tanh \eta t}{2qH(1 - \vartheta^2 + \delta\varepsilon)} \times \left\{(\eta^2 - \eta_0^2)t'\left[\frac{1}{2} + \frac{\cosh \eta t - 1}{\eta(T-t)\sinh \eta t}\right] + \frac{\eta^2 t'}{2}\right.$$
$$\left.\times \left[1 + \eta t' \coth \eta t + \frac{t'}{T-t}\right]\right\}\,,$$

$$R = \frac{1}{H}\left\{-\tanh \eta t \left[1 - \frac{\eta(T-t)(\cosh \eta t - 1)}{H \sinh \eta t \cosh \eta t}\right]\right. \tag{5.46}$$

$$\times \left[2\eta(T-t) + \frac{[1 + \eta_0^2 t'(T-t)](\cosh \eta t - 1)}{\sinh \eta t}\right. \tag{5.47}$$

$$\left. + \eta t' \left(1 + \frac{\eta(T-t)(\cosh \eta t + 1)}{\sinh \eta t}\right)\right] + \frac{\eta^3 (T-t)^2 t' (\cosh \eta t - 1)}{H \sinh \eta t \cosh^2 \eta t}$$

$$+ \eta(T-t)\tanh \eta t \left(\eta t' + \frac{\cosh \eta t - 1}{\sinh \eta t}\right) \tag{5.48}$$

$$\left. \times \left[\eta t' \frac{1 + \eta(T-t)\coth \eta t}{H \cosh \eta t} + \frac{\cosh \eta t - 1}{\eta(T-t)\sinh \eta t}\right]\right\}\,.$$

Formula (5.44) describes the emission of photons polarized perpendicular to the their exit plane. The origin of this radiation is closely connected with scattering in a media (at $q \to 0$ the radiation disappears), so it may be related to bremsstrahlung.

Formula (5.45), referring to the emission of photons polarized in the exit plane, contains contributions associated not only with bremsstrahlung but also with the transitional mechanism of radiation as well as with their mutual interference. The first term in (5.45) describes radiation in a vacuum through particle emission from matter into vacuum, the second term - the interference of radiation at emission, and radiation produced on the part of the particle trajectory in matter. The third term contains contributions to radiation, which are caused by scattering in matter as well as uniform motion to the point of exit from matter.

If we are concerned with the radiation spectrum of a particle passing through the plate of thickness $L = vT$ rather than that of a particle passing through the matter-vacuum boundary, then as seen from the appropriate calculations, the formulae for angular and spectral distributions of photons polarized perpendicular to the exit plane remain the same, but the terms of the form as given below are to be added to expression (5.45)

$$
\begin{aligned}
W'_{\parallel\vec{n}\omega} &= \frac{e^2\omega^2\vartheta^2}{4\pi^2 c}\left\{\frac{e^{-\omega\beta T\,\mathtt{Im}\varepsilon}}{\omega^2(1-\beta+\frac{1}{2}\vartheta^2)^2} + \frac{2e^{-\omega\beta T\,\mathtt{Im}\varepsilon}}{\omega(1-\beta+\frac{1}{2}\vartheta^2)}\mathtt{Im}\int_0^T \frac{dt}{\cosh^2\eta_2 t}\right. \\
&\times \exp\left[-i\omega t(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon^*) + \frac{\eta_2\vartheta^2}{4q(1-\vartheta^2+\delta\varepsilon^*)}(\eta_2 t-\tanh\eta_2 t)\right] \\
&+\frac{2}{(1-\beta+\frac{1}{2}\vartheta^2)}\mathtt{Im}\int_0^\infty \frac{dt'}{p_1\cosh\eta T}\left[1-\frac{\tanh\eta T}{p_1}\left(\eta t'+\frac{\cosh\eta T-1}{\sinh\eta T}\right)\right] \\
&\times \exp\left[-i\omega t'(1-\beta+\frac{1}{2}\vartheta^2) - i\omega T(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon)\right. \\
&+\frac{\eta\vartheta^2}{4q(1-\vartheta^2+\delta\varepsilon)} \\
&\left.\left.\times\left(\eta T-\frac{\tanh\eta T}{p_1}+\frac{(\eta t')^2\sinh\eta T+2\eta t'(\cosh\eta T-1)\left(\frac{\eta_0^2}{\eta^2}-1\right)}{p_1\cosh\eta T}\right)\right]\right\},
\end{aligned}
\tag{5.49}
$$

where

$$
p_1 = 1 + \frac{\eta_0^2 t'}{\eta}\tanh\eta T\,.
$$

The first term in (5.49) describes radiation at stopping, the second term, the interference between the radiation produced on the particle trajectory in vacuum (before it enters the matter) and the radiation in the matter. The third term is the interference between the radiations appearing on the particle trajectory in vacuum before the particle enters the plate and after it leaves it.

With increasing L, the contribution of $W'_{\parallel\vec{n}\omega}$ disappears leading to conversion of the formula describing the intensity of radiation produced in the plate into the one for the intensity of photons produced by a particle passing from matter into vacuum. This is understandable, as the radiation originating from the first vacuum–matter boundary is completely absorbed in the target if the plate thickness is much greater than the absorption depth of γ-quanta.

Consider some limiting cases for the derived general formulas (5.44)–(5.49).

1. Absorption of γ-quanta in the plate may be ignored. The plate thickness is much smaller than the photon absorption depth. Assuming that in (5.50)-(5.49) $\mathtt{Im}\,\varepsilon = 0$, one may obtain [3]

$$
\begin{aligned}
W_{\perp\vec{n}\omega} &= \frac{e^2\omega^2 q}{2\pi^2 c}\left\{\int_0^\infty dt\int_0^\infty dt'\frac{T}{p_0^2}\exp\left[i\omega(t-t')(1-\beta+\frac{1}{2}\vartheta^2)\right.\right.\\
&+\left.\frac{\vartheta^2\eta_0^4 T(t-t')^2}{4p_0 q}\right]+2\mathtt{Re}\int_0^T dt\int_0^\infty dt'\frac{T-t}{\tilde{H}^2\cosh^2\eta_0 t}\\
&\times\exp\left[-i\omega t'(1-\beta+\frac{1}{2}\vartheta^2)-i\omega t(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon')\right.\\
&+\left.\frac{\eta_0\vartheta^2}{4q}\left(\eta_0 t-\frac{\tanh\eta_0 t}{\tilde{H}}+\tilde{S}\right)\right]+2\mathtt{Re}\int_0^T dt\int_0^t d\tau\frac{T-t}{\tilde{p}^2\cosh^2\eta_0\tau}\\
&\times\left.\exp\left[-i\omega\tau(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon')+\frac{\eta_0\vartheta^2}{4q}\left(\eta_0\tau-\frac{\tanh\eta_0\tau}{\tilde{p}}\right)\right]\right\},
\end{aligned}
\tag{5.50}
$$

[3]Formulae (5.50), (5.51) coincide with the expressions derived by V.Ye. Pafomov when analyzing the process of radiation in a plate (see [Pafomov (1969)],§26, formulae (26.13)-(26.22)). Note, however, that the second and third summands in (26.15) as well as the fifth and the sixth ones in (26.16) contain errata. Formulae (5.50), (5.51) were obtained by Garibyan and Yan independently of us [Garibian and Yan (1976)].

$$
\begin{aligned}
W_{\parallel\vec{n}\omega} &= \frac{e^2\omega^2\vartheta^2}{4\pi^2 c}\left\{\int_0^\infty dt\int_0^\infty dt'\frac{1}{p_0^2}\left(\frac{1}{p_0}+2qT\vartheta^{-2}\right)\right.\\
&\times\exp\left[i\omega(t-t')(1-\beta+\frac{1}{2}\vartheta^2)+\frac{\vartheta^2\eta_0^4T(t-t')^2}{4p_0q}\right]\\
&+2\mathtt{Re}\int_0^T dt\int_0^\infty dt'\frac{1}{\tilde{H}^2\cosh^2\eta_0 t}\left[\frac{1+\eta_0 t'\tanh\eta_0 t}{\tilde{H}}+2q(T-t)\vartheta^{-2}\right]\\
&\times\exp\left[-i\omega t'(1-\beta+\frac{1}{2}\vartheta^2)-i\omega t(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon')\right.\\
&\left.+\frac{\eta_0\vartheta^2}{4q}\left(\eta_0 t-\frac{\tanh\eta_0 t}{\tilde{H}}+\tilde{S}\right)\right]\\
&+2\mathtt{Re}\int_0^T dt\int_0^t d\tau\frac{1}{\tilde{p}^2\cosh^2\eta_0\tau}\left[\frac{1}{\tilde{p}}+2q(T-t)\vartheta^{-2}\right]\\
&\times\exp\left[-i\omega\tau(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon')+\frac{\eta_0\vartheta^2}{4q}\left(\eta_0\tau-\frac{\tanh\eta_0\tau}{\tilde{p}}\right)\right]\\
&+\frac{1}{\omega^2(1-\beta+\frac{1}{2}\vartheta^2)^2}+\frac{2}{\omega(1-\beta+\frac{1}{2}\vartheta^2)}\mathtt{Im}\int_0^T\frac{dt}{\cosh^2\eta_0 t}\\
&\times\exp\left[-i\omega t(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon')+\frac{\eta_0\vartheta^2}{4q}(\eta_0 t-\tanh\eta_0 t)\right]\\
&+\frac{2}{\omega(1-\beta+\frac{1}{2}\vartheta^2)}\mathtt{Im}\int_0^\infty dt'\frac{1}{\tilde{p}_1^2\cosh^2\eta_0 T}\\
&\times\exp\left[-i\omega t'(1-\beta+\frac{1}{2}\vartheta^2)-i\omega T(1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon')\right.\\
&\left.\left.+\frac{\eta_0\vartheta^2}{4q}\left(\eta_0 T-\frac{\tanh\eta_0 T}{\tilde{p}_1}\right)\right]\right\}, \qquad (5.51)
\end{aligned}
$$

where

$$\eta_0=\sqrt{2i\omega\beta q(1-\frac{1}{2}\vartheta^2)};\quad p_0=1-\eta_0^2T(T-t')\,,$$

$$\tilde{p}_1=1+\eta_0 t'\tanh\eta_0 T;\ \tilde{p}=1+\eta_0(T-t)\tanh\eta_0\tau\,,$$

$$\tilde{H}=1+\eta_0^2(T-t)t'+\eta_0(T-t+t')\tanh\eta_0 t\,,$$

$$\tilde{S}=\frac{\vartheta^2\eta_0^2(T-t)t'\tanh\eta_0 t}{4\tilde{H}q}\left[1+\eta_0 t'\coth\eta_0 t+\frac{t'}{T-t}\right].$$

2. In the case of sufficiently high particle energies when the conditions $q\ll\omega(\mathtt{Im}\,\varepsilon)^2$ are fulfilled, i.e., $qL_c\ll\vartheta_\gamma^2$, where $L_c=\frac{1}{\omega\mathtt{Im}\varepsilon}$ is the absorption

depth of the γ-quantum; $\vartheta_\gamma^2 = 1/\omega L_c$ is the squared effective angle of γ-quantum radiation, all the functions appearing in (5.44)-(5.49) may be expanded in terms of a small argument q. The stated condition is satisfied, for example, for electrons with the energy $E \sim 1$ GeV when studying the Mossbauer radiation spectrum, or for the electrons whose energy $E > 10^{14}$ eV, when studying the spectrum of γquanta with $\omega \sim 1$ GeV. The resulting formulas have the form

$$\begin{aligned} W_{\perp\vec{n}\omega} = & \frac{e^2}{8\pi^2}\frac{|\varepsilon-1|^2 qT}{(1-\beta+\frac{1}{2}\vartheta^2)^2|1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon|^2} \\ & +\frac{e^2 q}{2\pi^2\omega}\frac{\mathtt{Im}\varepsilon(1-\beta+\frac{1}{2}\vartheta^2-\delta\varepsilon')}{(1-\beta+\frac{1}{2}\vartheta^2)|1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon|^4} \\ & +\frac{e^2}{2\pi^2\omega}\frac{q}{\mathtt{Im}\varepsilon|1-\beta+\frac{1}{2}\vartheta^2-\frac{1}{2}\delta\varepsilon|^2}\,, \end{aligned} \tag{5.52}$$

$$W_{\|\vec{n}\omega} = W_{\mathrm{tr}} + \quad (\text{terms} \quad \sim q)\,,$$

where W_{tr} is the contribution of transition radiation; $\delta\varepsilon' = \mathtt{Re}(\varepsilon-1)$.

According to (5.52), at $q \to 0$ the expression for radiation intensity includes a q-independent term (coinciding with the expression for transition radiation of a particle uniformly moving perpendicular to the boundary), and the terms proportional to q. The total intensity has a similar structure

$$W_{\vec{n}\omega} = W_{\|\vec{n}\omega} + W_{\perp\vec{n}\omega}\,.$$

In this approximation ($q \to 0$), the contribution to $W_{\|n\omega}$ of the terms proportional to q, is much smaller than the transition radiation. Here the term describing the transition radiation is maximum for the photon exit angles $\vartheta \sim m/E$. At the same time, the terms proportional to q and associated with the contribution to $W_{sn\omega}$ of the trajectories passing through matter (the last term in (5.52) lead to a broader angular distribution with the effective emission angle

$$\vartheta_\gamma = \frac{1}{\sqrt{\omega L_c}}\,.$$

3. The case of small plate thicknesses.

If $L \ll L_c$, (5.44)–(5.45) take the most simple form. Developing (5.44)–(5.45) as a series in powers of T, gives [Baryshevskii *et al.* (1977, 1976)]

$$W_{\perp\vec{n}\omega} = \frac{e^2}{2\pi^2 c}\frac{qT}{(1-\beta+\frac{1}{2}\vartheta^2)^2}\,,$$

$$W_{\|\vec{n}\omega} = \frac{e^2\vartheta^2}{16\pi^2 c}\frac{|\delta\varepsilon|^2\omega^2T^2}{(1-\beta+\frac{1}{2}\vartheta^2)^2} + \frac{e^2qT}{2\pi^2c}\frac{(1-\beta-\frac{1}{2}\vartheta^2)^2}{(1-\beta+\frac{1}{2}\vartheta^2)^4}\,.$$

Note that considerable fluctuations of the energy losses due to bremsstrahlung, generally speaking, cause changes in the spectra of transition radiation and bremsstrahlung even in a thin plate because the contribution to radiation at the frequency ω will also come from electrons with abruptly changed energy (due to radiation of a hard quantum). Thus, according to [Baryshevskii *et al.* (1977)] in a thin plate with due account of radiation losses

$$W_{\|\vec{n}\omega} = \frac{e^2\vartheta^2|\varepsilon-1|^2(\omega T)^2}{16\pi^2(1-\beta_0+\frac{1}{2}\vartheta^2)^2} + \frac{e^2qT(1-\beta_0-\frac{1}{2}\vartheta^2)^2}{2\pi^2(1-\beta_0+\frac{1}{2}\vartheta^2)^4}$$
$$+\frac{e^2\vartheta^2}{4\pi^2}\frac{T}{(1-\beta_0-\frac{1}{2}\vartheta^2)^2}\int_{E_1}^{E_0}\frac{(\beta_0-\beta)^2}{(1-\beta+\frac{1}{2}\vartheta^2)^2}\sigma(E_0|E)dE\,,$$

where $\beta_0 = \sqrt{1-\frac{m}{E_0}}$; $\sigma(E_0|E)dE = N\sigma_\gamma(E_0|E)$; $\sigma_\gamma(E_0|E)$ is the bremsstrahlung cross-section per unit interval of energies E of the electron with the initial energy E_0; E_1 is the limiting energy (on choosing it, see [Podgoretsky (1977a); Baryshevsky and Grubich (1979c)].

In the general case, the analysis of the formulas for $W_{s\vec{n}\omega}$ even in the absence of the energy losses is only possible when using numerical methods. Below we present the results of such an analysis at generating of resonance photons by an electron passing through the plate containing ^{138}W nuclei ($\omega = 46.5\,\mathrm{keV}$). The stated process is of great interest in connection with the possibility of creating the sources of resonance radiation with the help the beams of relativistic electrons [Perelshtein and Podgoretsky (1970)].

Particle scattering in a medium appreciably affects $W_{\|\vec{n}\omega}$, leading to the fact that at the angles $\vartheta \le m/E$ the radiation intensity of the waves with the polarization parallel to the exit plane of γ-quanta differs from the intensity of transition radiation Figure (5.1).

We also pay attention to the fact that at the angles $\vartheta \le m/E$, the major contribution to $W_{\|\vec{n}\omega}$ is made by the term associated with electron scattering in a medium and equal to $W_{\perp\vec{n}\omega}$. Numerical analysis of the formulas shows that in the range of electron energies ($E \ge 1\,\mathrm{GeV}$) we have discussed, for the values of the radiation angles of γ-quanta up to $\vartheta \sim \sqrt{\mathtt{Im}\varepsilon}$, the predominating contribution to $W_{\perp\vec{n}\omega}$ comes from the term, caused by appearance of the fan of trajectories of electron motion behind the plate due to multiple scattering in a medium. This contribution increases with the growth of the electron energy at fixed frequency of γ-quanta. For the

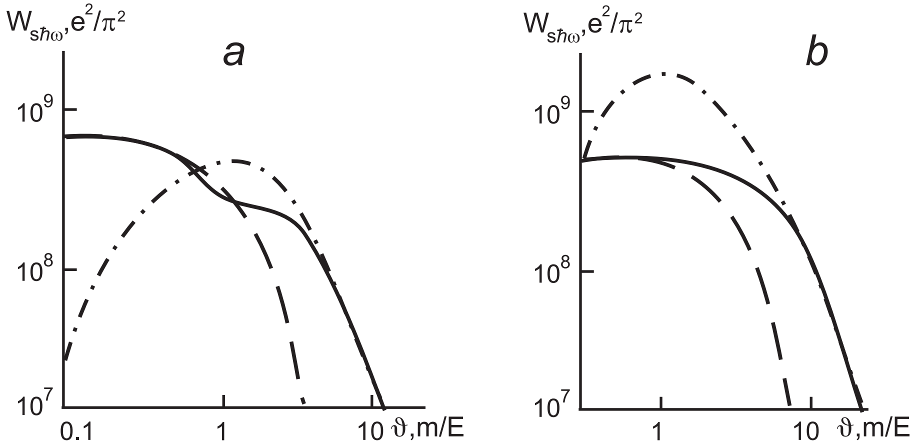

Fig. 5.1 Angular distribution for γ-quanta produced by the electron ($E = 40\,\mathrm{GeV}$) in the plates of thickness L_c (a), and $10L_c$ (b) the values of L_c and $10L_c$ are borrowed from [Baier *et al.* (1979)]. Solid lines — the density of the radiation energy $W_{\|\vec{n}\omega}$; Dashed–dot lines— transition radiation produced by the electron passing through the plate at a constant velocity directed normal to its surface; Dashed lines — the angular distribution of $W_{\perp\vec{n}\omega}$

angles $\vartheta > m/E$, the radiation intensity $W_{\|\vec{n}\omega}$ coincides with the intensity of transition radiation at normal transmission through the plate (see Figure (5.1)). Moreover, the increase in the plate thickness leads to broadening of angular distribution of the radiation energy density of γ-quanta (see Figure (5.1(b))).

We have also calculated the radiation intensity $W_{s\vec{n}\omega}$ for electrons with $E = 1$ GeV in a plate of thickness L_c. The angular distributions obtained are similar to those given in Figure (5.1) for electrons with the energy of 40 GeV. Numerical integration of formulae for non-resonance γ-quanta with the energy of 40 and 200 MeV, and electrons with the energy of 40 and 200 GeV, respectively has also been carried out. Calculations were made for tungsten plates of thicknesses $0.05\,L$ and $0.1\,L$. In this case a substantial contribution to the radiation intensity $W_{\perp\vec{n}\omega}$ proportional to q is made by the fan of vacuum trajectories.

Thus, the fan of vacuum trajectories appreciably changes the pattern of of angular and spectral distributions of the radiation intensity for a particle passing through matter-vacuum boundary.

Chapter 6

Scattering and Radiation in Crystals Exposed to Variable Fields

6.1 Generation of γ-quanta by Channeled Particles in the Presence of Variable Fields

Now let a crystal in which fast particle move be affected by a variable external field (electromagnetic or sound). The latter can influence the process of photon emission by a particle in two ways. On the one hand, it acts directly on the particle, causing forced vibrations in its channel, on the other hand, the field makes the nuclei swing. As a result, the channel where the particle moves starts bending, thus causing the appearance of a variable force which sets the particle into vibration.

As was repeatedly pointed out, electromagnetic radiation produced by spontaneous radiation transitions may be considered as the radiation of a certain oscillator (atom). Causing vibrations of the crystal nucleus, the external field leads to oscillations of the point of the equilibrium position of the oscillator. Suppose that the oscillation frequency of the equilibrium point under the external field is much less than that of the oscillator (i.e., the particle vibration frequency in the channel). In this case the oscillator follows the oscillations of the equilibrium point adiabatically. High-frequency vibrations of the charge, i.e., particle vibrations about the equilibrium position result in spontaneous radiation, discussed above; low-frequency oscillations, which are due to oscillations of the equilibrium point, lead to additional electromagnetic radiation. Consider the features of this radiation, following [Baryshevsky *et al.* (1980d,b)]. (This radiation was also discussed in [Plotnikov *et al.* (1979)] for a particular case of a standing acoustic wave).

Let a crystal be affected by an external wave. As a result, the centers of mass of the atoms execute forced oscillations $\vec{R}_i^{(s)} = \vec{r}_0^{(s)} \cos(\kappa \vec{R}_i - \Omega_s t)$, where $\vec{r}_0^{(s)}$ is the amplitude of forced oscillations; $\vec{R}_i$ - is the coordinate of the equilibrium point of the i-th nucleus; $\vec{\kappa}$ is the wave vector of the external wave; Ω_s is the external wave frequency. When a particle moves in the channel oriented along the z-axis, vibrations of nuclei cause oscillations of the equilibrium position of the oscillator corresponding to the particle in the xy plane as $\vec{r}_\perp^s = \vec{r}_{0\perp}^s \cos\Omega' t$. Here $\Omega' = \kappa_z c - \Omega_s$ is the oscillation frequency of the equilibrium position, c is the velocity of light. In the adiabatic case the particle trajectory in a transverse plane $\vec{r}_\perp(t)$ is determined by the sum $\vec{r}_\perp(t) = \vec{r}_\perp^s(t) + \vec{r}_\perp^c(t)$, where $\vec{r}_\perp^c(t)$ is the trajectory of the charged particle in the channel.

Let us give a more detailed consideration of particle motion in the channel exposed to, for example an ultrasonic wave. Assume that the wave moves in a crystal along the z-axis. The channel bends caused by the undulator may result in dechanneling. The particle will not leave the channel if the minimal radius of curvature of the channel $\rho_{\min}$ satisfies the inequality following from the equilibrium conditions

$$\frac{m\gamma v^2}{\rho_{\min}} \simeq \frac{m\gamma c^2}{\rho_{\min}} \leq |\nabla_\perp u(\vec{r}_\perp)|_{\max}\,, \tag{6.1}$$

where m is the particle rest mass; $\rho_{\min} = (r_{0\perp}^s \kappa^2)^{-1}$; $u(\vec{r}_\perp)$ is the potential energy of particle interaction with the crystal plane.

Suppose that we consider the particles with the amplitude of free vibrations in the channel $r_{0\perp}^c < d/2$ (d is the channel width). In this case the amplitude of ultrasonic vibrations should satisfy the condition $r_{0\perp}^s < 4u_{\max}/E\kappa^2$, E is the particle energy. For example, in silicon $r_{0\perp}^s < 10^{-4}$ cm for $E \sim 1$ GeV and $\Omega_s \sim 2\pi \cdot 10^7$ s^{-1}. Spectral distribution of radiation induced by oscillations of the center of equilibrium of the oscillator may be written in the form

$$\frac{dN}{d\omega} = \frac{e^2 L}{4\hbar c^4}(r_{0\perp}^s \Omega')^2 \left[1 - 2\frac{\omega}{\omega_m'} + 2(\frac{\omega}{\omega_m'})^2\right]\,, \tag{6.2}$$

where $\omega_m' = 2\Omega'\gamma^2$; L is the crystal thickness, $L \gg 1/\kappa$. If the potential $u(r_\perp)$ is harmonic, the formula for the spectrum of radiation produced by free vibrations of a particle in the channel is similar to (6.2). As a result, the relation of the intensity of radiation induced by the external wave to the intensity of spontaneous radiation can be written as follows

$$B \simeq \left(\frac{r_{0\perp}^s \Omega'}{r_{0\perp}^c \Omega}\right)^2 = \frac{4W_s c^2}{\rho v_s^3 \Omega^2 (r_{0\perp}^c)^2}, \tag{6.3}$$

where Ω and $r^c_{0\perp}$ are the frequency and the amplitude of particle vibrations in the channel; $W_s = \frac{1}{4}\rho\kappa^{-1}\Omega_s^3(r^s_{0\perp})^2$ is the power of the ultrasonic wave in W/cm^2, ρ is the density of the medium. Study radiation in the spectral range $\omega \sim 2\Omega'\gamma^2$. According to the condition (6.1),

$$B_{max} \sim \left(\frac{\Omega v_s}{\Omega_s c}\right) \gg 1.$$

So, for positrons with the energy $B \sim 1$ GeV at the ultrasonic wave power $W_s \sim 10^3$ W/cm^2 and frequency $\Omega_s \sim 2\pi 10^7$ s^{-1}, the vibration amplitude $r^s_{0\perp} \sim 10^{-5}$ and the relation $B \simeq 10$ for $r^c_{0\perp} \sim d/2$.

In the soft part of spectrum, it is important to take into account refraction and absorption of photons. he appropriate expression for the spectrum is analogous to that obtained above for spontaneous radiation, and it reads

$$\frac{dN}{d\omega} = \frac{e^2(r^s_{0\perp}\Omega')^2}{4\hbar c^4}\left[1 - 2\frac{\omega}{\omega'_m} + 2(\frac{\omega}{\omega'_m})^2\right] l_{\text{abs}}\left(1 - e^{-\frac{L}{l_{\text{abs}}}}\right), \tag{6.4}$$

where l_{abs} is the absorption length of the photon with frequency ω, $\Omega' = \kappa_z v_z - \Omega_s$. Note that the wave number is related to frequency as $\kappa = \kappa(\Omega_s)$. For the acoustic branch in the long wave limit

$$\kappa = \frac{\Omega_s}{v_s},$$

v_s is the velocity of sound. For light

$$\kappa = \frac{\Omega_s}{c} n(\Omega_s),$$

where $n(\Omega_s)$ is the refractive index.

When a channeled particle moves in a crystal in the presence of a light wave, it undergoes forced vibrations caused by the direct effect of the force from the wave. In this case the amplitude of forced vibrations is

$$x_s = \frac{eE_x}{2m\gamma}(\Omega^2 - \Omega'^2 - i\Omega'\Gamma)^{-1}, \tag{6.5}$$

where $E_x = E_0(1 - \beta n_0 \vec{n}_\gamma \vec{n}_e)$ is the x-component of the external field strength in the medium (the angle of the wave vector $\vec{\kappa}$ with the particle momentum $\vec{p}$ is assumed to be much less than unity or close to π).

The frequency of the emitted photon

$$\omega = \Omega'\left(1 - \frac{v_z}{c} n(\omega)\cos\vartheta\right)^{-1}.$$

As the amplitude of particle forced vibrations cannot exceed the channel width (and the radiation intensity is only determined by the vibration amplitude and frequency), the intensity of the radiation due to the electromagnetic wave cannot exceed the intensity of spontaneous radiation of a channeled particle, vibrating with the same frequency and amplitude.

6.2 Coherent Scattering of Photons by a Beam of Channeled Particles. The Effect of Super-radiation

We have already pointed out in (2.2) that a channeled particle may be considered as a fast atom. This allows us to state that under appropriate conditions for such particles it is possible to observe numerous effects known in atomic physics. Moreover, the similarity of the properties of a channeled particle and a fast atom enables using the results of the photon-atom interaction theory rather than carrying out new calculations in order to find any process (scattering, photon radiation or absorption). For this purpose suffice it in the beginning to consider the process in the coordinate system, where the initial longitudinal momentum of a particle is zero. In this system we deal with a resting atom, the cross-section (amplitude) of photon scattering by which is well known. Further, it is necessary to convert the scattering cross-section (amplitude) to the laboratory coordinate system according to simple rules (see, for example, [Goldberger and Watson (1984(@)], p.86-97) with due account of the fact that the atom corresponding to the channeled particle has a one-dimensional (axial channeling) or two-dimensional (planar channeling) momentum. For example, the amplitude of elastic coherent forward scattering of a photon by a channeled particle

$$f(\omega) = \frac{\sqrt{1-\beta^2}}{|1-\beta_z n(\omega)\cos\vartheta|} f(\omega'), \tag{6.6}$$

where $f(\omega')$ is the scattering amplitude in the rest system ($v_z = 0$)of the channeled particle; $\omega' = (1-\beta_z n(\omega)\cos\vartheta)\gamma\omega$ is the photon frequency in the system. The amplitude $f(\omega')$ has a usual Breit-Wigner form.

Using the optical theorem, we also immediately find the total cross-section of photon scattering by a channeled particle $\sigma = \frac{4\pi c}{\omega}\mathtt{Im} f(\omega)$.

Now estimate the addition $\delta\varepsilon$ to the dielectric permittivity of a crystal caused by photon interaction with a beam of particles. According to [Baryshevsky (1976); Lax (1951)] $\delta\varepsilon = \frac{4\pi\rho A c^2}{\omega^2} f(\omega)$, where ρ is the beam density; the difference of the constant A from unity is due to the difference between the mean field in the medium and the local one acting on a moving atom. For the media with ε close to unity, $A \simeq 1$. The effect of refraction by the beam is appreciable, when $\frac{1}{2}\frac{\omega}{c}\delta\varepsilon L \geq 1$. For a particle moving in a harmonic potential,

$$f(\omega') = -\frac{r_0}{2}\frac{\omega'}{\omega' - \Omega' - i\frac{\Gamma'}{2}},$$

where r_0 is the classical electron radius; Ω' and Γ' are the oscillation frequency of the oscillator and the width of the transition in the particle rest system, respectively. As a result, under the resonance conditions refraction is great, if $\frac{2\pi\rho c r_0}{\gamma\Gamma} \geq 1$; $\Gamma = \Gamma'\gamma^{-1}$ is the line width in the laboratory system. If the particle bunch thrown onto a crystal is accelerated as a whole, $\rho\gamma^{-1}$ is the bunch density before the acceleration ρ_0. As a result, $\rho_0 \geq \frac{\Gamma}{2\pi c r_0 L}$, i.e., $\rho_0 \geq 10^{13}$ for $\Gamma = 10^{12}$ s^{-1} and $L = 1$ cm. It should be noted that according to (6.6) the anomalous and complex Doppler effects lead to the fact that in a resonance with a moving oscillator an emitted hard photon appears along with a soft one. Therefore hard photons are also effectively refracted by a beam.

Since the amplitude of photon scattering by a channeled particle depends on the photon polarization, the channeled beam is an optically anisotropic medium. For example, in the case of planar channeling a beam is a birefringent medium. Under the stated conditions, the particle interaction through the field of photons is considerable. As a consequence, the formation of exciton polaritons in such a beam is possible, and when the beam is affected by a light pulse, the oscillators corresponding to channeled particles may be driven into the super-radiant state. In their system a boson avalanche may evolve, and eventually generation of ultrashort radiation pulses may occur [Baryshevsky (1980d,a)] (compare with similar phenomena in atomic physics [Allen and Eberly (1975); Bogdanov *et al.* (1979)]).

The presence of spin in electrons leads to spin-orbital level splitting at axial channeling, i.e., to the appearance of fine structure of the levels. That is why the effect of a circularly polarized electromagnetic wave on such particles will result in spin polarization of the electron beam (compare with the effects of electron polarization through photoionization of atoms [Delone and Fedorov (1979)]). Note that in the transition of channeled electrons between the states with different orbital moments, the efficiency of their interaction with a crystal changes sharply. s-electrons with enhanced density at nuclei dechannel faster than, for example, p-electrons. If channeled unpolarized electrons are, for example, in p-state, then a circularly polarized wave, causing the transition between one of the components of the fine structure of this state and s-state, will transfer a fraction of the electrons to s-state, from which they dechannel rapidly. As a result, the passing beam will appear to be partially polarized. If the crystal is thin enough to neglect dechanelling processes, in order to select polarized electrons, one can make use of the fact that the angular distributions of electrons passing through the crystal depend on the state in which they were in the crystal. A circularly polarized wave, transferring electrons from s-state to one of the states of the fine structure, will cause, for example, non-zero degree of electron polarization in the directions of the p-electron escape.

We also point out that, due to equally probable occupation of the state with different projection of the orbital moment on the axis, the degree of circular polarization of photons produced by a particle through axial channeling is practically zero. A circularly polarized wave, changing the occupation of different projections of the orbital moment, causes the emission of circularly polarized photons.

The probabilities of the induced processes discussed here are easy to find, according to the well known rules (see, for example, [Berestetsky *et al.* (1968)], §44, using the probability of a spontaneous process for polarized particles, whose explicit form is given in (3.2).

An intense light wave, causing the transitions between different states of a channeled particle will lead to the fact that the particle beam leaving the crystal will turn out to be spatially modulated (compare with the effect of modulation of a beam passing through a dielectric plate [Varshalovich and D'yakonov (1970, 1971)], and the modulation effect at electron diffraction in a single crystal [Fedorov (1980b)]). This new modulation mechanism exhibits high efficiency, and, due to fine splitting of levels, causes spatial modulation of the degree of polarization of the initially unpolarized particle beam (provided the crystal is illuminated (irradiated) by a circularly polarized wave).

As mentioned above, coherent occupation of the levels of transverse motion at the particle entering the crystal brings about beatings in the radiation intensity, depending on the target thickness. The degeneracy of the levels, likewise in atomic physics will give the opportunity to observe the burst in the intensity of radiation produced by channeled particles when the level crossing is stimulated, for example, by means of crystal bending.

6.3 Induced Scattering and Radiation under Diffraction Conditions

It is common knowledge (also see above) that, due to the periodic arrangement of atoms (nuclei) the energy spectrum of particles (γ-quanta) moving in a crystal exhibits the energy-band structure $E_{f\vec{k}}$ (f is the band number, $\vec{k}$ is the reduced quasi-momentum). For a particle with spin the spectrum also depends on the spin state of the incident beam [Baryshevsky (1976)]. The energy-band structure of the spectrum causes spontaneous transitions between the bands accompanied by the emission of photons, phonons, plasmons, etc., and, as a result, simulated transitions. Simulated transitions between the bands bring about resonance repolarization, modulation of a neutron beam, the change in the rate of nuclear reactions in crystals, and polarization of particles [Baryshevsky (1976, 1980b, 1979a)]. As an example consider simulated neutron transitions between the bands under the action of phonons , i.e., under ultrasonic pumping. By the action of ultrasound on a crystal nuclei in the equilibrium position start executing forced vibrations according to

$$\delta\vec{R}_i(t) = \vec{a}\sin(\vec{\kappa}\vec{R}_i - \Omega(\kappa)t + \delta), \tag{6.7}$$

where $\vec{a}$ is the amplitude of forced vibrations of the nucleus; $\vec{\kappa}$ is the wave vector of phonons; $\Omega(\kappa)$ is the phonon frequency; δ is the initial vibration phase; $\vec{R}_i$ is the equilibrium coordinate of the nucleus in the absence of phonons.

The Schrodinger equation describing diffraction of neutrons by a vibrating crystal has the form

$$i\hbar\frac{\partial\psi}{\partial t} = \left\{-\frac{\hbar^2}{2m}\Delta_r \ + \sum_i V[\vec{r} - \vec{R}_i - \vec{a}\sin(\vec{\kappa}\vec{R}_i - \Omega t + \delta)]\right\}\psi,$$

where V is the coherent potential of interaction of neutrons with nuclei.

In the expression for the potential (6.8) perform the summation over positions of nuclei. With this aim in view introduce the Fourier transform of the potential $V(q)$:

$$\begin{aligned} u(t) \equiv \sum_i V(\vec{r} - \vec{R}_i - \delta\vec{R}_i(t)) &= \frac{1}{(2\pi)^3}\sum_i \int d^3q V(\vec{q}) \\ &\times \exp\left\{i\vec{q}(\vec{r} - \vec{R}_i - \vec{a}\sin(\vec{\kappa}\vec{R}_i - \Omega t + \delta))\right\} \\ &= \frac{1}{(2\pi)^3}\sum_i \int d^3q V(\vec{q}) e^{i\vec{q}(\vec{r}-\vec{R}_i)}\left\{J_0(\vec{q}\vec{a})\right. \\ -2i\sum_{n=0}^{\infty} J_{2n+1}(\vec{q}\vec{a})&\sin[(2n+1)(\vec{\kappa}\vec{R}_i - \Omega t + \delta)] \\ &\left. +2\sum_{n=0}^{\infty} J_{2n}(\vec{q}\vec{a})\cos[2n(\vec{\kappa}\vec{R}_i - \Omega t + \delta)]\right\}. \end{aligned} \tag{6.8}$$

To perform the summation over i in (6.8), note that

$$\sum_i e^{-i(\vec{q}-n\kappa)\vec{R}_i} = \frac{(2\pi)^3}{v_0}\sum_\tau \delta(\vec{q} - n\vec{\kappa} - 2\pi\vec{\tau}), \tag{6.9}$$

where $2\pi\vec{\tau}$ is the reciprocal lattice vector; v_0 is the volume of the unit cell (it is assumed to be simple) in the crystal.

Further we shall consider diffraction by a set of planes characterized by such a vector $2\pi\vec{\tau}$ that $2\pi\vec{\tau}a \ll 1$ ($2\pi\tau \sim 10^8 \div 10^9$ cm^{-1}, $a \sim 10^{-10}$ cm). In this case in (6.8) it takes only to retain the terms containing $J_0(\vec{q}\vec{a}) \simeq 1$ and $J_1(\vec{q}\vec{a}) \simeq \frac{1}{2}\vec{q}\vec{a}$. As a result we have

$$u(t) = \frac{1}{v_0}\sum_{\tau} V(2\pi\vec{\tau})e^{i2\pi\vec{\tau}\vec{r}} + \delta V_{(+)}(\vec{r})e^{i\Omega t} - \delta V_{(-)}(\vec{r})e^{-i\Omega t};$$

$$\delta V_{(\pm)}(\vec{r}) = \frac{1}{2v_0}\sum_{\tau} V(2\pi\vec{\tau})(2\pi\vec{\tau}\vec{a})e^{i(2\pi\tau\mp\vec{\kappa})\vec{r}}e^{\pm i\delta}. \quad (6.10)$$

In view of (6.10) the the potential of neutron interaction with the crystal lattice moving under ultrasonic wave may be represented as a sum of two summands. The first one describes particle diffraction in a static grating, the second one - the time-periodic perturbation, which is the superposition of plane wave traveling in the crystal. Diffraction by these waves is possible as well as diffraction by static ones produced by the first summand, which, unlike diffraction in the static case, is accompanied by the change in the particle energy by the amount divisible into $\hbar\Omega$. As a consequence, the perturbation described by the second summand may cause resonant transitions between the energy band.

To find the probability of the interband transition per unit time under periodic perturbation, one should know stationary states of an unperturbed problem. (In the case in question the stationary wave functions describing the diffraction process in a crystal are well known [Baryshevsky (1976)]). If a crystal is a plate of thickness l, inside the crystal the wave function of the initial state in the two-wave Laue case has the form

$$\psi_k^{(+)} = \frac{1}{L^3}\sum_{\alpha=1}^{4} A_\alpha e^{i\vec{k}_\alpha\vec{r}}, \quad (6.11)$$

where L^3 is the normalization volume; $\vec{k}_1 = (\vec{k}_\perp, \vec{k}_{z1})$; $\vec{k}_2 = (\vec{k}_\perp, \vec{k}_{z2})$; $\vec{k}_3 = \vec{k}_1 + 2\pi\vec{\tau}$; $\vec{k}_4 = \vec{k}_2 + 2\pi\vec{\tau}$; $k_{z1} = k_z n_1(k_z)$; $k_{z2} = k_z n_2(k_z)$; k_z is the z-th component of the wave vector of neutrons in a vacuum; the expressions for refractive indices under diffraction conditions are given in [Baryshevsky (1976)]; $\vec{k}_\perp$ is the component of the wave vector of the incident wave, perpendicular to the z-axis (parallel to the crystal surface); $2\pi\vec{\tau}$ is the reciprocal lattice vector characterizing the family of diffracting planes.

In the final state one should use the wave function of the type $\psi_k^{(-)}$, under diffraction conditions having the form

$$\psi^{(-)}_{k'}(\vec{r}) = \frac{1}{\sqrt{L^3}} \sum_{\alpha=1}^{4} A'^*_{\alpha} \exp(i\vec{k}'^*_{\alpha}\vec{r}) \exp\left(-i\frac{k'}{\gamma'_0}\varepsilon'^*_{\alpha} l\right), \tag{6.12}$$

where $\varepsilon'_3 = \varepsilon'_1$; $\varepsilon'_4 = \varepsilon'_2$; k' is the wave number in a vacuum of the neutron which has undergone the transition; ε_α and γ_0 determining the refractive indices n_1 and n_2 are given in [Baryshevsky (1976)] (also see (4.4).

The probability of transition per unit time that is of interest to us is

$$W_{\vec{k}'\vec{k}} = \frac{2\pi}{\hbar} | < \psi^{(-)}_{\vec{k}'} | \delta V_{(\pm)} | \psi^{(+)}_{\vec{k}} > |^2 \delta(E_{\vec{k}'} \pm \hbar\Omega - E_k) \frac{L^3 d^3 k'}{(2\pi)^3} \tag{6.13}$$

where the plus (minus) sign refers to the transitions with the energy loss (acquisition); $E_{\vec{k}} = \hbar^2 k^2/2m$; $E_{\vec{k}'} = \hbar^2 k'^2/2m$.

Consider the matrix element appearing in (6.13). Integrated in it is performed with respect to the volume of the crystal plate. As the wave functions and perturbation $\delta V_{(\pm)}$ are the superpositions of plane waves, then integration over the crystal surface leads to appearance in the matrix element of two-dimensional δ-functions oft the form $\delta(\vec{k}'_\perp - 2\pi\vec{\tau}''_\perp \pm \vec{\kappa}_\perp - \vec{k}_\perp)$, fixing the component of the momentum parallel to the plate surface.

The stated δ-functions together with the δ-function with respect to energy included into (6.13) allow in (6.13) integration over d^3k'. As a result, one-dimensional integrals of the type as follows will remain in (6.13)

$$\begin{aligned} &\frac{1}{L}\int_0^l \exp\{-i(k'^*_{\alpha' z} \pm \kappa_z - k_{\alpha z})z\}\, dz \\ &= \frac{i}{L}\frac{\exp\{-i(k'^*_{\alpha' z} \pm \kappa_z - k_{\alpha z})l\} - 1}{k'^*_{\alpha' z} \pm \kappa_z - k_{\alpha z}}, \end{aligned} \tag{6.14}$$

which are the sharp functions of the difference of the z-projections of the wave numbers. Such integrals take on their maximum values when the difference of the real parts of the wave numbers vanishes. In other words, the process of neutron interaction with a vibrating crystal is governed by energy-momentum conservation law of the form

$$\begin{aligned} E_{k'} &= E_k \mp \hbar\omega; \\ \vec{k}'_\perp &= \vec{k}_\perp \mp \vec{\kappa}_\perp + 2\pi\vec{\tau}''_\perp \\ &\textit{or} \\ \vec{k}'_\perp = \vec{k}_\perp \mp \vec{\kappa}_\perp &+ 2\pi\vec{\tau}''_\perp + 2\pi\vec{\tau}_\perp \text{ and so on,} \end{aligned} \tag{6.15}$$

$$\mathtt{Re}(k'_{\alpha' z} = k_{\alpha z} \mp \kappa_z + 2\pi\tau_z)$$

or $\mathrm{Re}\, k'_{\alpha' z} = \mathrm{Re}\, k_{\alpha z} \mp \kappa_z + 2\pi\tau''_z + 2\pi_z$ and so on, where $\mp\vec{\kappa} + 2\pi\vec{\tau}''$ is the momentum due to δV interaction; $\vec{k}, \vec{k} + 2\pi\vec{\tau}$ is the primary momentum due to the particle.

If equalities (6.15) are fulfilled, then in the case when the crystal depth is less than the particle absorption depth, the integral in (6.14) equals lL^{-1}. Substitution of (6.14) into (6.13) demonstrates that the probability of transition per unit time integrated over d^3k' proves to be an oscillating function of the crystal thickness l with the spatial oscillation periods determined by the difference of the refractive indices of the plane waves involved in diffraction. The maximum value of the probability of transition is attained when (6.15) is fulfilled, being equal, for example, for neutrons exiting behind the crystal in the positive direction of the z-axis, in the case of the exact fulfillment of the Bragg conditions to

$$W = \int dW_{\vec{k}'\vec{k}} \simeq \frac{l^2}{\hbar^2 v_z L} \left| \frac{V(2\pi\tau'')(2\pi\vec{\tau}''\vec{a})}{2v_0} \right|^2 \tag{6.16}$$

where v_z is the z-th component of the particle velocity.

The experimentally observable quantity is the cross-section of the process $\sigma = L^3 W/v_z$, or the fraction of particles that have undergone a transition, per one incident particle: $\delta N = \sigma/L^2$. From (6.16) follows that

$$\delta N \simeq \frac{l^2}{\hbar^2 v_z^2} \left| \frac{V(2\pi\tau'')(2\pi\vec{\tau}''\vec{a})}{2v_0} \right|^2 . \tag{6.17}$$

According to [Baryshevsky (1976)] $V(2\pi\vec{\tau})$ may be expressed in terms of the amplitude of neutron scattering by a nucleus:

$$V(2\pi\tau) = -\frac{2\pi\hbar^2}{m} f e^{-W(\tau)}, \tag{6.18}$$

where f is the amplitude of coherent scattering of the neutron by the nucleus; $e^{-W(\tau)}$is the Debye-Waller factor.

In view of (6.17) the value of δN is maximum when the nuclei vibrate along the direction of τ, e.i., perpendicular to the planes by which the particle is diffracted. Note that the analogous result is also obtained in the case when the change in the particle energy through diffraction by a vibrating grating is ignored (static approximation, see [Entin (1979)]).

Estimate the magnitude of the effect. The scattering amplitudes are of the order of $10^{-12} \div 10^{-13}$ cm, the vibration amplitudes - $a \simeq 10^{-10}$ cm. Hence, for thermal neutrons ($v \simeq 10^5$ cm/sec) the fraction of particles that have undergone a transition is $\delta N \simeq 10^2 l^2$. From this follows that at crystal thicknesses as small as $l \simeq 10^{-1}$ cm all the particles undergo a transition with a change in energy. At large thicknesses the perturbation theory is not applicable. In this case it is helpful to consider the problem in terms of effective refractive indices in a rotating coordinate system [Baryshevsky (1976); Varshalovich and D'yakonov (1970)], or to usie the conception of quasi-energy.

Vibrations of nuclei in a crystal may be caused by either an ultrasonic or an electromagnetic wave. Under diffraction (channeling) of charged particles in a crystal the wave affects not only the nuclei but also the particle itself, bringing about the additional mechanism of interband transitions. Interband transitions of electrons induced by ultrasound (electromagnetic field) are accompanied by simulated radiation. Naturally, spontaneous interband transitions also exist.

Radiation through diffraction in the case of optical transitions between the neighboring bands in an infinite crystal without reference to spin structure of the bands was discussed in [Fedorov and Smirnov (1974); Fedorov *et al.* (1973); Fedorov (1980a)]. In view of the above analysis, taking into account a finite crystal thickness results in appearing of oscillations of the radiation intensity, depending on l and electron energies. The dependence of the band structure of electrons diffracting in a crystal on their spin will lead to the dependence of the intensity and polarization properties of radiation on the beam polarization state, as well as polarization of a non-polarized beam ((6.2)). Resonant interband transitions of electrons will cause the appearance of a spatially modulated beam behind the crystal. The beam modulation period will depend on the spin orientation. As a result, the initially non-polarized beam behind the crystal will prove to be spatially polarized in some regions of space. Of course, the aforesaid also refers to the electrons which moved in the channeling regime.

Let now γ-quanta be diffracted in a crystal (light in a liquid crystal or in some other periodic structure (array)). In this case even in a non-magnetic crystal in a wide energy range (from several kiloelectron-volts to tens and hundreds of gigaelectron-volts) there is band splitting, depending on the photon polarization state. Diffraction of Mossbauer γ-radiation in polarized crystals is considered in [Baryshevsky (1976)]. At two-wave diffraction the wave functions of γ-quanta are analogous to the functions in (4.37)-(4.40). For this reason the structure of the matrix element describing the transition of a γ-quantum from one band state to another is also similar to the structure of the matrix element appearing in (6.13). Consequently, ultrasonic (electromagnetic field) induces resonant repolarization of the diffracting beam of γ-quanta (light passing through a liquid crystal and etc.) under the condition determined by the conservation laws (6.15). The process of the interband transition of X-rays by the action of ultrasound without reference to the change in their frequency and polarization through the transition was treated in [Entin (1979)]. In the case of Mossbauer γ-quanta it is crucial that the change in the γ-quantum frequency through transition described by the conservation law should be taken into account.

Due to the close connection between the phenomena of diffraction and mirror reflection under diffraction conditions [Baryshevsky (1976)], analogous effects will manifest themselves for mirror reflected waves (neutrons, γ-quanta, light) too. In fact, the process of the interband transition of X-rays and γ-quanta under the electromagnetic wave causing vibrations of the crystal nuclei can be treated the process of coherent coalescence (splitting) of a γ-quantum and an optical photon.

6.4 Optical Anisotropy in a Rotating Coordinate System

It has been shown above that spectral-angular distribution of photons produced by particles passing through a crystal, depend considerably on the refracting properties of the medium. If a crystal is placed in an external variable field, its refracting properties change sharply. In particular, the effects caused by optical anisotropy of crystals in the γ-range acquire qualitatively new features, when the material is placed in a time-dependent external field (electromagnetic, sound).

To consider the essence of the arising phenomena, let us begin with a simple example of neutron refraction in a constant magnetic field on which a time-dependent transverse variable field is imposed [Baryshevsky (1979a)]. The Schrodinger equation describing the stated process has the form:

$$i\hbar\frac{\partial\psi}{\partial t} = \left\{-\frac{\hbar^2}{2m}\Delta_r - \vec{\mu}\vec{H}(\vec{r},t)\right\}\psi, \qquad (6.19)$$

where m is the neutron mass; $\vec{\mu} = \mu\vec{\sigma}$ is its magnetic moment; $\vec{\sigma}$ is the vector made up of the Pauli matrices $\sigma_x, \sigma_y, \sigma_z$; $\vec{H}(\vec{r},t)$ is the magnetic field acting on the neutron at point $\vec{r}$ at moment t with the components $H_x = H_\perp \cos\omega t$, $H_y = H_\perp \sin\omega t$, H_z is time-independent; ω is the rotation frequency of the transverse magnetic field.

Using the explicit form of $\vec{\sigma}$, one may obtain the following system of equations for the components ψ_1 and ψ_2 of the spinor wave function $\psi =$

$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$:

$$
\begin{aligned}
i\hbar\frac{\partial\psi_1}{\partial t} &= -\frac{\hbar^2}{2m}\Delta_r\psi_1 - \mu H_z\psi_1 - \mu H_\perp e^{-i\omega t}\psi_2; \\
i\hbar\frac{\partial\psi_2}{\partial t} &= -\frac{\hbar^2}{2m}\Delta_r\psi_2 + \mu H_z\psi_2 - \mu H_\perp e^{i\omega t}\psi_1;
\end{aligned}
\tag{6.20}
$$

Introduce new functions ψ_1 and ψ_2, using the following transformation

$$
\psi_1 = \varphi_1 \exp\left(-i\frac{\omega}{2}t\right); \;\; \psi_2 = \varphi_2 \exp\left(i\frac{\omega}{2}t\right). \tag{6.21}
$$

The transformation (6.21) is equivalent to that performing the conversion to the coordinate system rotating about the z-axis at the frequency ω [Slichter (1963)]. As a result, (6.20) goes over to the following system:

$$
\begin{aligned}
i\hbar\frac{\partial\varphi_1}{\partial t} + \frac{\hbar\omega}{2}\varphi_1 &= -\frac{\hbar^2}{2m}\Delta_r\varphi_1 - \mu H_z\psi_1 - \mu H_\perp\varphi_2; \\
i\hbar\frac{\partial\varphi_2}{\partial t} - \frac{\hbar\omega}{2}\varphi_2 &= -\frac{\hbar^2}{2m}\Delta_r\varphi_2 + \mu H_z\psi_2 - \mu H_\perp\varphi_1.
\end{aligned}
\tag{6.22}
$$

Introduction of the spinor function $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$, enables us to write (6.22) as follows

$$
i\hbar\frac{\partial\varphi}{\partial t} = -\frac{\hbar^2}{2m}\Delta_r\varphi - \vec{\mu}\vec{H}(\omega)\varphi, \tag{6.23}
$$

where $\vec{H}(\omega)$ has the components $H_x(\omega) = H_\perp$, $H_y(\omega) = 0$, $H_z(\omega) = H_z - \frac{\hbar\omega}{2\mu}$, and at the initial instant of time the function φ is

$$
\varphi(t_0) = \left\{ \begin{array}{c} \psi_1(t_0)\left(i\frac{\omega}{2}t_0\right) \\ \psi_2(t_0)\left(-i\frac{\omega}{2}t_0\right) \end{array} \right\}.
$$

Thus, the problem of refraction of a neutron wave in a time-dependent magnetic field has reduced to the problem of wave refraction in a constant effective magnetic field $\vec{H}(\omega)$ depending on frequency ω.

Due to the complete equivalence of equation (6.23) and the equations describing neutron motion in a time-independent magnetic field $\vec{H}(\vec{r})$, all the conclusions concerning the laws of refraction and mirror reflection in it hold true, however, with a considerable difference that both the refractive index and the amplitude of the reflected neutron wave now become dependent on the external field frequency ω.

The situation when $H_z \gg H_\perp$ seems to be of particular interest. In this case at the frequency $\omega = \frac{2\mu H_z}{\hbar}$ the component of the effective field H_z

vanishes, and the effective field $H(\omega)$ equals $H_\perp$, which is much less than the value of the magnetic field in the absence without resonance. Hence, the refractive index (coefficient of mirror reflection) will appear to be smaller. For instance, if without a rotating field, the magnetic field was so great that the neutrons experienced total mirror reflection from it, under the resonance conditions, the neutrons will pass through the area occupied by the magnetic field. Similarly, the polarization state of the neutron beam will prove to be strongly dependent on the frequency of a variable field.

Now let neutrons (electrons and so on) be incident onto a single crystal with polarized electrons (nuclei). Then the area occupied by the crystal may be described in terms of a spatially periodic effective magnetic field $\vec{B}(\vec{r})$. If the crystal is placed in the external rotating magnetic field (or excite a circular sound wave in it), then a spatially periodic ω-dependent field $\vec{B}(\vec{r},\omega)$ emerges in a rotating system. Mathematical formulation of the particle beam propagation in the periodic field $\vec{B}(\vec{r},\omega)$ is completely equivalent to that describing the phenomena of refraction, diffraction and mirror reflection of particles in single crystals in the absence of a variable field [Baryshevsky (1976, 1979a)]. Therefore the formulae for the refractive indices of a crystal placed in a variable field under diffraction conditions are similar [Baryshevsky (1976, 1979a)].

It is common knowledge that under diffraction of particles in crystals the effect of anomalous transmission (anomalous suppression of inelastic processes, nuclear reactions) arises [Pinsker (1974); Afanasiev and Kagan (1965)]. In the case under consideration, due to the frequency dependence of the periodic field $\vec{B}(\vec{r},\omega)$, a new phenomenon appears: the effect of anomalous transmission of particles (γ-quanta) through crystals, which depends on the frequency of the external field (electromagnetic, sound). (The probability of inelastic processes and nuclear reactions also depends significantly on the frequency of the external field). It is important to emphasize that effect of anomalous transmission and reaction suppression depending on the frequency of the external field occurs for both instantaneous particle (γ-quantum) intensity and the intensity averaged over the alteration period of the external variable field.

Note that, as shown in [Baryshevsky (1979c); Baryshevskii (1981)], even in non-magnetic unpolarized crystals placed in an external magnetic field, one may observe multi-frequency precession of neutron spin and H-dependent effect of suppression of nuclear reactions. When the crystal is exposed to an external variable field (magnetic, sound) the effects depending on the field frequency emerge: anomalous suppression of nuclear reactions (analogous to that considered above) and multi-frequency precession of a neutron spin.

Thus, an external variable field sharply changes refractive properties of a crystal under diffraction, which eventually manifests directly in the process of radiation. By way of example, consider diffraction of neutrons in a constant magnetic field.

According to [Baryshevsky (1976)] the system of equations for the neutron wave function $\psi(\vec{r})$ describing the dynamic diffraction in an arbitrary magnetically-ordered crystal with polarized nuclei has the form

$$\left(\frac{k^2}{k_0^2}-1\right)\varphi(\vec{k})-\sum_{\tau}\hat{g}(\vec{\tau})\varphi(\vec{k}-2\pi\vec{\tau})=0;$$

$$\varphi(\vec{r})=\sum_{\tau}\varphi(\vec{k}+2\pi\vec{\tau})\exp[i(\vec{k}+2\pi\tau)\vec{r}]; \tag{6.24}$$

$$\hat{g}(\vec{\tau})=\hat{g}_{nuc}(\vec{\tau})+\hat{g}_{mag}(\vec{\tau})$$

$$=\frac{4\pi}{\Omega_0 k_0^2}\sum_j(\hat{f}_{j_{nuc}}(\vec{\tau})+\hat{f}_{j_{mag}}(\vec{\tau}))e^{-2\pi\tau\vec{r}_j}, \tag{6.25}$$

where $\hat{g}(\tau)$ is the structure amplitude; $\hat{f}_j(\tau)$ is the amplitude of coherent scattering by the j-th center included in the unit cell; $\vec{r}_j$ is the coordinate of the j-th center; the summation is performed over all the scatterers constituting the unit cell; Ω_0 is the unit cell volume; $\vec{k}_0$ is the wave vector of the neutron incident on a crystal.

At $\vec{\tau}\neq 0$ the amplitude of coherent magnetic scattering is defined by the expression [Baryshevsky (1976)]

$$\hat{f}_{j\,mag}(\vec{\tau})=-4\pi\mu_n\left[\frac{(\vec{\sigma}\vec{\tau})(\vec{\tau}\vec{\mu}_j)}{\tau^2}-\vec{\sigma}\vec{\mu}_j\right]F_j(\vec{\tau})e^{-W_j(\tau)}. \tag{6.26}$$

At $\vec{\tau}=0$ the magnetic contribution to $\hat{g}(0)$ has the form

$$\hat{g}_{mag}(0)=\frac{2m\mu_n}{\hbar^2k_0^2}\vec{\sigma}\vec{B}, \tag{6.27}$$

where $\vec{B}$ is the macroscopic magnetic field of the target; m is the particle mass.

In the case of a non-magnetic unpolarized crystal placed in a constant magnetic field of strength $\vec{H}$, the structure amplitudes of (6.25) can be written as follows:

$$\hat{g}(0)=\frac{4\pi}{\Omega_0k_0^2}\sum_j f_{j_{nuc}}(0)+\frac{2m\mu_n}{\hbar^2k_0^2}\vec{\sigma}\vec{H}, \tag{6.28}$$

$$\hat{g}_{\tau\neq 0}(\vec{\tau}) = g(\vec{\tau}) = \frac{4\pi}{\Omega_0 k_0^2} \sum_j f_{j_{nuc}}(\vec{\tau}) e^{-i2\pi\tau r_j}. \tag{6.29}$$

Choose the quantization axis parallel to the direction of the field $\vec{H}$. As a result, the operator system (6.24) will reduce to two independent systems of equations for either neutron spin component, parallel φ_+ and antiparall φ_- to the quantization axis:

$$\left(\frac{k^2}{k_0^2} - 1\right) \varphi_\pm(\vec{k}) - \sum_\tau g_\pm(\vec{\tau}) \varphi_\pm(\vec{k} - 2\pi\vec{\tau}) = 0,$$
$$g_\pm(0) = g_{nuc}(0) \pm \frac{2m\mu_n}{\hbar^2 k_0^2} H, \;\; g_\pm(\vec{\tau}) = g_{nuc}(\vec{\tau}). \tag{6.30}$$

The system of equations (6.17) has a standard form for the dynamical diffraction theory. This enables us to immediately write the expression for the wave function of a neutron that has passed through the crystal plate of thickness l [Baryshevsky (1976)]:

$$\psi(\vec{r}) \begin{pmatrix} c_+\psi_+(\vec{r}) \\ c_-\psi_-(\vec{r}) \end{pmatrix},$$

where $\begin{pmatrix} c_+ \\ c_- \end{pmatrix}$ is the spin wave function of the neutrons incident of the plate; the z-axis is directed along the quantization axis;

$$\psi_\sigma(\vec{r}) = \frac{(2\varepsilon_2^\sigma - g_\sigma(0)) \exp\left(ik_0\varepsilon_1^\sigma \frac{l}{\gamma_0}\right) - (2\varepsilon_1^\sigma - g_\sigma(0)) \exp\left(ik_0\varepsilon_2^\sigma \frac{l}{\gamma_0}\right)}{2(\varepsilon_2^\sigma - \varepsilon_1^\sigma)} e^{-i\vec{k}_0\vec{r}}$$
$$- \frac{\beta g(\vec{\tau})}{2(\varepsilon_2^\sigma - \varepsilon_1^\sigma)} \left[\exp\left(ik_0\varepsilon_1^\sigma \frac{l}{\gamma_0}\right) - \exp\left(ik_0\varepsilon_2^\sigma \frac{l}{\gamma_0}\right)\right] e^{i(\vec{k}_0 + 2\pi\vec{\tau})\vec{r}},$$
$$\varepsilon_{1(2)}^\sigma = \frac{1}{4}\{(1+\beta) g_\sigma(0) - \beta\alpha$$
$$\pm\sqrt{[\beta\alpha + g_\sigma(0)(1-\beta)]^2 + 4\beta g(\tau) g(-\tau)}\Big\}; \tag{6.31}$$

$\sigma = \pm$ ((+) corresponds to the neutrons with spin parallel to $\vec{H}$, (−) corresponds to the neutrons with the opposite spin direction); $\gamma_0 = \vec{k}_0\vec{n}/k_0$; $\vec{n}$ is the normal to the crystal surface; $\alpha = 2\pi\vec{\tau}(2\pi\vec{\tau} + 2\vec{k}_0)/k_0^2$ is the quantity characterizing deviation from the exact Bragg conditions; $\beta = \gamma_0/\gamma_1$; $\gamma_1 = (\vec{k}_0 + 2\pi\vec{\tau})\vec{n}/|\vec{k}_0 + 2\pi\vec{\tau}|$. In the case of the symmetric Laue diffraction $\gamma_0 = \gamma_1$, $\beta = 1$ and

$$\varepsilon_{1(2)}^\sigma = \frac{1}{4}\left\{2g_0(0) - \alpha \pm \sqrt{\alpha^2 + 4g(\vec{\tau})g(-\vec{\tau})}\right\}. \tag{6.32}$$

First consider how the magnetic field influences the diffracted neutrons. Using the expression for the wave function (6.21), write the expression for the intensity of the diffracted wave:

$$I_d = |c_+\psi_+|^2 + |c_-\psi_-|^2 = |c_+|^2\beta^2 \left|\frac{g(\tau)}{A_+}\right|^2$$
$$\times \left\{\exp\left(-k_0\mathtt{Im}2\varepsilon_2^+\frac{l}{\gamma_0}\right) + \exp\left(-k_0\mathtt{Im}2\varepsilon_1^+\frac{l}{\gamma_0}\right)\right.$$
$$\left.-2\cos\left(k_0\mathtt{Re}\frac{A_+}{2}\frac{l}{\gamma_0}\right)\exp\left[-k_0\mathtt{Im}(\varepsilon_1^+ + \varepsilon_2^+)\frac{l}{\gamma_0}\right]\right\}$$
$$+|c_-|^2\beta^2\left|\frac{g(\tau)}{A_-}\right|^2\left\{\exp\left(-k_0\mathtt{Im}2\varepsilon_2^-\frac{l}{\gamma_0}\right)\right.$$
$$+\exp\left(-k_0\mathtt{Im}2\varepsilon_1^-\frac{l}{\gamma_0}\right) - 2\cos\left(k_0\mathtt{Re}\frac{A_-}{2}\frac{l}{\gamma_0}\right)$$
$$\left.\times\exp\left[-k_0\mathtt{Im}(\varepsilon_1^- + \varepsilon_2^-)\frac{l}{\gamma_0}\right]\right\}, \tag{6.33}$$

where

$$A_\pm = \left\{[g_\pm(0)(1-\beta) + \beta\alpha]^2 + 4\beta g(\vec{\tau})g(-\vec{\tau})\right\}^{1/2}. \tag{6.34}$$

First of all, note that in the presence of the external magnetic field, oscillations of the intensity of the diffracted wave (the pendulum effect), unlike those in the case when $H = 0$, occur at two spatial frequencies

$$\kappa_1 = k_0\mathtt{Re}\frac{A_+}{2};\ \kappa_2 = k_0\mathtt{Re}\frac{A_-}{2}. \tag{6.35}$$

In symmetric diffraction the frequencies κ_1 and κ_2 coincide and do not depend on the value of the magnetic field:

$$\kappa_1 = \kappa_2 = \kappa = k_0\mathtt{Re}\frac{1}{2}\sqrt{\alpha^2 + 4g(\vec{\tau})g(-\vec{\tau})} \tag{6.36}$$

If the neutron beam is polarized parallel to the magnetic field ($c_+ = 1$, $c_- = 0$), then I_d oscillates at the frequency κ_1, if antiparallel, I_d oscillates at the frequency $\kappa_2 \neq \kappa_1$.

Let an unpolarized neutron beam fall upon a crystal. In this case I_d is given by expression (6.33), where $|c_+|^2 = |c_-|^2 = 1/2$. As $\kappa_1 \neq \kappa_2$, at certain values of the magnetic field H the situation is possible, when the contribution to I_d coming from one of the neutron spin components appears to be zero. Hence, at such values of H the diffracted beam will be fully polarized.

The intense neutron beam fully polarized along the magnetic field will be obtained at the exit from the crystal plate provided that one of the summands in (6.33) takes on its maximum value, with the second summand taking on its minimum value. For simplicity, assume that the crystal is non-absorptive, and the exponential factors in (6.33) are equal to unity.

When in (6.33) the cosine in the augend equals -1, and in the addend - $+1$, we obtain a beam fully polarized along the field. The neutron beam fully polarized opposite the field is obtained when the cosine in the augend becomes $+1$, and in the addend it quals -1. In the general case this condition may be written as follows with due account of the explicit form for frequencies κ_1 and κ_2

$$\text{for}\, p_z = -1 \begin{cases} \mathtt{Re} k_0 \frac{1}{2} A_+ \frac{l}{\gamma_0} = 2\pi N, \\ \mathtt{Re} k_0 \frac{1}{2} A_- \frac{l}{\gamma_0} = 2\pi N' + \pi; \end{cases} \tag{6.37}$$

$$\text{for}\, p_z = 1 \begin{cases} \mathtt{Re} k_0 \frac{1}{2} A_+ \frac{l}{\gamma_0} = 2\pi N + \pi, \\ \mathtt{Re} k_0 \frac{1}{2} A_- \frac{l}{\gamma_0} = 2\pi N'; \end{cases} \tag{6.38}$$

where N and N' are integral numbers.

Find the phase difference η between the components of the wave function of neutrons corresponding to the parallel and anti-parallel spin states when passing through the plate of thickness l, the magnetic field strength H being equal to

$$\eta = \frac{1}{2} k_0 \frac{l}{\gamma_0} \mathtt{Re}(A_+ - A_-). \tag{6.39}$$

Estimation of the expression (6.39) shows that in the asymmetric diffraction case at $(1-\beta) \simeq 10^{-1}$, $l = 0.1$ cm, $\lambda \sim 1\,\text{Å}$ and $g(0) \simeq 10^{-6}$, the phase difference is π at the magnetic field strength of the order of 1000 Gs. Hence, at such strength of the magnetic field one may obtain fully polarized neutron beams with the possibility to specify the polarization direction by changing the direction of the magnetic field.

By varying the value of the magnetic field strength, it is also possible to modulate the intensity of the diffracted beam. The degree of modulation can be close to 100%. Indeed, choose the plate thickness l so that in the absence of the magnetic field the intensity of the diffracted beam would be zero. Then, as follows from (6.33), with the external magnetic field imposed, the intensity becomes non-zero. With the strength of the external magnetic field vanishing, the frequencies κ_1 and κ_2 become equal to each other, and the intensity of the diffracted beam starts oscillating at one frequency only, which is defined by equality (6.36), so we have an ordinary pendulum effect.

Now consider the influence of the magnetic field on the beam absorption in a crystal. From (6.31) follows that at $g_+(0) \neq g_-(0)$ the imaginary parts of $\varepsilon^{+}_{1(2)}$ and $\varepsilon^{-}_{1(2)}$ will be different, and what is more, they will reveal different dependence on the value of the magnetic field:

$$\mathtt{Im}\varepsilon^{\sigma}_{1(2)} = \frac{1}{4}(1+\beta)\mathtt{Im}g_{\sigma}(0) \pm \Big[(\mathtt{Re}A_{\sigma})^2 + (\mathtt{Im}A_{\sigma})^2\Big]^{1/2} \sin\left(\frac{1}{2}\arctan\frac{\mathtt{Im}A_{\sigma}}{\mathtt{Re}A_{\sigma}}\right), \tag{6.40}$$

where A_{σ} are given by equality (6.34).

Note that if $\mathtt{Im}A_{\sigma} = 0$, then

$$\mathtt{Im}\varepsilon^{\sigma}_{1} = \mathtt{Im}\varepsilon^{\sigma}_{2}. \tag{6.41}$$

This is attained at the value of the magnetic field

$$H = -(\sigma) \times \frac{2(1-\beta)\beta\alpha\mathtt{Im}g(0) + (1-\beta)^2\mathtt{Im}g^2(0) + 4\beta g(\tau)g(-\tau)}{2(1-\beta)^2\mathtt{Im}g(0)} \times \frac{\hbar^2 k_0^2}{2m\mu_n}, \tag{6.42}$$

where (σ) indicates the neutron spin state for which (6.42) holds.

As a result, in the case of the asymmetric Laue diffraction under consideration, the effect of the anomalous transmission of particles through a crystal, and, hence, the yield of nuclear reactions will depend on the strength of the external magnetic field.

Analyze polarization characteristics of the diffracted neutron beam in more detail.

To fix the idea, we shall assume that the polarization vector of neutrons incident on a crystal $\vec{p}_0$ is directed perpendicular to the quantization axis, i.e. to the z-axis (the z-axis is directed parallel to $\vec{H}$). The direction of $\vec{p}_0$ is chosen as the x-axis so that $c_+ = c_- = 1\sqrt{2}$. Using (6.31), immediately find the components p_x and p_y of the neutron polarization vector in the diffracted wave:

$$
\begin{aligned}
p_x = \ & \frac{\beta^2}{4}\frac{|g(\tau)|^2}{|(\varepsilon_2^+ - \varepsilon_1^+)(\varepsilon_2^- - \varepsilon_1^-)|}\left\{\cos\left[k_0\mathtt{Re}(\varepsilon_1^+ - \varepsilon_1^-)\right.\right.\\
& \left.\times\frac{l}{\gamma_0} + \delta\right]\exp\left[-k_0\mathtt{Im}(\varepsilon_1^+ + \varepsilon_1^-)\frac{l}{\gamma_0}\right]\\
& - \cos\left[k_0\mathtt{Re}(\varepsilon_1^+ - \varepsilon_2^-)\frac{l}{\gamma_0} + \delta\right]\\
& \times\exp\left[-k_0\mathtt{Im}(\varepsilon_1^+ + \varepsilon_2^-)\frac{l}{\gamma_0}\right]\\
& - \cos\left[k_0\mathtt{Re}(\varepsilon_2^+ - \varepsilon_1^-)\frac{l}{\gamma_0} + \delta\right]\\
& \times\exp\left[-k_0\mathtt{Im}(\varepsilon_2^+ + \varepsilon_1^-)\frac{l}{\gamma_0}\right]\\
& + \cos\left[k_0\mathtt{Re}(\varepsilon_2^+ - \varepsilon_2^-)\frac{l}{\gamma_0} + \delta\right]\\
& \left.\times\exp\left[-k_0\mathtt{Im}(\varepsilon_2^+ + \varepsilon_2^-)\frac{l}{\gamma_0}\right]\right\}, \qquad (6.43)
\end{aligned}
$$

where $\delta = \delta_+ - \delta_-$ and the following notation is used

$$
\frac{g(\tau)}{2(\varepsilon_2^\pm - \varepsilon_1^\pm)} = \left|\frac{g(\tau)}{2(\varepsilon_2^\pm - \varepsilon_1^\pm)}\right| e^{i\delta_\pm}.
$$

The component p_y is obtained from p_x by replacing cos with $-\sin$.

Using (6.30) and (6.31) the differences of the values of ε appearing in (6.33) are written as follows:

$$
\begin{aligned}
\varepsilon_1^+ - \varepsilon_1^- &= G + \frac{1}{4}(A_+ - A_-);\\
\varepsilon_1^+ - \varepsilon_2^- &= G + \frac{1}{4}(A_+ + A_-);\\
\varepsilon_2^+ - \varepsilon_1^- &= G - \frac{1}{4}(A_+ + A_-);\\
\varepsilon_2^+ - \varepsilon_2^- &= G - \frac{1}{4}(A_+ - A_-);\\
G &= \frac{m\mu_n}{\hbar^2 k_0^2}H(1+\beta) \qquad (6.44)
\end{aligned}
$$

In the case of the symmetric Laue diffraction when $\beta = 1$, A_+ and A_- are equal. And the neutron polarization vector undergoes beatings with changes in H at one frequency, determined by the Larmour spin precession frequency in a magnetic field.

At the asymmetric Laue diffraction ($\beta \neq 1$) the situation changes drastically. $A_+ \neq A_-$, and with the changes in H, the neutron polarization vector undergoes beating at four different frequencies determined by the differences (6.44):

$$\omega_1 = \frac{\hbar k_0^2}{m}\mathtt{Re}(\varepsilon_1^+ - \varepsilon_1^-);\ \omega_2 = \frac{\hbar k_0^2}{m}\mathtt{Re}(\varepsilon_1^+ - \varepsilon_2^-);$$
$$\omega_3 = \frac{\hbar k_0^2}{m}\mathtt{Re}(\varepsilon_2^+ - \varepsilon_1^-);\ \omega_4 = \frac{\hbar k_0^2}{m}\mathtt{Re}(\varepsilon_2^+ - \varepsilon_2^-). \quad (6.45)$$

From (6.43) and (6.44) follows that at relatively small magnetic fields ($H = 10^3 \div 10^4$ Gs) the effect of multi-frequency precession in a crystal should be clearly observed even at $l \sim 10^2 \div 10^{-1}$ cm.

Using the expression for the wave function (6.31), we also write the expression for the p_z component of the polarization vector of a diffracted wave:

$$p_z = |\psi_+|^2 - |\psi_-|^2 = \beta^2\left|\frac{g(\tau)}{A_+}\right|^2\left\{\exp\left[-k_0\mathtt{Im}2\varepsilon_2^+\frac{l}{\gamma_0}\right]\right.$$
$$+\exp\left[-k_0\mathtt{Im}2\varepsilon_1^+\frac{l}{\gamma_0}\right] - 2\cos\left(k_0\mathtt{Re}\frac{A_+}{2}\frac{l}{\gamma_0}\right)$$
$$\left.\times\exp\left[-k_0\mathtt{Im}(\varepsilon_2^+ + \varepsilon_1^+)\frac{l}{\gamma_0}\right]\right\} - \beta^2\left|\frac{g(\tau)}{A_-}\right|^2$$
$$\times\left\{\exp\left[-k_0\mathtt{Im}2\varepsilon_2^-\frac{l}{\gamma_0}\right] + \exp\left[-k_0\mathtt{Im}2\varepsilon_1^-\frac{l}{\gamma_0}\right]\right.$$
$$\left.-2\cos\left(k_0\mathtt{Re}\frac{A_-}{2}\frac{l}{\gamma_0}\right)\exp\left[-k_0\mathtt{Im}(\varepsilon_2^- + \varepsilon_1^+)\frac{l}{\gamma_0}\right]\right\}. \quad (6.46)$$

Comparison of (6.33) and (6.46) shows that the longitudinal component of the polarization vector of the diffracted beam oscillates at the same two frequencies κ_1 and κ_2 as the intensity of the diffracted beam does.

Further consider the expressions for the components of the polarization vector and the intensity of the diffracted wave when absorption can be ignored (the crystal thickness is much smaller than the absorption depth, but of the order or greater than the spatial precession period; this requirement can always be met for neutrons as $\mathtt{Re}g \gg \mathtt{Im}g$, if only the neutron energy does not lie in the resonance region). Then equating in (6.33) the exponential factors to unity, gives the following expressions for the components of the polarization vector:

$$p_x = 2\beta^2\frac{|g(\tau)|^2}{|A_+A_-|}\cos\left(k_0G\frac{l}{\gamma_0} + \delta\right)\left\{\cos k_0\mathtt{Re}\left[G + \frac{1}{4}\right.\right.$$
$$\left.\left.\times(A_+ - A_-)\right]\frac{l}{\gamma_0} - \cos k_0\mathtt{Re}\left[G + \frac{1}{4}(A_+ + A_-)\right]\frac{l}{\gamma_0}\right\}; \quad (6.47)$$

p_y is obtained from p_x by substituting $\cos\left(k_0 G\frac{l}{\gamma_0}+\delta\right)$ for $-\sin\left(k_0 G\frac{l}{\gamma_0}+\delta\right)$. From this

$$p_x + ip_y = 2\beta^2 \frac{|g(\tau)|^2}{|A_+A_-|} \exp\left[-i\left(k_0 G\frac{l}{\gamma_0}+\delta\right)\right] \times \left\{\cos k_0 \mathtt{Re}\left[G+\frac{1}{4}(A_+ - A_-)\right]\frac{l}{\gamma_0} - \cos k_0 \mathtt{Re}\left[G+\frac{1}{4}(A_+ + A_-)\right]\frac{l}{\gamma_0}\right\}. \tag{6.48}$$

According to (6.47) and (6.48) the longitudinal component of the neutron polarization vector rotates at the frequency $\Omega = (1+\beta)\mu_n H/\hbar$ in the plane xy, i.e., in the plane normal to the magnetic field. In the case of asymmetric diffraction $\beta \neq 1$ the rotation frequency of the neutron polarization vector Ω does not coincide with the Larmour frequency $2\mu_n H/\hbar$. Thus, under diffraction conditions the spin rotation frequency depends not only on the value of the magnetic field but also on the angle of incidence on the crystal and the orientation of the crystal surface with respect to crystallographic axes.

As follows from (6.48) rotation is accompanied by the oscillation of the magnitude of the longitudinal component of the polarization vector at the frequencies ω_1 and ω_2.

With the help of (6.46) and (6.33) we obtain the following expressions for the longitudinal component of the polarization vector p_z and the intensity of the diffracted wave I_d for a thin plate:

$$p_z = 2\beta^2\left|\frac{g(\tau)}{A_+}\right|^2\left[1-\cos\left(k_0\mathtt{Re}\frac{1}{2}A_+\frac{l}{\gamma_0}\right)\right] -2\beta^2\left|\frac{g(\tau)}{A_-}\right|^2\left[1-\cos\left(k_0\mathtt{Re}\frac{1}{2}A_-\frac{l}{\gamma_0}\right)\right]; \tag{6.49}$$

$$I_d = 2\beta^2\left|\frac{g(\tau)}{A_+}\right|^2\left[1-\cos\left(k_0\mathtt{Re}\frac{1}{2}A_+\frac{l}{\gamma_0}\right)\right] +2\beta^2\left|\frac{g(\tau)}{A_-}\right|^2\left[1-\cos\left(k_0\mathtt{Re}\frac{1}{2}A_-\frac{l}{\gamma_0}\right)\right]; \tag{6.50}$$

It is clear from (6.49) and (6.50) that density oscillations of the components of the wave function, which correspond to the states with spin parallel and antiparallel to the magnetic field will occur at two different frequencies κ_1 and κ_2.

Now we shall consider thoroughly the oscillations of the transverse component of the polarization vector. With this aim in view recast (6.48) as follows:

$$p_x + ip_y = 2\beta^2 \frac{|g(\tau)|^2}{|A_+A_-|} \exp\left[-i\left(k_0 G\frac{l}{\gamma_0} + \delta\right)\right]$$
$$\times 2\sin\left(k_0 \mathtt{Re}\frac{1}{4}A_- \frac{l}{\gamma_0}\right) \sin\left[k_0 \mathtt{Re}\left(G + \frac{1}{4}A_+\right)\frac{l}{\gamma_0}\right]. \qquad (6.51)$$

Analyzing expressions (6.49)-(6.51), one may see that the oscillation frequencies of the transverse component $p_x + ip_y$ of the polarization vector in the presence of the external magnetic field do not coincide with those of the longitudinal component of the polarization vector p_z. Hence, it is always possible to select such a value of the magnetic field (at given $g(\tau)$, l, α and β) that the transverse component will vanish, while the longitudinal component will be non-zero. A coherent neutron beam, initially fully polarized along the x-axis, under diffraction conditions may become partially or fully polarized along the z-axis.

How does absorption affect rotation of the neutron polarization vector? Let a crystal thickness be larger than the absorption depth of a rapidly damped wave, but smaller than the absorption depth of an anomalously transmitted wave. In this case p_x and p_y may be written in the form

$$p_x = \beta^2 \frac{|g(\tau)|^2}{|A_+A_-|} \cos\left\{k_0 \mathtt{Re}\left[G - \frac{1}{4}(A_+ - A_-)\right]\frac{l}{\gamma_0} + \delta\right\}, \qquad (6.52)$$

p_y is obtained by replacing cos with $-\sin$. Write the explicit form of the spin rotation frequency ω_4:

$$\omega_4 = \frac{\hbar k_0^2}{m}\mathtt{Re}\left\{\frac{m\mu_n}{\hbar^2 k_0^2}H(1+\beta)\right.$$
$$-\frac{1}{4}\sqrt{[g_+(0)(1-\beta) + \beta\alpha]^2 + 4\beta g(\tau)g(-\tau)}$$
$$\left.+\frac{1}{4}\sqrt{[g_-(0)(1-\beta) + \beta\alpha]^2 + 4\beta g(\tau)g(-\tau)}\right\}. \qquad (6.53)$$

Pay attention to the fact that in this case the spin rotation frequency no longer demonstrates linear dependence on the value of the magnetic field, which now becomes more complicated, as described by expression (6.53).

The spin phenomena investigated also occur in the case of the symmetric Laue diffraction, if the field boundary is not parallel to the crystal surface. Now consider the case when a diffracted wave exits through the same crystal surface on which the initial beam falls, i.e., consider diffraction reflection of neutrons from a non-magnetic crystal placed in a constant homogeneous magnetic field.

As the set of dynamic equations (6.30) in question is perfectly analogous in form to that describing diffraction in a crystal in the absence of a magnetic field, we can immediately write down the coefficient of diffraction reflection for each spin component of the neutron wave [Baryshevsky (1976)]

$$R_{\pm} = \beta^2 |g(\tau)|^2 \times \left| \frac{1 - \exp\left[i(\varepsilon_2^{\pm} - \varepsilon_1^{\pm}) k_0 \frac{l}{\gamma_0}\right]}{[2\varepsilon_1^{\pm} - g_{\pm}(0)] - [2\varepsilon_2^{\pm} - g_{\pm}(0)] \exp\left[i(\varepsilon_2^{\pm} - \varepsilon_1^{\pm}) k_0 \frac{l}{\gamma_0}\right]} \right|^2 \quad (6.54)$$

where $\varepsilon_{1(2)}^{\pm}$ are specified by equality (6.31).

For simplicity, consider the symmetric Bragg case, when $\beta = -1$. Then, according to (6.32),

$$\varepsilon_{1(2)}^{\pm} = \frac{1}{4}[\alpha \pm \sqrt{[2g_{\pm}(0) - \alpha]^2 - 4g(\tau)g(-\tau)}]. \quad (6.55)$$

From (6.54) follows that when the following conditions are fulfilled

$$\left(\alpha - 2g(0) - \frac{4m\mu_n}{\hbar^2 k_0^2} H\right)^2 < 4g(\tau)g(-\tau);$$
$$\left(\alpha - 2g(0) + \frac{4m\mu_n}{\hbar^2 k_0^2} H\right)^2 > 4g(\tau)g(-\tau) \quad (6.56)$$

the reflection coefficient $R_+ = 1$, while $R_- \gg 1$. And vice versa, if the conditions

$$\left(\alpha - 2g(0) - \frac{4m\mu_n}{\hbar^2 k_0^2} H\right)^2 > 4g(\tau)g(-\tau);$$
$$\left(\alpha - 2g(0) + \frac{4m\mu_n}{\hbar^2 k_0^2} H\right)^2 < 4g(\tau)g(-\tau) \quad (6.57)$$

are satisfied, then $R_- = 1$, and $R_+ \ll 1$.

The phenomena considered above also occur in a wave passing through a crystal in the incident direction of the initial (primary) beam. Outside the diffraction conditions the intensity of the diffracted wave diminishes rapidly. At the same time the refractive index of the wave propagating in the initial direction contains the admixture owing to the existence of diffraction, for example,

$$\varepsilon_1^{\sigma} = \frac{1}{2} g_{\sigma}(0) + \frac{\beta g(\tau) g(-\tau)}{2(\beta\alpha + g_{\sigma}(0)(1 - \beta))} + \ldots \quad (6.58)$$

Therefore even away from diffraction, the neutron spin rotates at the frequency different from the Larmour one. The contribution to the refractive index of a particle (γ-quantum) passing through a crystal due to the summand of the type as considered in (6.58) affects the optical anisotropy of crystals in a hard spectrum, and depends, in particular, on variable fields acting on the crystal. (Under the diffraction conditions these fields considerably modify $\varepsilon^{\sigma}_{1(2)}$ [Baryshevsky (1979a)]).

Chapter 7

Interference of Independently Generated Beams of γ-quanta

7.1 Interference of Independently Generated Photons

The interference phenomena in beams of light generated by independently emitting sources have been widely debated in literature. The study of such phenomena in the X-ray band would make it possible to carry out direct measurements of the phases of scattering amplitudes and structure amplitudes in crystals. However, as shown in [Baryshevskii and Podgoretskii (1968)] it is impossible to perform such measurements with conventional sources of X-ray and γ radiation. Nevertheless, according to [Baryshevsky and Feranchuk (1980a)], high intensity and pointed directivity of radiation produced be relativistic particles in crystals give hope for experimental detection of the interference phenomenon of independently generated beams of γ-quanta.

In the beginning consider the nature of the phenomenon. Let us have two excited atoms with energies E_a and E_b located at points $\vec{r}_a$ and $\vec{r}_b$, and two atoms (two elementary (simple) counters) located at points $\vec{r}_c$ and $\vec{r}_d$. One and the same final state of the system (photon registration by the counters at specified instants of time t and $\vec{\tau}$), due to the identity of incident particles, is achieved by two possible ways: $(a \to c,\ b \to d)$ and $(b \to c,\ a \to d)$. Observation cannot distinguish these two regimes. Therefore the probability $P_{ab}(\vec{r}_c\,,\,t\,;\,\vec{r}_d\,,\,\tau)$ that at time t radiation will interact with the atom located at point $\vec{r}_c$, and at time τ, with the atom located at point $\vec{r}_d$, contains the interference term and can be represented in the form

$$P_{ab}(\vec{r}_c\,,\,t\,;\,\vec{r}_d\,,\,\vec{\tau}) = A\left|\frac{\exp[i(k_a r_{ac} - \omega_a t + \delta_a)]}{r_{ac}}\right.$$
$$\times\frac{\exp[i(k_b r_{bd} - \omega_b \tau + \delta_b)]}{r_{bd}} + \frac{\exp[i(k_a r_{ad} - \omega_a \tau + \delta_a)]}{r_{ad}}$$
$$\left.\times\frac{\exp[i(k_b r_{bc} - \omega_b t + \delta_b)]}{r_{bc}}\right|^2 , \qquad (7.1)$$

where A is the constant insignificant for the case in question.

The expression of the type $r_{ac}^{-1}\exp[i(k_a r_{ac} - \omega_a t + \delta_a)]$ is the wave function of a photon with the wave number $k_a = \omega_a/c$, which is emitted at point $\vec{r}_c$, where $r_{ac} = |\vec{r}_a - \vec{r}_c|$. For simplicity, it is assumed that the atoms emit monochromatic radiation of the same polarization. According to (7.1), the probability $P_{ab}(\vec{r}_c\,,\,t\,;\,\vec{r}_d\,,\,\tau)$ is independent of random phases δ_a and δ_b. If the distance between the atoms of either pair is assumed to be much shorter than the distance R between the pairs, then (7.1) can be written in the form

$$P_{ab}(\vec{r}_c\,,\,t\,;\,\vec{r}_d\,,\,\vec{\tau}) \approx \frac{2A}{R^4}\,\{1 + \cos\left[k_a(r_{ac} - r_{ad})\right.$$
$$\left. + k_b(r_{bd} - r_{bc}) - (\omega_a - \omega_b)(t - \tau)\right]\}\,, \qquad (7.2)$$

where $r_{ac} = |\vec{r}_a - \vec{r}_c|$ etc. But for the particle identity, the first term in the braces in (7.2) would describe the probability of joint registration of the two photons; the second term describes the change of this probability due to the identity. As one may see, taking into account the particle identity, leads to the fact that the probability $P_{ab}(\vec{r}_c, t; \vec{r}_d, \vec{\tau})$ is the oscillating function of the coordinates that undergoes time beatings at the difference of the frequencies of the emitted photons. Since real sources and detectors contain many pairs of atoms, (7.2) should be summed over these pairs. In this case, the interference term, which is of interest to us, does not vanish unless the cosine appearing in (7.2) undergoes oscillations with the change in the positions of atoms within the volumes of the sources S and detectors D. Hence, the following inequality should hold for any pair of atoms within the limits of S and D

$$k_a(r_{ac} - r_{ad}) + k_b(r_{bd} - r_{bc}) \ll 1\,. \qquad (7.3)$$

The condition (7.3) under which the interference of independent beams does not disappear, may as well be written as follows

$$\vartheta_S \vartheta_D \ll \lambda/R\,, \qquad (7.4)$$

$$l_S l_D \ll \lambda R\,, \tag{7.5}$$

where $\vartheta_{S(D)} = l_{S(D)}/R$ is the angle at which the source (detector) with the lateral dimension $l_s(l_d)$ is visible from the detector (source) located at a distance R from $S(D)$; λ is the radiation (emission) wave length (it is supposed that $\Delta\lambda/\lambda \ll 1$).

If inequalities (7.3)–(7.5) are fulfilled, and $t = \tau$ (simultaneous registration of coincidences), then the second term in (7.2) equals the first term. Consequently, the coincidence count probability for identical particles is different from the result obtained by classical count by not more than a factor of two. From this also follows that the possibility to observe the interference is determined by the possibility to register coincidences in the classical situation. If the intensity of the sources is such that random coincidences of particles without reference to the identity occur in the given experiment, the interference phenomena are observed.

Since the number of coincidences obtained during the time T fluctuates, the intensity of the sources and the time T should be such that the average number of coincidences $\langle N\rangle$ during the stated time would be greater than the magnitude of fluctuations in the number of coincidences $\delta N = \sqrt{\langle N^2\rangle - \langle N\rangle^2} \sim \sqrt{\langle N\rangle} \sim \sqrt{n_1 n_2 \tau_c T}$ ($n_1(n_2)$ is the number of particles registered by counter 1(2) per unit time; τ_c is the resolution time of the coincidence circuit).[1] Hence, the inequality

$$\frac{\langle N\rangle}{\delta N} \simeq \sqrt{n_1 n_2 \tau_c T} > 1$$

should hold. If ρ is the surface intensity of the source and η is the efficiency of the counters, then $n_{1,2} \simeq \rho l_S^2 (l_D^2/R^2)\eta$. When the condition (7.5) is fulfilled, we obtain $n_{1,2} \leq \rho\lambda^2\eta$. Thus, finally we have

$$\langle N\rangle/\delta N \simeq \eta\rho\lambda^2\sqrt{\tau_c T} > 1\,, \tag{7.6}$$

which coincides with the expression in [Goldberger and Watson (1965)] for the case when the length of the train of the incident waves is comparable with the time τ_c.

Expressions (7.1), (7.2) are derived under the assumption that the atoms of the source are fixed and undisturbed. However, generally speaking, in real conditions it is not the case. Thermal motion and collisions of atoms in a source leads to the frequency modulation of the emitted photons. In this general case, the photon wave function may be represented as follows:

$$\Psi_a(t) \sim \exp\left\{-i[\omega_a t + \Omega_a(t)]\right\}\,, \tag{7.7}$$

[1] To be more specific, consider the case of small counting rate of the coincidence circuit. Otherwise, we may talk of the correlation function rather than of the number of coincidences.

where $\Omega_a(t)$ is the phase change of the photon due to thermal motion and collisions of the emitting atom a.

If the dimensions of the sources and detectors satisfy inequalities (7.3)–(7.5), then using the wave functions of the type (7.7), one can write the following expression for the probability $P_{ab}(\vec{r}_c\,, t\,; \vec{r}_d\,, \tau)$ averaged over the states of atoms a and b in the source:

$$\begin{aligned} &P_{ab}(\vec{r}_c\,, t\,; \vec{r}_d\,, \tau) = \text{const}\,\langle|\exp\{-i[\omega_a t + \Omega_a(t)]\} \\ &\times\exp\{-i[\omega_b\tau + \Omega_b(\tau)]\} + \exp\{-i[\omega_a\tau + \Omega_a(\tau)]\} \\ &\times\exp\{-i[\omega_b t + \Omega_b(t)]\}\,|^2\rangle\,, \end{aligned} \tag{7.8}$$

here angle brackets mean averaging.

Equality (7.8) includes the quantity

$$G_{ab}(t,\tau) = \langle\exp\{-i[\Omega_a(t) - \Omega_a(\tau) + \Omega_b(\tau) - \Omega_b(t)]\}\rangle$$

characterizing the kinetic processes in the source. As is seen from (7.8), when studying photon correlations, the probability of registration of delayed coincidences by two counters only depends on mutual correlations between atoms a and b. If we measured triple or higher fold coincidence events, the corresponding probabilities would only depend on mutual correlations between three and more atoms. This is slightly different from the situation arising in studying correlations in the radiation scattered by a certain target for the case when the energy spectrum of the scattered radiation being measured depends also on time correlations of the state of one atom. For simplicity, let us further assume that the correlations between atoms a and b may be neglected (for example, investigating the radiation of a gaseous source). Then

$$\begin{aligned} G_{ab}(t,\tau) &= G_a(t,\tau)G_b^*(t,\tau)\,, \\ G_{a(b)}(t,\tau) &= \langle\exp\left\{-i[\Omega_{a(b)}(t) - \Omega_{a,(b)}(\tau)]\right\}\rangle\,. \end{aligned}$$

For homogeneous systems, $G_a(t,\tau) = G_b(t,\tau) = G(t,\tau)$.

In most cases of practical interest $G(t,\tau)$ may be represented as follows:

$$G(t-\tau) = \exp\left\{-w^2(t-\tau)/4\right\}\,, \tag{7.9}$$

where $w^2(t-\tau)/2 = \langle[\Omega_a(t) - \Omega_a(\tau)]^2\rangle$.

Using (7.9), from (7.8) we obtain the equality

$$\begin{aligned} &P_{ab}(t-\tau) = \text{const}\,\left\{1 + \exp\left\{-\mathtt{Re}[w^2(t-\tau)]/2\right\}\right. \\ &\left.\times\cos[(\omega_a - \omega_b)(t-\tau)]\right\}\,. \end{aligned} \tag{7.10}$$

Sum (7.10) over all the pairs of atoms in the source[2] and the detector, assuming that the source emits with equal probability the photons of only two frequencies ω_1 and ω_2. As a result, we obtain the below expression for probability $P(t-\tau)$ that one photon will be registered at moment t, and the other one, at moment τ:

$$P(t-\tau) = \text{const}\left\{1 + \exp\left[-\frac{\mathtt{Re}w^2(t-\tau)}{2}\right]\right.$$
$$\left.\times \cos^2\left[\frac{\omega_1-\omega_2}{2}(t-\tau)\right]\right\}. \tag{7.11}$$

Thus, the curve of delayed coincidences undergoes modulated beatings, depending on the the delay time $\theta = |t-\tau|$ at the frequency equal to the difference of frequencies ω_1 and ω_2. The frequency of beatings can, in principle, be controlled by means of various external influences, e.g., by placing the source to the external magnetic field.

Now determine the total number of coincidences N in the given experiment, i.e., determine the area under the curve of delayed coincidences if the maximum delay time used in the experiment is θ_m. For this we integrate (7.11) over $\theta = t-\tau$ within the interval $[0, \theta_m]$, which gives the expression of the form

$$N = \text{const}\left\{\theta_m + \int_0^{\theta_m} \exp\left[-\frac{\mathtt{Re}w^2(\theta)}{2}\right]\cos^2\left[\frac{\omega_1-\omega_2}{2}\theta\right] d\theta\right\}. \tag{7.12}$$

The first term proportional to θ_m would correspond to the number of coincidences if the photons could be distinguished; the second one gives the addition to N, appearing due to the particle identity. Hence, taking into account the identity leads to the fact that the area under the curve of delayed coincidences is not proportional to θ_m, as it would be for distinguishable particles.

Consider (7.12) for two limiting cases: in the first one the perfect gas with temperature Q acts as a source, in the second one the source is such that the major role in modulating the radiation frequency is played by collisions, whose influence will be taken into account in the collision approximation. In the former case, $\mathtt{Re}[w^2(\theta)] = k^2\bar{v}^2\theta^2$, in the latter case $\mathtt{Re}[w^2(\theta)] = 4\rho\sqrt{\bar{v}^2}\sigma\theta$; here $\bar{v}^2$ is the mean–square thermal velocity of atoms, ρ is the atomic density in the gas, and σ is the collision cross section.

[2]Such summation can be made if the photon density in the source is such that the simulated emission of atoms can be neglected.

Substitution of the stated expression for $w^2(\theta)$ into (7.12) gives the following expressions for the two cases:

$$N = \text{const}\left\{\theta_m + \int_0^{\theta_m} \exp - \left(-\frac{k^2\bar{v}^2}{2}\theta^2\right) \times \cos^2\left[\frac{\omega_1-\omega_2}{2}\theta\right] d\theta\right\}, \tag{7.13}$$

$$N' = \text{const}\left\{\theta_m + \frac{1}{2\Gamma} + \frac{\Gamma}{2[(\omega_1-\omega_2)^2+\Gamma^2]} - \frac{\exp(-\Gamma\theta_m)}{2\Gamma}\left(1+\frac{a\Gamma\cos[(\omega_1-\omega_2)\theta_m+\varphi]}{(\omega_1-\omega_2)^2+\Gamma^2}\right)\right\}. \tag{7.14}$$

where $ae^{i\varphi} = \Gamma + i(\omega_1-\omega_2)$; $\Gamma = 2\rho\sqrt{\bar{v}^2}\sigma$ is the impact width of the level[3].

If $\theta_m(\bar{v}^2k^2)^{1/2} \gg 1$ and $\Gamma\theta_m \gg 1$, (7.13) and (7.14) may be recast as follows

$$N \simeq \text{const}\,\theta_m\left\{1+\sqrt{\frac{\pi}{\bar{v}^2\theta_m^2k^2}} + \sqrt{\frac{\pi}{\bar{v}^2\theta_m^2k^2}}\exp\left[-\frac{(\omega_1-\omega_2)^2}{\bar{v}^2k^2}\right]\right\}, \tag{7.15}$$

$$N \approx \text{const}\,\theta_m\left\{1+-\frac{1}{2\Gamma\theta_m}+\frac{1}{2\Gamma\theta_m}\frac{\Gamma^2}{(\omega_1-\omega_2)^2+\Gamma^2}\right\}. \tag{7.16}$$

Thus, the area under the delayed-coincidence curve depends on the difference $\omega_1-\omega_2$ and on the mechanism of the radiation frequency modulation. When $\omega_1=\omega_2$, expressions (7.15) and (7.16) differ from the result obtained for classical particles by the magnitudes $2\sqrt{\pi/\bar{v}^2\theta_m^2k^2}$ and $1/\Gamma\theta_m$, respectively. If $(\omega_1-\omega_2)^2/\bar{v}^2k^2 \gg 1$ or $|\omega_1-\omega_2| \gg \Gamma$, the number of coincidences exceeds the classical result by $\sqrt{\pi/\bar{v}^2\theta_m^2k^2}$ and $1/2\Gamma\theta_m$, respectively. This means that when the stated inequalities are fulfilled, the photons of frequency ω_1 and photons of frequency ω_2 may be considered non-identical. At the same time, the photons of the same frequency (either ω_1 or ω_2), of course remain identical to one another, which is manifested in the fact that the magnitudes of N and N' differ from those predicted for classical particles.

Let us note in conclusion that it would be tempting to carry out such experiments not only for optical photons but also for, e.g., Mössbauer γ-quanta. However, due to the short wavelength of γ-quanta, it is practically

[3]When the impact and the Doppler width of the level can be neglected, (7.14), where Γ is the natural width of the level, holds true for the area of the delayed coincidence curve.

impossible nowadays to realize the conditions (7.5) and (7.6) using conventional sources.

Indeed, from (7.5) we obtain that if for light at $\lambda \simeq 10^{-5}$ cm and $l_S \sim l_D \sim 10^{-1}$ cm, it should be $R \geq 10^3$ cm, then for γ-quanta ($\lambda \sim 10^{-8}$ cm) at the same dimensions of the source and the detector, it should be $R \geq 10^6$ cm.

A more detailed treatment shows that this problem might be avoided, using some artificial procedures. But even stricter requirements are imposed by inequality (7.6). From it follows that with other conditions being equal, the observation time T_γ in the X-ray spectrum should be by several orders of magnitude greater than the corresponding time T_c in the optical spectrum ($T_\gamma \approx (\lambda_c/\lambda_\gamma)^4 T_c$, i.e., $T_\gamma \approx 10^{12} T_c$). The aforesaid also refers to the case when a scattering target is placed between the sources and detectors, as it may be treated just as a source of scattered waves. Serious problems also exist for other types of radiation (electrons and neutrons).

7.2 Interference of γ-quanta Generated by the Beams of Relativistic Particles

Quite a different situation arises when radiation produced by relativistic particles is used as a source[Baryshevsky and Feranchuk (1980a)]. There are two possible kinds of experiment: (a) radiation is produced when a relativistic particle passes through a crystal, (b) synchrotron radiation is diffracted in the Mossbauer crystal.

Recall (see(3.3) that radiation is a crystal is formed through two mechanisms: parametric one, and radiative transitions between the levels (regions) of transverse motion. The emerging γ-quanta move within a narrow angle along the direction of the particle motion and along the direction determined by the reciprocal lattice vector $2\pi\vec{\tau}$. The number of resonance γ-quanta, produced by one electron in a crystal, due to the parametric effect, for the forward direction equals ($\hbar = c = 1$)

$$N_\gamma^{(\tau)} = e^2 \frac{\Gamma}{\omega_p}, \tag{7.17}$$

for radiation in the direction of diffraction

$$N_\gamma^{(\tau)} = e^2 \frac{g_{00}''(\omega_p)}{\sqrt{|\Delta|}} \sqrt{g_{00}''(\omega_p)} \frac{\Gamma}{|2\pi\tau|}; \ \frac{m^2}{E^2} < |g_{00}(\omega_p)|, \tag{7.18}$$

where $\Delta = g_{00}g_{11} - g_{01}g_{10}$.

Angular divergence of the quanta produced (see (4.7)) for the forward direction has the magnitude of the order of

$$\theta^{(0)} \le \sqrt{\left(\frac{m}{E}\right)^2 + |g_{00}|}. \tag{7.19}$$

Angular divergence of the photons emitted along the direction of diffraction is much less

$$\theta^{(\tau)} \le \sqrt{|\Delta|}. \tag{7.20}$$

In the case in question the linear dimensions of the source are defined by the width d of the electron beam incident on the crystal ($l_S = d$). Linear dimensions of the input window of the detector, where the photons produced by the particle get equal

$$l_D = d + R\theta^{(0,\tau)}. \tag{7.21}$$

As a result, the condition (7.5) may be written in the form

$$d^2 + dR\theta^{(0,\tau)} \le \lambda R. \tag{7.22}$$

In observation of the interference phenomena in radiation propagating along the direction of particle motion, the condition (7.22) is difficult to fulfil. For example, for a crystal of ^{57}Fe and the electron energy $E \sim 1$ GeV, the width of the electron beam should be less than 10^{-4} cm. At the same time, due to the fact that $\theta^{(\tau)} \ll \theta^{(0)}$, when observing interference in the direction of diffraction under the same conditions $d \sim 0.1$ cm.

Further we shall consider the possibility of observation of interference in the direction of diffraction, assuming that the condition (7.22) is fulfilled. Let the resolution time of the coincidence circuit τ_c is less than the length of the train of the obtained γ-quantum, i.e., of the order of magnitude $\tau_c \le 1/\Gamma$. Taking into account that the number of particles passing through a crystal in one second is I/e (I is the current strength), we have for the number of quanta produced in the crystal in one second:

$$n = \frac{I}{e} N_\gamma^{(\tau)}. \tag{7.23}$$

Due to a small angular divergence of the radiation produced in the crystal, all the photons get into the detector. Therefore when the condition (7.22) is fulfilled using the parametric effect, we find the following estimate for the observation time of the interference pattern T:

$$T > \frac{|2\pi\tau|^2 |\Delta|}{\eta^2 \Gamma (g''_{00})^3 I^2 e^2}. \tag{7.24}$$

When $I = 10\mu A = 10^{-5}\, A$, $\eta \simeq 1$, $E = 1$ GeV, from (7.24) follows the estimate $T \geq 10^4$ s for ^{57}Fe: $T \geq 10^2$ s for ^{137}W.

Now consider the possibility of observation of the independently generated photons, using diffraction of synchrotron radiation in a crystal containing resonance nuclei. Applying the conditions (7.5), (7.6) and the expression for the intensity of the synchrotron radiation (see, for instance, [Feranchuk (1979a)]), one may obtain the expression for the observation time

$$T \geq \frac{3}{16\pi^2 e^4} \frac{S^2 \omega_p^4}{\Gamma n_e^2}, \tag{7.25}$$

where S is the area of the electron beam cross section; n_e is the number of electrons in the accelerator. When $S = 10^{-3}$cm, $r = 10^3$ cm, $n_e = 10^{12}$ (or $I = 0.1\, A$), the estimate is $T \geq 10^3$ s.

The stated time may be appreciably reduced ($T \sim 1$ s), using radiation of the powerful storage rings like those discussed in [Kapitsa (1979)], which are to be constructed. Such times are also achieved with the help of the parametric effect at the electron current of the order of 10^{-4} A. The phenomena being analyzed may be applied for direct phase analysis by introducing Mössbauer nuclei into the structure in question. If this method is hampered, the Mössbauer crystal may be used as the source of radiation, which is subsequently diffracted by the examined substance.

Chapter 8

Theory of Measurement of Nuclear Reaction Times Using Shadow Effect. Yield of Reactions Induced by High-energy Particles in Crystals

8.1 Quantum Theory of Reactions Induced by Channeled Particles

Particle motion in a single crystal is accompanied by numerous inelastic processes and reactions. The investigation of these processes and reactions provides important information about crystal structure and the properties of nuclei. In particular, the shadow effect is widely used to explore the nuclear reaction times τ in the range $\tau \leq 10^{-16}$ s [Karamyan *et al.* (1973)]. When interpreting the results obtained, it is supposed that what is measured in the experiments under discussion is the nucleus lifetime.

Although, analyzing the fluctuations of effective cross sections of reactions, Lyuboshitz and Podgoretky [Lyubosihtz and Podgoretskii (1976); Lyubosihtz (1978a,b)] showed that in strong overlap of the levels the law of the compound nucleus decay becomes appreciably nonexponential. It was also stated that in this case the process of inelastic scattering can be divided into instantaneous diffraction scattering and fluctuation scattering, associated with the decay of the compound systems.

According to [Lyubosihtz and Podgoretskii (1976); Lyubosihtz (1978a,b)] the characteristic time duration of the fluctuating part of the reaction is determined by the mean interlevel distance rather than by the level width of a compound nucleus. (The whole analysis in [Lyubosihtz and Podgoretskii (1976); Lyubosihtz (1978a,b)] was carried out by using packets.)

Within the framework of a stationary quantum mechanical theory of scattering, we have demonstrated that, by applying monochromatic states, it is also possible to define the nuclear decay law [Baryshevsky and Tkacheva (1978); Baryshevsky (1979b)]. Moreover, angular distribution of secondary particles, studied in the experiments on shadow effect [Karamyan *et al.* (1973)] turned out to be determined by the correlation function of the reaction amplitudes, which in the case of strong overlap of the levels coincides with the function introduced in [Lyubosihtz and Podgoretskii (1976); Lyubosihtz (1978a,b)].

In this regard it is worthy of mention that the possibility of using the formulae employed in the experiments on the shadow effect for establishing the relationship between the angular distribution of secondary particles and the law of the compound nucleus decay in the case under consideration requires additional analysis. This circumstance is attributed to the fact that until now the theory describing the method for measuring nuclear reaction times has been practically completely based on using classical models involving a number of uncertain parameters (variables) (such as the chain cutoff radius; for more details, see [Karamyan *et al.* (1973)]). Whereas an essential element of quantum consideration of the effect given in [Yazaki and Yoshida (1974)] is the assumption that the motion of finite particles is classical.

Below is presented a quantum mechanical theory of scattering which enables deriving formulae relating the angular distribution of particles - the reaction products - to the position distribution function of the compound nucleus without using under-substantiated model approximations. It is also shown that in excitation of a group of levels, the position distribution function of the compound nucleus undergoes spatial beating with the period determined by the interlevel distance. This enables using shadow effect for investigation not only the level width but also the interlevel distance. Applicability to the shadow effect of the hypothesis of the rapidly established statistical equilibrium in the transverse plane of the phase space of the particle leaving the crystal, widely used within the framework of the classical approach is validated [Karamyan *et al.* (1973)]. Using the formal theory of reactions makes it possible to directly apply the obtained results to electron-nuclear reactions induced by relativistic particles (e.g. electrons and positrons) too, if by the particle mass we mean its relativistic mass.

So, let a particle a be incident on a crystal, causing a nuclear reaction $A(a,b)$ in it. Consider the angular distribution of particles b. According to the general theory of reactions (see, for example, [Goldberger and Watson (1984(@)]) the cross section of this process may be written in the form:

$$\frac{d\sigma_{ab}}{d\Omega_b} = \frac{m_a m_b}{(2\pi\hbar^2)^2 k_a} \times \sum_B \int k_b dE_b \delta(E_{bB} - E_{aA}) |\langle \varphi_{bB}^{(-)} | \mathcal{F} | \varphi_{aA}^{(+)} \rangle|^2 \,, \tag{8.1}$$

where $m_{a(b)}$ is the mass of the particle $a(b)$; $k_{a(b)}$ is the wave number of particle $a(b)$; E_{aA} is the energy of the initial state; E_{bB} is the energy of the final state; $\mathcal{F}$ is the scattering operator; $\varphi^{(+)}$ is the wave function of the initial state taking account of the interaction of particles a and A with the crystal, having at infinity the asymptotics which contains diverging waves; $\varphi^{(-)}$ is the same for the final state with the asymptotics containing converging waves.

The shadow effect is applied to investigation of the duration time of nuclear reaction $\tau \leq 10^{-16}$ s [Karamyan *et al.* (1973)]. As in this case the width of nuclear levels which are involved in the reaction (and the energy of particles) is much greater than the characteristic vibration frequencies of nuclei in the crystal, in order to find the operator $\mathcal{F}$, the impulse approximation may be used. According to this approximation [Goldberger and Watson (1984(@)], it it is assumed that $\mathcal{F}$ coincides with the scattering operator describing the reaction $A(a,b)B$ with free particles, i.e., particles that do not interact with the crystal:

$$\langle \vec{k}_b, \vec{k}_B | \mathcal{F} | \vec{k}_a, \vec{k}_A \rangle = (2\pi)^3 \delta(\vec{k}_b + \vec{k}_B - \vec{k}_a - \vec{k}_A) \langle \vec{k}_b, \vec{k}_B | T | \vec{k}_a, \vec{k}_A \rangle \,, \tag{8.2}$$

where the amplitude $\langle \vec{k}_b, \vec{k}_B | T | \vec{k}_a, \vec{k}_A \rangle = T_{ba}(\vec{\kappa}_f, \vec{\kappa}, \varepsilon)$ depends of the relative momenta of the initial ($\vec{\kappa}$) and final ($\vec{\kappa}_1$) states , and the energy $\varepsilon = \hbar^2\kappa^2/2\mu$ of the relative motion in the initial state ($\mu = m_a M_A/(m_a + M_A)$ is the reduced mass in the initial state). As a result, (8.1) may be written as follows

$$\frac{d\sigma_{ab}}{d\Omega_b} = \frac{m_a m_b}{(2\pi\hbar^2)^2 (2\pi)^{12} k_a} \int \cdots \int k_b dE_b d^3k_B d^3k_b' d^3k_a' \times d^3k_A \delta(E_{bB} - E_{aA}) |\langle \varphi_{\vec{k}_b}^{(-)} | \vec{k}_b' \rangle \delta(\vec{k}_b' + \vec{k}_B - \vec{k}_a' - \vec{k}_A) \times \langle \vec{k}_b', \vec{k}_B | T | \vec{k}_a', \vec{k}_A \rangle \langle \vec{k}_a' | \varphi_{\vec{k}_a}^{(+)} \rangle \langle \vec{k}_A | \Phi_A \rangle|^2 \,, \tag{8.3}$$

where $|\varphi_{\vec{k}_a}^{(+)}\rangle$ is the wave function of particle a incident on the crystal; $|\varphi_{\vec{k}_b}^{(-)}\rangle$ is the wave function of outcoming particle b; $|\Phi_A\rangle$ is the wave function of the nucleus.

Further we shall consider quite thin crystals so that we could neglect energy losses of particles a and b participating in the reaction (according to [Kagan and Kononets (1973)] for protons with the energies $E = 0.4$ MeV in Si, the thickness is $l \sim 5 - 7 \times 10^3$ Å, for α particles with the energies $E = 2$ MeV in Au, the thickness is $l \sim 3 \times 10^3$ Å). In this case the wave functions describing the phenomenon of channeling of particles incident on the crystal and leaving it may be found, using the method presented in (). For example, inside the crystal the wave function may be given as

$$\varphi^{(+)}_{\vec{k}_a} = \sum_n c_{n\vec{k}_a} \Psi_{n\vec{\kappa}_a}(\vec{\rho}) \exp(ik_{azn}z),$$

$$c_{n\vec{k}_a} = \frac{(2\pi)^2}{\Omega_0} \int_S \exp(i\vec{k}_{a\perp}\vec{\rho}')\Psi^*_{n\vec{\kappa}_a}(\vec{\rho}')d^2\rho', \tag{8.4}$$

where the z-axis of the coordinate system is directed parallel to the family of axes (planes) along which the particle is channeled; $\Psi_{n\vec{\kappa}_a}(\vec{\rho})$ is the Bloch function; the coordinate $\vec{\rho} = (x, y)$; n is the index of the energy zone of the particle's transverse motion; $\vec{k}_{a\perp}$ is the component of vector $\vec{k}_a$ perpendicular to the z-axis; $\vec{\kappa}_a$ is the reduced wave vector, corresponding to $\vec{k}_{a\perp}$;

$$k_{azn} = \sqrt{k_a^2 - \frac{2m_a}{\hbar^2}\varepsilon_n(\vec{\kappa}_a)}\,,$$

$\varepsilon_n(\vec{\kappa}_a)$ is the energy of the transverse motion of the particle in zone n; Ω_0 is the volume of the two–dimensional unit cell of the crystal in plane ρ. Integration with respect to $d^2\rho'$ is performed over the two–dimensional unit cell in plane ρ.

The state $\varphi^{(-)}$ is found, using the relation $\varphi^{(-)}_{\vec{k}}(\vec{r}) = \varphi^{(+)*}_{-\vec{k}}(\vec{r})$. From the form of the wave functions $\varphi^{(+)}_{\vec{k}_a}(\varphi^{(-)}_{\vec{k}_b})$ follows that the matrix elements $\langle\vec{k}'_a|\varphi^{(+)}_{\vec{k}_a}\rangle$ and $\langle\varphi^{(-)}_{\vec{k}_b}|\vec{k}'_b\rangle$ make the intermediate momenta $\vec{k}'_a$ and $\vec{k}'_b$ close to $\vec{k}_a$ and $\vec{k}_b$ with the accuracy of the order of $1/l$ for $k'_{a\parallel}$ and $k'_{b\parallel}$ (l is the crystal thickness, the symbol $\parallel$ denotes the components of the momentum parallel to the axis (plane) along which the particle is channeled), which is much smaller than $k_{a(b)}\vartheta_L$, where ϑ_L is the Lindhard angle. The uncertainty of momentum k_A is of the order of $1/r_0$, where r_0 is the vibration amplitude of the nucleus.

Pay attention to the fact that the reaction amplitude T_{ba} can be presented in the form

$$T_{ba}(\vec{\kappa}_f, \vec{\kappa}, \varepsilon) = T_D(\vec{\kappa}_f, \vec{\kappa}, \varepsilon) + \sum_\lambda \frac{\gamma_{\lambda b}(\vec{\kappa}_f)\gamma_{\lambda a}(\vec{\kappa})}{\varepsilon(\kappa) - E_\lambda + \frac{1}{2}i\Gamma_\lambda}\,, \tag{8.5}$$

where T_D is the amplitude of the direct reaction; $\gamma_{\lambda b}(\gamma_{\lambda a})$ in the general case, are complex quantities, which, at weak overlap of the levels, coincide with partial widths for the transitions $\lambda \to b$, $\lambda \to a$; E_λ is the energy of the resonance λ; Γ_λ is its width. The dependence of T_D and $\gamma_{\lambda a(b)}$ on the momenta is determined by the spatial domain of the order of the nuclear dimension in size. For this reason, the momentum uncertainty of the incident and outcoming particles, which is caused by their interaction with the crystal ($\leq 10^{10}\,\mathrm{cm}^{-1}$), can be ignored in T_D and $\gamma_{\lambda a(b)}$. As a consequence, $\vec{\kappa}_f$ and $\vec{\kappa}$ in them may be equated to the vacuum values of the momentum of particles (b — $\vec{k}_b$) and (a — $\vec{k}_a$), respectively.

Note also that in integration with respect to the component of the relative momentum, which is parallel to the incident direction of the primary particle, the contribution from the resonant denominator in (8.5) will be determined by the residues at points

$$\kappa_\parallel = \left(\frac{2\mu}{\hbar^2}E_\lambda - \kappa_\perp^2 - i\frac{\mu\Gamma_\lambda}{\hbar^2}\right)^{1/2}$$
$$\simeq \left(\frac{2\mu}{\hbar^2}E_\lambda\right)^{1/2}\left(1 - \frac{\hbar^2\kappa_\perp^2}{4\mu E_\lambda} - i\frac{\Gamma_\lambda}{4E_\lambda}\right).$$

As the transverse relative momentum

$$\vec{\kappa}_\perp = \frac{M_A}{m_a + M_A}\vec{k}'_{a\perp} - \frac{m_a}{m_a + M_A}\vec{k}_{a\perp}$$

is limited by the matrix elements, the contribution of $\kappa_\perp$ to the real part of the pole appears to be small and should be ignored.

Thus, one may consider that in integration with respect to the intermediate momentum, the amplitude T_{ba} depends only on the component of the relative momentum $\kappa_\parallel$ that is parallel to the momentum of the incident particle:

$$\vec{\kappa}_\parallel = \frac{M_A}{m_a + M_A}k'_{a\parallel} - \frac{m_a}{m_a + M_A}k_{a\parallel}.$$

As a result, we obtain the following expression for the differential cross section:

$$\frac{d\sigma_{ab}}{d\Omega_b} = D\int d^3r_b|\varphi^{(-)}_{\vec{k}_b}(\vec{r}_b)|^2 Q(\vec{r}_b)\,, \tag{8.6}$$

where the constant

$$D = \frac{2\pi m_a m_b}{(2\pi\hbar^2)^2 k_a}\,,$$

$$Q(\vec{r}_b) = \left| \int dr_{A\|} \left\{ \int d\kappa_\| \frac{m_a + M_A}{m_a} T_{ba}(\kappa_\|) \right.\right.$$
$$\left. \times \exp\left[-i\frac{m_a + M_A}{m_a}\kappa_\|(r_{b\,\|} - r_{A\,\|}) \right] \right\}$$
$$\times \varphi^{(+)}_{\vec{k}_a}\left(\vec{r}_{b\perp}; \frac{m_a + M_A}{m_a} r_{b\,\|} - \frac{M_A}{m_a} r_{A\,\|} \right)$$
$$\left. \times \Phi_A(\vec{r}_{b\perp} - \vec{R}_{i\perp};\, r_{A\,\|} - z_i) \right|^2 .$$

Expression (8.6) should be averaged over the coordinates of the equilibrium positions of the excited nuclei $\vec{R}_i$, which are assumed to be uniformly distributed over the crystal volume. Upon such averaging equality (8.6) may be written as follows

$$\frac{d\sigma_{ab}}{d\Omega_b} = D \int d^3 r_b n_{\vec{k}_b}(\vec{r}_b) \tilde{Q}(\vec{r}_b)\,, \tag{8.7}$$

$$\tilde{Q}(\vec{r}_b) = \sum_f n_{\vec{k}_a f}(\vec{r}_{b\perp}) \Phi_A^2(\vec{r}_{b\perp})$$
$$\times \left| \int dr_{A\,\|} \left\{ \int d\kappa_\| \frac{m_a + M_A}{m_a} T_{ba}(\kappa_\|) \right.\right.$$
$$\left. \times \exp\left[-i\frac{m_a + M_A}{m_a}\kappa_\|(r_{b\,\|} - r_{A\,\|}) \right] \right\}$$
$$\left. \times \exp\left(-ik_{\|\,af}\frac{M_A}{m_a} r_{A\,\|} \right) \Phi_A(r_{A\,\|}) \right|^2 , \tag{8.8}$$

where

$$n_{\vec{k}_b}(\vec{r}_b) = \sum_n |c_{n_{\vec{k}_b}}|^2 |\psi_{n\vec{\kappa}_b}(\vec{r}_b)|^2\,, \tag{8.9}$$

$$n_{\vec{k}_a f}(\vec{r}_{b\perp}) = |c_{f_{\vec{k}_a}}|^2 |\psi_{f\vec{\kappa}_a}(\vec{r}_{b\perp})|^2\,, \tag{8.10}$$

$n_{\vec{k}_b}(\vec{r}_b)$ only depends on the components of vectors $\vec{r}_b$ and $\vec{k}_b$ perpendicular to the direction of the axes (planes) along which particle b is channeled and has the meaning of density distribution of particles b in the transverse plane relative to the stated axes (planes); $n_{\vec{k}_a f}(\vec{r}_{b\perp})$ only depends on the projection of vector $\vec{r}_{b\perp}$ which is perpendicular to the direction of the axes along which the incident particle a moves and has the meaning of density distribution of particles a occupying the transverse energy level f in the plane perpendicular to the channeling axes (planes).

Discuss the derived expression in more detail. The quantity $d\sigma_{ab}/d\Omega_b$ determines up to a constant the angular distribution of the flow J of particles b produced through the reaction. On the other hand, the flow J can be found by solving the Schrödinger equation of the form

$$(\Delta_r + k^2 - u(\vec{r}))\psi(\vec{r}) = q(\vec{r}), \tag{8.11}$$

where $q(\vec{r})$ is the current distribution amplitude of the source of particles;

$$u(\vec{r}) = \frac{2m}{\hbar^2}V(\vec{r}),$$

$V(\vec{r})$ is the potential in which the emitted particle moves.

The solution of (8.11) has the form

$$\psi(\vec{r}) = \int G(\vec{r}, \vec{r}\,')q(\vec{r}\,')d^3r\,', \tag{8.12}$$

where $G(\vec{r}, \vec{r}\,')$ is the Green function of (8.11).

According to [Baryshevsky (1976)], in moving in an arbitrary potential

$$\lim_{r\to\infty} G(\vec{r}, \vec{r}\,') = -\frac{1}{4\pi}\frac{\exp(ikr)}{r}\psi_{\vec{k}}^{(-)*}(\vec{r}\,'). \tag{8.13}$$

As a result,

$$\psi(\vec{r}) = -\frac{1}{4\pi}\frac{\exp(ikr)}{r}\int \psi_{\vec{k}}^{(-)*}(\vec{r}\,')q(\vec{r}\,')d^3r\,'. \tag{8.14}$$

From (8.12) follows that the flow of particles produced by the source is:

$$J = \frac{\mathrm{const}}{r^2}\left|\int \psi_{\vec{k}}^{(-)*}(\vec{r}\,')q(\vec{r}\,')d^3r'\right|^2. \tag{8.15}$$

Let us average (8.15) over the distribution of currents in the source and assume that the source is spatially incoherent, i.e.,

$$\langle q(\vec{r}\,')q(\vec{r}\,'')\rangle = Q(\vec{r}\,')\delta(\vec{r}\,' - \vec{r}\,'').$$

As a consequence,

$$J = \frac{\mathrm{const}}{r^2}\int \left|\psi_{\vec{k}}^{(-)}(\vec{r}\,')\right|^2 Q(\vec{r}\,')d^3r'. \tag{8.16}$$

Comparison of (8.16) with (8.6) and (8.7) gives that $\tilde{Q}(\vec{r}_b)$ has a meaning of distribution density of the emitting points of the source, which in our case are the nuclei produced via coalescence of particles a and A.

Note that the quantity $n_{\vec{k}_b}(\vec{r}_b)$ that appeared upon averaging of the cross-section over the positions of nucleus A is, in fact, the diagonal element of the density matrix of particles b, which determines their distribution in the transverse plane of the channel. In terms of classical theory, this means that the angular distribution of the emitted particles is defined by the statistical equilibrium density in the phase space of the transverse particle motion (in [Ryabov (1975)] direct calculation showed that within the classical limit, the density $n_{\vec{k}_b}(\vec{r}_b)$ coincides with the classical equilibrium density). Thus, the assumption about the fast established statistical equilibrium in the transverse plane of the phase space, regarded as a hypothesis in the classical derivation of angular distributions, is quite substantiated from the quantum viewpoint.

Substitute expression (8.5) for the reaction amplitude into formula (8.8) defining the distribution of the emitting points $\tilde{Q}(\vec{r}_b)$:

$$\begin{aligned}\tilde{Q}(\vec{r}_b) = \sum_f n_{\vec{k}_a f}(\vec{r}_{b\perp})\Phi_A^2(\vec{r}_{b\perp}) \Bigg| & T_D \exp\left(-ik_{a\parallel f}\frac{M_A}{m_a}r_{b\parallel}\right) \\ & \times\Phi_A(r_{b\parallel}) - i\pi\sum_\lambda \frac{2M_A\gamma_{\lambda b}\gamma_{\lambda a}}{\hbar^2\kappa'_\lambda} \\ & \times\int \exp\left[-i(\kappa'_\lambda - i\kappa''_\lambda)\frac{m_a+M_A}{m_a}|r_{b\parallel}-r_{A\parallel}|\right. \\ & \left. -ik_{af\parallel}\frac{M_A}{m_a}r_{A\parallel}\right]\Phi_A(r_{A\parallel})dr_{A\parallel}\Bigg|^2 ,\end{aligned} \tag{8.17}$$

where

$$\kappa'_\lambda = \sqrt{\frac{2\mu}{\hbar^2}E_\lambda}\,, \quad \kappa''_\lambda = \sqrt{\frac{2\mu}{\hbar^2}}\frac{\Gamma_\lambda}{4\sqrt{E_\lambda}}\,, \quad \kappa_\lambda = \kappa'_\lambda - i\kappa''_\lambda\,.$$

According to (8.17), the distribution of the emitting point is formed by the superposition of damped waves and stretches in the incident direction of the primary particle (integration in (8.17) is, in fact performed over all $r_{A\parallel} \leq r_{b\parallel}$; at $r_{A\parallel} > r_{b\parallel}$ the integrand oscillates extremely rapidly and the stated domain of integration can be discarded).

As would be expected, the rate of wave damping is determined by the lifetime of the compound nucleus at level λ and by its velocity. Indeed, the index of power of the damped exponent is:

$$\delta = k''_\lambda\frac{m_a+M_A}{m_a} = \frac{1}{2}\frac{\mu}{\hbar^2\kappa'_\lambda}\Gamma_\lambda\frac{m_a+M_A}{m_a}\,.$$

Integration over $dr_{A\parallel}$ leads to the fact that

$$\kappa'_\lambda = k_{a\parallel}\frac{\mu}{m_a}$$

with the accuracy up to the momentum associated with the thermal vibrations of nucleus A in the lattice. Hence, one can write

$$\delta = \frac{1}{2}\frac{\Gamma_\lambda}{\hbar}\frac{m_a + M_A}{\hbar k_{a\|}}\,, \quad \text{i.e.,} \quad \delta = \frac{1}{2}\frac{1}{v_c \tau_\lambda}\,,$$

where

$$\frac{1}{\tau_\lambda} = \frac{\Gamma_\lambda}{\hbar}$$

and

$$v_c = \frac{\hbar k_{a\|}}{m_a + M_A}$$

is the velocity of the compound nucleus.

Note that according to (8.17), the interference of the direct and resonance scattering channels has an appreciable influence on the shape of $\tilde{Q}(\vec{r}_b)$ at short lifetimes of the compound nucleus, when the magnitude of its spatial displacement is of the order of the vibration amplitude of the nucleus in a crystal. In this case, the first and second terms in (8.17) overlap most strongly, causing a significant deviation from a conventionally used exponential distribution law even when only one level is excited

$$\tilde{Q}(\vec{r}_b) \sim \exp\left(-\frac{r_{b\|} - r_{A\|}}{v_c \tau}\right)\,.$$

If a group of levels is excited, the superposition of waves λ entering into (8.17) brings about spatial oscillations of the distribution $\tilde{Q}(\vec{r}_b)$. The oscillation period $l_{\lambda\lambda'}$ is defined by the energy difference of the excited resonances:

$$l_{\lambda\lambda'} = 2\pi v_c \hbar / E_\lambda - E_{\lambda'}\,.$$

Thus, the shadow effect is applicable for determining the lifetime of a compound nucleus and the distance between the levels (at $E_\lambda - E_{\lambda'} \geq \Gamma_\lambda$, $\Gamma_{\lambda'}$) even in the case when a monochromatic particle beam is incident on the crystal. Note that the quantity $\Delta E \sim \hbar v_c / r_0$ acts as effective nonmonochromaticity, r_0 is the vibration amplitude of the nuclei in the lattice.

Now assume that a certain group of resonance levels is excited by a beam of particles which have the energy spread considerably exceeding the maximum distance between these levels. Then the cross-section, and hence, (8.17) should be averaged over the stated spread, which comes to integration of (8.17) with respect to $k_{a\|\gamma}$. As a result, (8.17) simplifies, taking the form

$$\begin{aligned}\tilde{Q}(\vec{r}_b) = \sum_f n_{\vec{k}_a f}(\vec{r}_{b\perp})\Phi_A^2(\vec{r}_{b\perp}) \{ &|T_D|^2 \Phi_A^2(r_{b\|}) \\ &-\frac{4\pi^2}{\Delta k_\|}\frac{m_a}{M_A}|T_D|\sum_\lambda \frac{2M_A\gamma_{\lambda b}\gamma_{\lambda a}}{\hbar^2\kappa'_\lambda}\sin\delta_D \Phi_A^2(r_{b\|}) \\ &+\frac{2\pi^3}{\Delta k_\|}\frac{m_a}{M_A}\int_{-\infty}^{r_{b\|}} \left|\sum_\lambda \frac{2M_A\gamma_{\lambda b}\gamma_{\lambda a}}{\hbar^2\kappa'_\lambda}\right. \\ &\left.\times\exp\left[-\frac{i}{\hbar}\left(E_\lambda - i\frac{1}{2}\Gamma_\lambda\right)\frac{r_{b\|}-r_{A\|}}{v_c}\right]\right|^2 \\ &\times\Phi_A^2(r_{A\|})dr_{A\|}\} \;, \end{aligned} \tag{8.18}$$

where $T_D = |T_D|\exp(i\delta_D)$; $\Delta k_\|$ is the domain of averaging. The oscillations appearing in (8.18) are the time–to–space conversion of a well-known phenomenon of time oscillations in the radiation intensity, which arise through level excitation by a non-monochromatic packet. In the experiments on the shadow effect they permit studying not only the lifetime of the levels but also the distance between them.

If a large number of neighboring levels are excited in the reaction, in practice to explicitly find the sums involved in (8.18) is a complicated task. However, if assumed that the level are randomly distributed over the excitation region, the expression for $\tilde{Q}(\vec{r}_b)$ may be derived by averaging (8.8) over the distribution of these levels (according to [Lyubosihtz and Podgoretskii (1976); Lyubosihtz (1978a,b)], averaging of the cross-section over the energy spread in the beam and over the level distribution leads to one and the same result). As a consequence, the average value of $\langle T_{ba}^*(\kappa_\|)T_{ba}(\kappa'_\|)\rangle$ will enter into the reaction cross section in (8.7).

Suppose that the levels are statistically independent, as well as the energy spread in the beam is much greater than the average interlevel distance and the width of the levels, but it does not exceed the interval over which the levels are concentrated. In this case the average value of $\langle T_{ba}^*(\kappa_\|)T_{ba}(\kappa'_\|)\rangle$ is the function of the difference of its arguments [Lyubosihtz and Podgoretskii (1976); Lyubosihtz (1978a,b)]:

$$\langle T_{ba}^*(\kappa_\|)T_{ba}(\kappa'_\|)\rangle = g_{ba}(\kappa_\| - \kappa'_\|)\,. \tag{8.19}$$

Note that in [Lyubosihtz and Podgoretskii (1976); Lyubosihtz (1978a,b)] the amplitude correlation function is presented as the function of energies, rather than the function of the wave numbers of relative motion. Integration in (8.7) and (8.8) with respect to $\kappa_{\|}$ and $\kappa'_{\|}$ gives

$$\frac{d\sigma_{ab}}{d\Omega_b} = (2\pi)^2 D \int d^3 r_b n_{\vec{k}_b}(\vec{r}_b)\langle Q(\vec{r}_b)\rangle \,, \tag{8.20}$$

$$\langle Q(\vec{r}_b)\rangle = n_{\vec{k}_a}(\vec{r}_{b\perp})\Phi_A^2(\vec{r}_{b\perp}) \int_{-\infty}^{r_{b\|}} g_{ba}(r_{b\|} - r_{A\|}) \times \Phi_A^2(r_{A\|}) dr_{A\|} \,, \tag{8.21}$$

$$g_{ba}(r_{b\|} - r_{A\|}) = \frac{1}{2\pi}\int g_{ba}(\kappa) \times \exp\left[-i\kappa\frac{m_a + M_A}{m_a}(r_{b\|} - r_{A\|})\right]\frac{m_a + M_A}{m_a} d\kappa \,, \tag{8.22}$$

$$n_{\vec{k}_a}(\vec{r}_{b\perp}) = \sum_f n_{\vec{k}_a f}(\vec{r}_{b\perp}) \,. \tag{8.23}$$

When the crystal is illuminated by particles under the conditions when channeling phenomenon for them is absent, the density $n_{\vec{k}_a}(\vec{r}_{b\perp})$ is independent of the coordinate, being a constant. If the time in the delay-time distribution function, obtained in [Lyubosihtz and Podgoretskii (1976); Lyubosihtz (1978a,b)] is expressed in terms of the travel distance, i.e. $t = (r_{b\|} - r_{A\|})/v_c$, then the stated function coincides with the function $g_{ba}(r_{b\|} - r_{A\|})$, appearing in (8.21). From this follows that

$$g_{ba}(r_{b\|} - r_{A\|}) = g_{ba}\left(\frac{r_{b\|} - r_{A\|}}{v_c}\right) .$$

Recall now that the density $n_{\vec{k}_b}(\vec{r}_b)$ only depends on the component $\vec{\rho}$ of vector $\vec{r}_b$, which lies in the plane perpendicular to the axes (planes) along which particle b is channeled. Therefore, if we introduce the coordinate system with the x-, y-axes lying in this plane, (8.20) may be written as follows:

$$\frac{d\sigma_{ab}}{d\Omega_b} = (2\pi)^2 D \int d^2 \rho_b n_{\vec{k}_b}(\vec{\rho}_b)\langle \widetilde{Q(\vec{\rho}_b)}\rangle \,, \tag{8.24}$$

where $\langle \widetilde{Q(\vec{\rho}_b)}\rangle = \int dz_b \langle Q(\vec{\rho}_b)\rangle$ describes the distribution of the emitting points in the transverse plane $\vec{\rho}_b$.

The integral in (8.24) only by the specific expression $\langle \widetilde{Q(\rho_b)}\rangle$ differs from the integral, determining averaged over the crystal thickness yield of nuclear reaction $I(\vec{k}_{b\perp})$ which is excited upon entering the crystal of particle b with the momentum $\vec{k}_{b\perp}$ transverse with respect to the axes along which the identical (akin) particle emitted by a nucleus is channeled (see, for example, [[Kagan and Kononets (1973)], formula (4.12)], where the quantity $I(\vec{k}_{b\perp})$ includes the squared absolute value of the wave function of the excited nucleus instead of $\langle\widetilde{Q(\rho_b)}\rangle$). In the case when nuclear reaction times are too short for the compound nucleus to get displaced over the distance larger than the vibration amplitude r_0 of particle A, the quantity $\langle\widetilde{Q(\rho_b)}\rangle$ is "smeared" over the region with spatial dimensions of the order of r_0. As a consequence, the angular distribution of the number of particles which have left the crystal coincide in form with the angular dependence of the yield $I(\vec{k}_{b\perp})$ of nuclear reactions. According to [Kagan and Kononets (1973)], the distribution of $I(\vec{k}_{b\perp})$ is minimal for entrance angles close to zero and grows with the entrance angles approaching the Lindhard angle, forming a breastwork due to the contribution from the over-barrier states. With further increase in the entrance angle it drops, approaching the magnitude characteristic of a disordered medium. Hence, the angular distribution of the particles leaving the crystal will also be the same as described above, which agrees with the experimentally observed pattern [Karamyan *et al.* (1973)].

With the increase in the reaction time, the function $\langle\widetilde{Q(\rho_b)}\rangle$ becomes more and more smeared and the shadow depth decreases (with the increase in the vibration amplitude of the nuclei, the depth of the minimum in the nuclear reaction yield $I(\vec{k}_{b\perp})$ diminishes [Kagan and Kononets (1973)]).

Thus, the expressions derived above in the general case solve the problem of the relationship between the angular distributions of the particles which have left the crystal and the function g_{ba}.

Expression (8.24) may be further particularized by substituting a quasiclassical expression for the particle distribution density $n_{\vec{k}_b}(\rho_b)$. It should be pointed out here that though the motion of a heavy particle or, for example, a relativistic positron is quasiclassical, in analyzing the reaction yield (the intensity of the particles produced), one should use quantum mechanical expressions (8.24) rather than classical formulas. This can be explained by fact that the function $Q(\rho)$ is generally nonzero in a classically inaccessible for positively charged particles range of the potential of particle interaction with the plane (or axis). It immediately follows from the representation form of (8.24) that the reaction yield depends on the sign of the particle charge. In the case of positrons, the maximum of the density $n_{\vec{k}_b}(\rho_b)$ does not coincide with that of function $Q(\rho_b)(|\Phi_A(\rho_b)|^2)$. For electrons, the overlap of functions $n_{\vec{k}}(\rho)$ and $Q(\rho)(|\Phi_A(\rho)|^2)$ is most complete. As a consequence, in the case of electrons, the integral of the form (8.24) is maximal, i.e., the yield of nuclear reactions is maximal.

Now discuss in more detail the features of the electron and positron distribution over the levels. With this aim in view, pay attention to the fact that in the quasi-classical limit, passing from summation to integration over the entering points and vice versa allows giving a simple geometric interpretation of the particle distribution over the levels. Indeed, consider the distribution of incident particles in a unit cell. In the planar case this is the distribution over the domain of length a. The probability to find a particle within the interval dx of this domain is dx/a or

$$\frac{1}{a}dx = \frac{1}{a}\frac{dx}{d\varepsilon_n}\frac{d\varepsilon_n}{dn}dn = \frac{2\pi}{aT_n|V'|}dn\,,$$

where it is taken into account that

$$\varepsilon_n = \frac{p_x^2}{2m} + V(x)\,, \quad \frac{d\varepsilon_n}{dn} = \frac{2\pi}{T_n}\,, \quad V' = \frac{dV}{dx}\,.$$

In other words, we may consider the equation

$$\varepsilon_n = \frac{p_x^2}{2m} + V(x)$$

as the one performing the conversion from variables $x(n)$ to variables $n(x)$ (similarly, in the axial case). From this follows (also see (1.3) that, for example, at zero entrance angle in the the case of positrons, the largest fraction of particles is concentrated at the bottom of the well. For electrons in a planar potential, the largest fraction of particles is outside the narrow part of the well, and, consequently, the largest fraction of electrons is concentrated at the levels near the top of the well. Analogous estimations are also possible for the axial case.

Chapter 9

Spin Rotation and Radiative Self-Polarization of Particles Moving in Bent Crystals

9.1 Spin Rotation of Relativistic Particles Passing Through a Crystal

With the growth in energy of particles their spin precession frequency in external fields diminishes, in the ultra-relativistic case being determined by the anomalous magnetic moment [Berestetsky *et al.* (1968)]. As a result, for example, in a magnetic field H of strength 10^4 Gs the electron (proton) spin precession frequency $\omega = 2\mu' H/\hbar$ (μ' is the anomalous part of the magnetic moment) is 10^8 s^{-1}, and the spin rotation angle over one centimeter path length l is just $\vartheta = \omega \frac{l}{c} \approx 10^{-2}$ rad. It turns out, however, that at particle channeling in a crystal there appears precession leading to the spin rotation angle of the order of hundreds of radians over one centimeter path length [Baryshevsky (1979c,d)].

If a crystal is nonmagnetic, then the equation for the spin polarization vector $\vec{\zeta}$ may be written in the form (see, for example, [Berestetsky *et al.* (1968)], §41)

$$\frac{d\vec{\zeta}}{dt} = \frac{2\mu'}{\hbar}[\vec{E}(\vec{\zeta}\vec{n}) - \vec{n}(\vec{\zeta}\vec{E})]\,, \tag{9.1}$$

where $\vec{E}$ is the electric field at the point of particle location; $\vec{n} = \vec{v}/c$; $\vec{v}$ is the particle velocity.

Intracrystalline fields $\vec{E}$ are large, reaching the values of 10^7 CGSE and even greater. Therefore from (9.1) follows that for constant intracrystalline fields, the spin precession frequency could reach $10^{11}\,$s^{-1} and the angle ϑ could be of the order of $10\,$rad/cm.

However, when a particle moves through a crystal in arbitrary direction, the field $\vec{E}$, likewise in an amorphous medium, takes on random values at the particle location point. As a consequence, such a field causes spin depolarization.

Under channeling conditions the situation is basically different. If the crystal bending radius is ρ_0, then the beam of protons with the energy of $1 - 10^2$ GeV will change its direction following the crystal bend up to the radii of curvature of $\rho_0 \sim 1$ cm [Tsyganov (1976a,b); Kaplin and Vorobiev (1978); Baryshevsky *et al.*Baryshevsky, Dubovskaya and Feranchuk (1978)], i.e., the particle will move along a curved path. The stated motion is due to a constant mean electric field acting on a particle in a bent crystal [Tsyganov (1976a,b)]. The magnitude of the field reaches 10^9 SGSE.

Equation (9.1) for a particle moving in a crystal, for example, in a planar channel, bent to a radius of curvature ρ_0 around the y-axis, has a form ($v_y = 0$, $E_y = 0$, the trajectory lies in the x, z plane)

$$\frac{d\zeta_{x(z)}}{dt} = \pm\frac{2\mu'}{\hbar}(E_x n_z - E_z n_x)\zeta_{x(z)}\,. \tag{9.2}$$

The position vector $\vec{\rho} = (x, z)$ of a particle in such a channel rotates about the y-axis with the frequency $\Omega = c/\rho_0$. Its magnitude oscillates about the particle equilibrium position ρ_0' in the channel with the frequency Ω_k, amplitude α, and initial phase δ. In the explicit form $x = \rho(t)\cos\Omega t$, $z = \rho(t)\sin\Omega t$, $\rho(t) = \rho_0' + \alpha\cos(\Omega_k t + \delta)$. We point out that, due to the presence of centrifugal forces in a bent crystal, the equilibrium point ρ_0' does not coincide with the position ρ_0 of the minimum of the electrostatic potential $\varphi(\rho)$ of the channel, as it occurs in a straight channel. For example, when moving in a harmonic well

$$\varphi = -\frac{k}{e}\frac{(\rho-\rho_0)^2}{2}\,, \quad \rho_0' - \rho_0 = -\frac{2E}{k\rho_0}\,,$$

E is the particle energy.

Integration of (9.2) in the polar coordinate system gives ($|\vec{\zeta}| = 1$, $\vec{E} = -\vec{\nabla}\varphi$)

$$\zeta_{z(x)} = \frac{\cos}{\sin}\left\{\frac{2\mu'\Omega}{\hbar c}\int_0^t \rho\frac{d\varphi}{d\rho}dt' + \arctan\frac{\zeta_x(0)}{\zeta_z(0)}\right\}\,. \tag{9.3}$$

For a harmonic well, (9.3) accurate up to the terms of the order $(\rho_0' - \rho_0)/\rho_0$ and $\alpha\rho_0^{-1} \ll 1$ can be written in the form

$$\zeta_{z(x)}(t) = \frac{\cos}{\sin}\left\{\omega t + \beta[\sin(\Omega_k t + \delta) - \sin\delta] + \arctan\frac{\zeta_x(0)}{\zeta_z(0)}\right\}\,, \tag{9.4}$$

where

$$\omega = \frac{2\mu'}{\hbar}E(\rho_0')$$

and

$$E(\rho_0') = -\frac{k}{e}(\rho_0' - \rho_0)$$

is the electric field at the location point of the particle center of equilibrium in a bent crystal;

$$\beta = -\frac{2\mu' ka}{\hbar e \Omega_k}\,.$$

The coefficient β in (9.4) is small (for Si the coefficient $k = 4 \cdot 10^{17}\,\mathrm{eV/\,cm^2}$, $\Omega_k \simeq 10^{13}\,\mathrm{s^{-1}}$ for protons with $E \sim 100\,\mathrm{GeV}$, as a result, $\beta =\simeq 10^{-2}$). Neglecting the term containing β, we obtain that the spin rotates with frequency ω (with growing energy $\Omega_k \sim 1/\sqrt{E}$, the coefficient $\beta \sim \sqrt{E}$ increases, and the spin rotation turns into oscillations at frequency ω and the frequencies multiple of Ω_k). Due to a large magnitude of the field $E(\rho_0')$ curving the particle trajectory ($E(\rho_0') \sim 10^7 - 10^9$ CGSE), the frequency $\omega \simeq 10^{11} - 10^{13}\,\mathrm{s^{-1}}$ and the rotation angle $\vartheta \sim 10 - 10^3\,\mathrm{rad/\,cm}$.

If the radius of curvature $\rho_0 \to \infty$ (a straight channel), then only spin oscillations due to the term containing β remain. In this case a significant spin rotation occurs only at high energies (at low energies it is absent).

9.2 Spin Rotation at Deflection of a Charged Relativistic Particle in the Electric Field

For relativistic particles moving in an arbitrary electric field, there is a simple relation between the spin precession angle and the change in the direction of particle momentum [Lyubosihtz (1980a)]. In the case of planar channeling, this relationship enables one to determine the spin rotation angle in the effect considered in (9.1) without turning to particular models describing the distribution of the intracrystalline field. Below when considering this problem, we shall follow the line of reasoning given by Lyuboshitz in [Lyubosihtz (1980a)].

We shall proceed from the Bargmann-Michel-Telegdi equation [Berestetsky *et al.* (1968)] describing the spin behavior of a relativistic particle moving quasiclassically in an external electric field. Let m be the particle mass; e its charge, ζ the spin polarization vector referred to an "instantaneous" rest system; γ the Lorentz factor; $\vec{l}$ the unit vector in the velocity direction; g the gyromagnetic ratio (by definition, the magnetic moment $\mu = \frac{eg}{2mc}\hbar s$, where s is the particle spin). According to [Berestetsky *et al.* (1968)],

$$\frac{d\vec{\zeta}}{dt} = [\vec{\Omega}\vec{\zeta}],$$

where t is the time in the lab reference frame,

$$\vec{\Omega} = -\frac{e}{2mc} g \left\{ \vec{H} - \frac{\gamma - 1}{\gamma} \vec{l}(\vec{H}\vec{l}) + \left[\vec{E}\frac{\vec{v}}{c} \right] \right\} - (\gamma - 1) \left[\vec{l}\frac{d\vec{l}}{dt} \right] , \tag{9.5}$$

$\vec{E}$ and $\vec{H}$ are the strengths of the electric and magnetic fields at the particle location point. The first term in (9.5) for the angular velocity of precession $\vec{\Omega}$ may be written as

$$-\frac{e}{2mc\gamma} g \vec{H}^* ,$$

where $\vec{H}^*$ is the magnetic field strength in the intrinsic frame of reference; the term

$$-(\gamma - 1) \left[\vec{l}\frac{d\vec{l}}{dt} \right] ,$$

corresponds to the Thomas spin precession [Möller (1972)].

From the equation of motion

$$\frac{d\vec{p}}{dt} = e\vec{E} + \frac{1}{c}[\vec{v}\vec{H}] \tag{9.6}$$

follows that the instantaneous angular velocity of rotation of a particle momentum is defined by formula

$$\vec{\Omega}_0 = \left[\vec{l}\frac{dl}{dt} \right] = -\frac{e}{mc\gamma} \left\{ \vec{H} - \vec{l}(\vec{H}\vec{l}) + \frac{\gamma^2}{\gamma^2 - 1} \left[\vec{E}\frac{\vec{v}}{c} \right] \right\} . \tag{9.7}$$

Comparison of (9.5) and (9.7) shows that in the absence of a magnetic field vectors $\vec{\Omega}$ and $\vec{\Omega}_0$ are parallel (or antiparallel) to one another and are related as

$$\vec{\Omega} = \left[(g - 2)\frac{\gamma^2 - 1}{2\gamma} + \frac{\gamma - 1}{\gamma} \right] \vec{\Omega}_0 \tag{9.8}$$

or

$$\vec{\Omega} = \left[\frac{1}{2}(g - 2)\gamma + \frac{\gamma}{\gamma + 1} \right] \frac{v^2}{c^2} \vec{\Omega}_0 . \tag{9.9}$$

It is clear that if the trajectory of a charged particle in the electric field is a plane curve, vectors $\vec{\Omega}(t)$ and $\vec{\Omega}_0(t)$ have constant direction along the normal $\vec{n}$ to the plane of motion ($\vec{\Omega}_0(t) = \Omega_0(t)\vec{n}$, $\vec{\Omega}(t) = \Omega(t)\vec{n}$). In this case the angle of the polarization vector precession around the normal $\vec{n}$ is

$$\theta(t) = \int_0^t \left[(g - 2)\frac{\gamma^2(t') - 1}{2\gamma(t')} + \frac{\gamma(t') - 1}{\gamma(t')} \right] \frac{d\theta_0(t')}{dt'} dt' , \tag{9.10}$$

where $\theta_0(t) = \int_0^t \Omega_0(t')dt'$ is the angle between the particle initial momentum and its momentum at time t.

If the kinetic energy of a particle moving along the trajectory practically does not change, the relation between the angles of spin and momentum rotation is defined by formula

$$\theta = \left[(g-2)\frac{\gamma^2-1}{2\gamma} + \frac{\gamma-1}{\gamma}\right]\theta_0 \,. \tag{9.11}$$

In the nonrelativistic case

$$\theta = \frac{1}{2}(g-1)\frac{v^2}{c^2}\theta_0 \,. \tag{9.12}$$

Note that for sufficiently small sections of the trajectory, the relation (9.11) also holds true even when the direction of vectors $\vec{\Omega}_0$ and $\vec{\Omega}$ changes with time. In this case the axis of spin rotation through the angle θ is perpendicular to the plane containing the initial and final momenta of the particle. [1]

It is essential that allowing for radiative damping practically does not change the relations derived. Indeed, radiative deceleration comes to the appearance of an additional electric field in the intrinsic reference frame of a charged particle. This field is unlikely to affect the magnetic moment, so formula (9.5) for the angular velocity of precession does not change. On the other hand, the retardation force in the lab reference frame, which is to

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[1]For a spin wave function, the equation of precession in the electric field has a form

$$i\frac{\partial\Psi(t)}{\partial t} = (\vec{\Omega}(t)\hat{s})\Psi(t)\,,$$

where $\vec{\Omega}(t)$ is defined according to (9.8–(9.9), $\hat{s}$ is the spin operator. In non-planar motion, the operators $\vec{\Omega}(t)\hat{s}$ taken at different instants of time do not commute with one another, and the symbolic representation of the solution is as follows

$$\Psi(t) = \hat{T}\exp\left(-i\int_0^t \hat{s}\vec{\Omega}(t')dt'\right)\Psi(0)\,,$$

where $\hat{T}$ is the chronological operator [Kagan and Kononets (1973)]. In the first approximation of the perturbation theory

$$\Psi(t) = \left(1 - i\hat{s}\int_0^t \vec{\Omega}(t')dt'\right)\Psi(0)\,.$$

For the polarization vector this corresponds to the equality

$$\vec{\xi}(t) = \vec{\xi}(0) + \left[\int_0^t \vec{\Omega}(t')dt'\vec{\xi}(0)\right]\,.$$

.

be introduced into the right-hand side of equation (9.6) at $\vec{H} = 0$ has the form [Landau and Lifshitz (1967)]

$$\vec{f} = \frac{2e^4}{3m^2c^5}\gamma^2 v\vec{l}\left\{\vec{E}^2 - \frac{v^2}{c^2}(\vec{E}\vec{l})^2\right\}$$

(here the terms negligibly small in comparison with the Lorentz force are discarded). As the retardation force $\vec{f}$ is directed opposite to the velocity, it makes zero contribution to angular velocity

$$\vec{\Omega}_0 = \left[\vec{l}\frac{d\vec{l}}{dt}\right] .$$

Hence, vectors $\vec{\Omega}$ and $\vec{\Omega}_0$ still satisfy relations (9.8)-(9.9).

In the presence of an external magnetic field the parallelism of vectors $\vec{\Omega}$ and $\vec{\Omega}_0$ is, generally speaking, violated, except for the case of motion in a transverse magnetic field at $\vec{E} = 0$ when

$$\vec{\Omega} = \left(\frac{g-2}{2}\gamma + 1\right)\vec{\Omega}_0.$$

It is easy to see that in the ultra-relativistic limit ($\gamma \gg 1$) at arbitrary fields $\vec{E}$ and $\vec{H}$, the following approximate equality holds accurate up to the terms $eH/mc\gamma^2$

$$\vec{\Omega} = \vec{\Omega}_0\left(\frac{g-2}{2}\gamma + 1\right) + \frac{ge}{2mc\gamma}\vec{l}(\vec{H}\vec{l})\,. \tag{9.13}$$

Consider some particular applications of formulae (9.8)-(9.11).

Motion in a homogeneous electric field. Let at $t = 0$ a particle be at the origin or coordinates, the initial momentum $p = mv\gamma$ be directed along the y-axis, and the electric field strength - along the x-axis. Then the calculation from formula (9.10) gives the following expression for the angle of spin rotation about the z-axis:

$$\theta = \frac{eEy(t)}{2mc^2}(g-2) + \arccos\frac{\gamma + \cosh\frac{eEy(t)}{pc}}{\gamma\cosh\frac{eEy(t)}{pc} + 1}, \tag{9.14}$$

where

$$y(t) = \frac{pc}{eE} arcsinh\frac{eE(t)}{mc\gamma}$$

The deflection angle of the particle in the electric field is

$$\theta_0 = \arctan\frac{eEt}{p}. \tag{9.15}$$

If the particle kinetic energy varies insignificantly, then $\theta_0 \simeq \frac{eEt}{p} \simeq \frac{eEy}{pv} \ll 1$, and formula (9.14) goes over to (9.11).

Planar channeling in bent crystals. Curving the trajectory of a charged particle moving along the bent channel is due to the existence of the perpendicular to the momentum mean electric field, whose magnitude can reach $10^7 - 10^8$ SGSE. In [Baryshevsky (1979c,d)] is shown that, due to this fact, when ultra-relativistic particles are channeled in bent crystals the rotation angle of the polarization vector takes on large values (see (9.1)). It is interesting that this angle may be found from formulae (9.11) or (9.10), without turning to particular models describing the distribution of the intracrystalline field. Indeed, suppose that the momentum of a channeled particle is parallel to the bending plane. Then the particle deflection angle θ_0 coincides with the crystal bending angle, and the spin rotation axis is perpendicular to the plane of bending. For a proton the radiation energy losses are vanishingly small, and relation (9.11) holds true. At $\gamma \gg 1$, find $\left(\frac{g-2}{g} = 1.79\right)$:

$$\theta = \left(1 + \frac{g-2}{2}\gamma\right)\theta_0 = (1 + 1.91\varepsilon)\theta_0, \qquad (9.16)$$

where ε is the proton energy, GeV. According to (9.16) the proton spin rotates in the same direction as the momentum does. At $\varepsilon = 10$ GeV the spin precession angle is 20 times as large as the momentum deflection angle.

Note that at the given radius of curvature R the maximum energy of particles, which are also "captured" into the channeling regime in a bent crystal, is $eE_{max}R$, where E_{max} is the maximum strength of the electric field. As $\theta_0 = y/R$, where y is the length of the trajectory, the spin rotation angle of the proton, corresponding to the maximum energy is only determined by the values of E_{max} and y:

$$\theta_{max} = \frac{g-2}{2}\frac{eE_{max}y}{mc^2} \simeq 5.79 \cdot 10^{-7} E_{max} y.$$

If $E_{max} = 10^7$ SGSE, $y = 1$ cm, $R = 100$ cm, then $\varepsilon \sim 300$ GeV and $\theta_{max} \sim 6$ rad. This value agrees with the estimates given in [Baryshevsky (1979c,d)] and (9.1).

In the case of channeling of positrons, radiation losses at achievable energies can be significant, and searching for the spin rotation angle one should use relation (7.6), which takes account of the change in the kinetic energy in motion. The corresponding design equation takes the form $\left(\frac{g-2}{2} \simeq 1.16 \cdot 10^{-3}\right)$

$$\theta = (1 + 2.27\bar{\varepsilon})\theta_0, \qquad (9.17)$$

where $\bar{\varepsilon} = \frac{c}{l}\int_0^{l/c}\varepsilon(t)dt$ is the mean value of energy, GeV.

Scattering by the electrostatic (Coulomb) potential. At quasi-classical scattering of a charged particle at the angle θ_0 in a static field of the system of charges, rotation of the polarization vector is defined by formula (9.10). Integration in (9.10) is made along the unclosed trajectory, uniquely determined by the scattering angle and plane. The rotation axis of the polarization vector is probably perpendicular to the scattering plane. When speaking about scattering at small angles, within the region of particle motion the potential energy is small in comparison with the kinetic one, and thus, the connection between the spin rotation angle and the scattering angle is specified by relation (9.11).

For the Coulomb scattering the latter statement also holds true beyond pure classical description of a particle motion in the electric field, which have been used until now. In this case the main contribution to the amplitude of scattering at the angles $\theta_0 \ll 1$ comes from the region of high impact parameters $\rho \gg \hbar/p$, where the particle potential energy is much smaller than its kinetic energy. Therefore we shall apply the eikonal approach, enabling representation of the amplitude of scattering at small angles as follows [Landau and Lifshitz (1977)]

$$a(\theta_0) = -\frac{ik}{2\pi}\int_0^\infty \rho d\rho \left\{ (e^{iS(\rho)\hbar} - 1)\int_0^{2\pi} e^{-ik\theta_0\rho\cos\psi}d\psi \right\} \tag{9.18}$$

Here $k = p/\hbar$; $S(\rho) = \frac{1}{v}\int_{+\infty}^{-\infty} u(\rho_1 z)dz$ is the difference of the classical action integrals for a straight trajectory with the impact parameter ρ with and without interaction; ψ is the angle of vector ρ, perpendicular to the particle momentum with the scattering plane. Formula (9.18) is valid for both non-relativistic and relativistic energies. To take into account spin precession in the electric field, let us multiply the function $\exp(iS(\rho)/\hbar)$ in (9.18) by the rotation matrix

$$\hat{R}(\theta(\rho)) = \exp\left(-i\hat{s}\frac{[\vec{k}\vec{\rho}]}{k\rho}\theta\rho\right), \tag{9.19}$$

where $\theta(\rho)$ is the spin rotation angle corresponding to the motion of the charged particle along the classical (close to straight) trajectory with the impact parameter ρ; $\hat{s}$ is the spin operator. It has been shown above that the angle $\theta(\rho)$ is connected with the angle of the momentum deflection $\theta_0(\rho)$ for the same trajectory by relation (9.11). On the other hand, the angle $\theta_0(\rho)$ is determined in terms of the action function:

$$\theta_0(\rho) = \frac{1}{\hbar k}\frac{d}{d\rho}S(\rho). \tag{9.20}$$

Thus, provided that the angles $\theta(\rho)$ and $\theta_0(\rho)$ are small in (9.18) $\exp(iS(\rho)/\hbar)$ should be replaced by

$$\exp(iS(\rho)/\hbar)\hat{R}(\theta(\rho)) \simeq \exp(iS(\rho)/\hbar)\left[1 - ib\frac{dS(\rho)}{d\rho}\right.$$
$$\left.\times\frac{\cos\psi}{\hbar k}\hat{s}_z + ib\frac{dS(\rho)}{d\rho}\frac{\sin\psi}{\hbar k}\hat{s}_y\right], \qquad (9.21)$$

where

$$b = \left(\frac{g-2}{2}\frac{\gamma^2-1}{\gamma} + \frac{\gamma-1}{\gamma}\right) \qquad (9.22)$$

(it is assumed that the z-axis is directed parallel to the normal to the scattering plane, the x-axis - along the initial momentum of a particle). Upon integration with respect to the angle ψ, the formula for the scattering amplitude takes the form

$$\hat{A}(\theta_0) = a(\theta_0) + b\hat{s}_z\int_0^\infty J_1(k\theta_0\rho)\left[\frac{d}{d\rho}\exp(iS(\rho)/\hbar)\right]\rho d\rho;$$
$$a(\theta_0) = -ik\int_0^\infty J_0(k\theta_0\rho)\left[\exp(iS(\rho)/\hbar) - 1\right]\rho d\rho. \qquad (9.23)$$

Using well known relations for the Bessel function

$$\frac{d}{d\rho}(\rho J_1(k\theta_0\rho)) = k\theta_0\rho J_0(k\theta_0\rho); \int_0^\infty J_0(k\theta_0\rho)\rho d\rho = \frac{2\delta(\theta_0^2)}{k^2},$$

we obtain with the accuracy up to the terms of the order of θ_0^2

$$\hat{A}(\theta_0) = a(\theta_0)(1 - i\hat{s}_z b\theta_0). \qquad (9.24)$$

From this the angle of spin rotation about the z-axis is $b\theta_0$.

It may be argued that this result within the range of angles $\theta_0 \ll 1$, $\theta = b\theta_0 \ll 1$ is not bound by any additional conditions. Within the quasi-classical limit the requirement of smallness is only imposed on the scattering angle θ_0, while the spin rotation angle at ultra-relativistic energies may take on any values.

Such consideration has nothing to do with the use of relativistic equations, being applicable to particles with arbitrary spin and gyromagnetic ratio. In the case of scattering of electrons with not very high energies ($\gamma \simeq 10^3$) in a Coulomb field of a nucleus of charge ze, the anomalous magnetic moment of the electron may be neglected, which according to (9.23) and (9.24) gives

$$A(\theta_0) \simeq \frac{2ze^2}{pv\theta_0^2}\frac{\Gamma\left(1 - i\frac{ze^2}{\hbar v}\right)}{\Gamma\left(1 + i\frac{ze^2}{\hbar v}\right)}\exp\left(2i\frac{ze^2}{\hbar v}\ln\frac{\theta_0}{2}\right)$$
$$\times\left[1 - i\hat{\sigma}_z\frac{\gamma-1}{2\gamma}\theta_0 + \theta(\theta_0^2)\right], \qquad (9.25)$$

where $\hat{\sigma}_z$ is the Pauli matrix. And the spin rotation angle is

$$\theta = \frac{\gamma - 1}{\gamma}\theta_0. \tag{9.26}$$

Relations (9.25) and (9.26) may be obtained independently on the basis of the solution of the Dirac equation in the limit (in extreme case) $\theta_0 \ll 1$ (see[Gluckstern and Lin (1964)]). At non-relativistic energies $\theta = \frac{v^2}{2c^2}\theta_0$, and at ultra-relativistic energies $\theta = \theta_0$, which corresponds to helicity conservation.

In conclusion we shall point out an interesting consequence of relation (9.8): if the gyromagnetic ratio satisfies the condition

$$1 < g < 2, \tag{9.27}$$

then at the energy $\varepsilon_0 = \frac{g}{g-2}mc^2$, the electric field does not influence particle spin at all (the angular velocity of precession vanishes, though the magnetic moment is nonzero). This is a purely relativistic effect caused by cancellation between the "dynamic" and Thomas precessions. At $\varepsilon < \varepsilon_0$, spin rotates in the same direction as the momentum does, at $\varepsilon > \varepsilon_0$, it rotates in the opposite direction. For example, a deuteron with $g = 1.72$, as well as some nuclei (e. g. ^6Li with $g = 1.64$), satisfies the condition (9.27). [2] For a deuteron $\varepsilon_0 = 11.5$ GeV. Analogous phenomenon also occurs in a transverse magnetic field, providing that $0 < g < 2$. The energy at which the polarization vector preserves constant direction in this case equals

$$\tilde{\varepsilon}_0 = \frac{2}{2-g}mc^2$$

(for a deuteron, for example, $\tilde{\varepsilon}_0 = 13.4\,\text{GeV}$).

9.3 Depolarization of Fast Particles Moving in Matter

As it has been shown (see (9.2), at small deflection of charged particles from the initial direction in the magnetic field, the spin polarization vector $\vec{\zeta}$ rotates around the normal to the plane passing through the initial and final momenta $\vec{p}_0$ and $\vec{p}_1$ through the angle

$$\theta = \left[(g-2)\frac{\gamma^2 - 1}{2\gamma} + \frac{\gamma - 1}{\gamma}\right]\theta_0. \tag{9.28}$$

Here θ_0 is the angle of momentum $\vec{p}_0$ with momentum $\vec{p}_1$; γ is the Lorentz factor; g is the gyromagnetic ratio (by definition the magnetic moment

[2] For nuclei the quantity g is associated with the so-called nuclear gyromagnetic ratio by formula $g = g_{\text{nuc}}A/z$ (A is the number of nucleons in a nucleus, z is the atomic number).

$\mu = \frac{e\hbar}{2mc} gs$, where e is the charge, m is the mass, s is the spin of the particle). At small change in the kinetic energy, providing that $\theta \ll 1$, $\theta_0 \ll 1$, formula (9.28) holds true irrespective of the character of the intermediate motion of the particle in question.[3] It is easy to see that when $\theta \ll 1$, the angle of deflection of the polarization vector from the initial direction $\vec{\zeta}_0$ is

$$\theta = \theta \sin \psi, \tag{9.29}$$

where ψ is the angle of $\vec{\zeta}_0$ with vector $[\vec{p}_0 \vec{p}_1]$. Relations (9.28), (9.29) allow calculating the degree of depolarization of the charged fast particle moving in a macroscopic medium [Lyubosihtz (1980b)]. Below we shall follow the same line of reasoning as given in [Lyubosihtz (1980b)].

Consider the case of longitudinal polarization ($\psi = \pi/2$). It is clear that at multiple scattering of a particle in the Coulomb field of nuclei and electrons the mean values of the transverse components of the momentum and polarization vector are zero. Thus, vector $\langle \vec{\zeta}_{\|} \rangle$ preserves its direction. Despite the fact that $\langle \vec{\zeta}_{\perp} \rangle = 0$, the quantity $\langle \tilde{\theta}^2 \rangle = \langle \theta^2 \rangle = \langle \vec{\zeta}_{\perp}^2 \rangle / \zeta_{\|}^2$ is nonzero. As a result, the particle undergoes depolarization (the value of $|\vec{\zeta}_{\|}|$ decreases).

According to (9.28) the mean-square angle of deflection of the polarization vector $\vec{\zeta}_{\|}$ from the initial direction when the particle is passing through a thin layer of matter Δl is related to the mean-square angle of the multiple Coulomb scattering in this layer as

$$\langle \tilde{\theta}^2 \rangle = \langle \theta^2 \rangle = \left[\frac{g-2}{2} \frac{\gamma^2 - 1}{\gamma} + \frac{\gamma - 1}{\gamma} \right]^2 \langle \theta_0^2 \rangle \tag{9.30}$$

It is known that $\langle \theta_0^2 \rangle$ is described with good accuracy by the expression [Rossi and Greisen (1948); Bricman *et al.* (1978)]

$$\langle \theta_0^2 \rangle = z^2 \left(\frac{E_s}{m} \right)^2 \frac{\gamma^2}{(\gamma^2 - 1)^2} \frac{\Delta l}{L_{rad}}, \tag{9.31}$$

where $z = e/e_0$ is the ratio of the particle charge to the electron charge; m is the particle mass; $E_s = 21$ MeV; L_{rad} is the radiation length for an electron. Substituting (9.31) into (9.30) and taking into account that at small $\langle \tilde{\theta}^2 \rangle$ the degree of polarization is

$$\eta = 1 - \langle \cos \tilde{\theta} \rangle \simeq \frac{1}{2} \langle \tilde{\theta}^2 \rangle, \tag{9.32}$$

[3] In the particular case of quasi-classical motion along the plane trajectory the angles θ and θ_0 in (9.28) may take on any value (see [Lyubosihtz (1980a)]).

we come to the formula describing depolarization of longitudinally polarized particles:

$$\eta_{\parallel} = \frac{1}{2} z^2 \left(\frac{E_s}{m}\right)^2 \left[\frac{g-2}{2} + \frac{1}{\gamma+1}\right]^2 \frac{\Delta l}{L_{rad}}, \tag{9.33}$$

If a particle is polarized in the direction perpendicular to the momentum, then at fixed angle ψ of the polarization vector $\vec{\zeta}_{\perp}$ with the normal to the scattering plane $\vec{\eta}$, in view of (9.29), $\eta_{\perp} = \eta_{\parallel} \sin^2 \psi$. At averaging over the azimuth angle, a factor 1/2 appears.

Thus, when polarized particles pass through the layer of matter, their depolarization in the transverse direction is half as much as depolarization in the longitudinal direction $\eta_{\perp} = \frac{1}{2}\eta_{\parallel}$. This leads to the fact that in the general case the initial angle Φ of the polarization vector with the momentum increases by

$$\Delta\Phi = \frac{1}{4}\eta_{\parallel} \sin 2\Phi \tag{9.34}$$

($\eta_{\parallel}$ is determined from formula (9.33)). $\Delta\Phi$ reaches its maximum value at $\Phi = \pi/4$ and vanishes at $\Phi = 0$ and $\Phi = \pi/2$. And the degree of depolarization

$$\eta = \left(1 - \frac{1}{2}\sin^2\Phi\right)\eta_{\parallel} = (2 - \sin^2\Phi)\eta_{\perp}. \tag{9.35}$$

According to (9.33) and (9.35), at non-relativistic energies

$$\eta = \frac{1}{8} z^2 \left(1 - \frac{1}{2}\sin^2\Phi\right) \left(\frac{E_s}{m}\right)^2 (g-1)^2 \frac{\Delta l}{L_{rad}}, \tag{9.36}$$

whereas at ultra-relativistic energies

$$\eta = \frac{1}{8} z^2 \left(1 - \frac{1}{2}\sin^2\Phi\right) \left(\frac{E_s}{m}\right)^2 (g-2)^2 \frac{\Delta l}{L_{rad}}, \tag{9.37}$$

We point out that the basic formula (9.33) holds for a layer of fixed thickness, passing through which a particle loses a small fraction of its energy, and the condition $\eta \ll 1$ should also be satisfied. With the latter condition preserved, it is easy to take into account the energy losses by substituting expression (9.33) into the integral

$$\eta_{\parallel} = \frac{1}{2} z^2 \left(\frac{E_s}{m}\right)^2 \int_0^{\Delta l} \left[\frac{g-2}{2} + \frac{1}{\gamma(x)+1}\right]^2 \frac{dx}{L_{rad}}, \tag{9.38}$$

where $\gamma(x)$ is the Lorentz factor of the particle at the distance x from the front boundary of matter. Here relations (9.34) and (9.35) remain valid, as well as expression (9.36) for non-relativistic energies.

From (9.38) follows that the degree of depolarization of the longitudinally polarized protons or antiprotons ($|z| = 1$, $\frac{g-2}{2} = 1.79$, $m = 938$ MeV) is described by expression

$$\eta_{p\|} = 0.8 \cdot 10^{-3} \int_0^{\Delta l} \left(1 + \frac{0.28}{1 + 0.53T(x)}\right)^2 \frac{dx}{L_{rad}}, \qquad (9.39)$$

where $T(x) = E(x) - m_p c^2$ is the kinetic energy, GeV. It is easy to see that on the nuclear collision length in lead $\left(\frac{\Delta l}{L_{rad}} \sim 20\right)$ protons with the energy $T \geq 1$ GeV proton get depolarized by $1.5 - 2\%$.

In a similar manner one can estimate the degree of depolarization of a passing beam of neutral particles with the magnetic moment $\mu = \frac{e_0 \hbar}{2m_p c} g_a s_a$ (for a neutron $g_n = -3.82$, and for a Λ-particle $g_\Lambda = -1.2$). Indeed, in the first approximation the neutral particle moves in the same electric field as the charged particle deflected through small angles. In the given electric field the spin rotation angles of the particle in question are related to those of the proton as (see [Lyubosihtz (1980a)]).

$$b(\gamma) = g_a / \left(g_p - \frac{2\gamma}{\gamma + 1}\right). \qquad (9.40)$$

In view of (9.40), (9.35) and (9.38) the degree of depolarization of arbitrary polarized neutral particles is energy-independent and described by the expression

$$\begin{aligned} \eta_a &= \left(1 - \frac{1}{2}\sin^2\Phi\right) \int_0^{\Delta l} b^2(\gamma) \frac{\partial \eta_{p\|}}{\partial z} dx \\ &= \frac{1}{8}\left(1 - \frac{1}{2}\sin^2\Phi\right)\left(\frac{E_s}{m_p}\right)^2 g_a^2 \frac{\Delta l}{L_{rad}}. \qquad (9.41) \end{aligned}$$

This result may also be obtained in a different way, considering the change in polarization at Schwinger scattering of a neutral particle with a nonzero magnetic moment in the Coulomb nuclear field. In the unit event of Schwinger scattering the polarization vector of scattered particles $\vec{\zeta} = -\vec{\zeta}_0 + 2\vec{n}(\vec{\zeta}_0 \vec{n})$, where $\vec{n}$ is the unit vector along the normal to the scattering plane [Berestetsky *et al.* (1968)]. Upon averaging over the azimuth angle we have $\vec{\zeta}_\| = -\vec{\zeta}_{0\|}$, $\vec{\zeta}_\perp = 0$. From this follows that in the layer of thickness Δl

$$\eta_{a\|} = 2\eta_{a\perp} = 2N\left(\int \sigma_{Sch}(\theta_0) d\Omega\right)\Delta l.$$

Here N is the number of nuclei per unit volume;

$$\sigma_{Sch}(\theta_0) = \frac{1}{4}\left(\frac{ze^2}{m_p c^2 \theta_0}\right)^2 g_a^2 F^2(m_a\sqrt{\gamma^2 - 1}\theta_0);$$

F is the form factor including electron screening of the nuclear field and the influence of the finite size of a nucleus. Integration with respect to the solid angle gives (compare with similar calculations in [Rossi and Greisen (1948)])

$$2N \int \sigma_{Sch}(\theta_0) d\Omega \Delta l \simeq \frac{1}{8} \left(\frac{E_s}{m} \right)^2 g^2 \frac{\Delta l}{L_{rad}}.$$

From (9.41) follows, in particular, that the degree of depolarization of longitudinally polarized neutrons on the radiation length $\eta_{n\|} = 9 \cdot 10^{-4}$; for Λ-hyperons $\eta_{\Lambda\|} = 0.9 \cdot 10^{-4} \Delta l / L_{rad}$.

In view of the smallness of factor $g - 2$ the energy-dependence of depolarization of μ-mesons and electrons is more appreciable than that of the protons. For longitudinally polarized μ-mesons ($m_\mu = 105.6$ MeV, $(g-2)/2 = 1.16 \cdot 10^{-3}$), formula (9.38) has the form

$$\eta_{\mu\|} = 5 \cdot 10^{-3} \int_0^{\Delta l} \left(2.32 \cdot 10^{-3} + \frac{1}{1 + 4.7 T(x)} \right)^2 \frac{dx}{L_{rad}}, \tag{9.42}$$

where $T(x)$ is the kinetic energy of the μ-meson, GeV. The degree of depolarization of μ-mesons passing through the layer of lead can reach 10%, while for media with a small atomic numbers it is as low as a fraction of a percent. As for electrons, the approach developed here is only applicable provided that $\Delta l \ll L_{rad}$, $T \gg 15\sqrt{\frac{\Delta l}{L_{rad}}}$ MeV. And

$$\eta_{e\|} = 220 \left(2.32 \cdot 10^{-3} + \frac{1}{T} \right)^2 \frac{\Delta l}{L_{rad}}, \tag{9.43}$$

At the energies $T < 10\sqrt{\frac{\Delta l}{L_{rad}}}$ electrons become completely depolarized.

9.4 Oscillations of Polarization of a Fast Channeled Particle Caused by its Quadrupole Moment

In (9.1) we considered the effect of spin rotation of a relativistic particle channeled in a non-magnetic bent crystal, which is caused by the action of the crystal electric field (also responsible for the rotation of a particle momentum) on tits dipole magnetic moment. It turns out that for particles (nuclei, ions) with spin $I \geq 1$ the presence of multipole moments, first of all, the quadrupole one, results in spin rotation even at motion in a straight channel [Baryshevsky and Sokolsky (1980)].

First consider a nonrelativistic particle with the quadrupole moment Q. In view of the quasi-classical character of its motion in the channel, we may write the equation of motion for its moment as follows:

$$\frac{d\hat{I}_i}{dt} = \frac{e}{3\hbar}\varepsilon_{ikl}\varphi_{kn}\hat{Q}_{ln}\,, \tag{9.44}$$

where $\hat{I}_i$ is the operator of the particle spin projection;

$$\hat{Q}_{ln} = \frac{3Q}{2I(2I-1)}\left\{\hat{I}_{ln} - \frac{2}{3}I(I+1)\delta_{ln}\right\}$$

is the operator of its quadrupole moment, $\hat{I}_{ln} = \hat{I}_l\hat{I}_n + \hat{I}_n\hat{I}_l$;

$$\varphi_{ln} = \frac{\partial^2\varphi}{\partial x_l \partial x_n}$$

is the the second-derivative of the electrostatic potential of the channel at the point of particle location; ε_{ikl} is the totally antisymmetric unit tensor. It is essential that due to the Lorentz factor compensation through relativistic transformation of φ_{ik} and t, equations (9.44) are applicable for a relativistic channeled particle as well. The change in polarization over the unit length of the particle flight is energy-independent. In the case of a particle moving near the channel center, it is possible to employ the harmonic approximation for φ. Here the quantities φ_{ik} do not depend on the coordinates, and the solution of (9.44) simplifies considerably. So, for a particle with spin $I = 1$ moving in the direction of the z-axis (x, y are the principal axes of the tensor φ_{ik}), we get

$$\begin{aligned}
\hat{I}_x(t) &= \hat{I}_x(0)\cos\alpha\omega t + \hat{I}_{yz}(0)\sin\alpha\omega t\,,\\
\hat{I}_y(t) &= \hat{I}_y(0)\cos\omega t - \hat{I}_{zx}(0)\sin\omega t\,,\\
\hat{I}_z(t) &= \hat{I}_z(0)\cos(1-\alpha)\omega t + \hat{I}_{xy}(0)\sin(1-\alpha)\omega t\,,
\end{aligned} \tag{9.45}$$

$$\begin{aligned}
&\hat{I}_{xx}(t) = \hat{I}_{xx}(0),\ \hat{I}_{yy}(t) = \hat{I}_{yy}(0),\ \hat{I}_{zz}(t) = \hat{I}_{zz}(0),\\
&\hat{I}_{xy}(t) = \hat{I}_{xy}(0)\cos(1-\alpha)\omega t - \hat{I}_z(0)\sin(1-\alpha)\omega t\,,\\
&\hat{I}_{yz}(t) = \hat{I}_{yz}(0)\cos\alpha\omega t - \hat{I}_x(0)\sin\alpha\omega t\,,\\
&\hat{I}_{zx}(t) = \hat{I}_{zx}(0)\cos\omega t - \hat{I}_y(0)\sin\omega t\,,
\end{aligned} \tag{9.46}$$

where $\omega = \frac{eQ}{2\hbar}\varphi_{xx}$; $\alpha = \frac{\varphi_{yy}}{\varphi_{xx}}$. From relations (9.45), (9.46) follows that when a fully polarized particle with spin directed along the z-axis enters a crystal, the mean values of the following projections of spin and quadrupolarization will change with time

$$\langle\hat{I}_z(t)\rangle = \cos(1-\alpha)\omega t\,, \quad \langle\hat{Q}_{xy}(t)\rangle = -\frac{3}{2}Q\sin(1-\alpha)\omega t\,. \tag{9.47}$$

Similarly, at the initial polarization: (a) in the x-direction

$$\langle \hat{I}_x(t) \rangle = \cos \alpha\omega t\,, \quad \langle \hat{Q}_{yz}(t) \rangle = -\frac{3}{2} Q \sin \alpha\omega t\,, \tag{9.48}$$

(b) in the y-direction

$$\langle \hat{I}_y(t) \rangle = \cos \omega t\,, \quad \langle \hat{Q}_{zx}(t) \rangle = \frac{3}{2} Q \sin \omega t\,. \tag{9.49}$$

Thus, polarization of the particle with $Q \neq 0$ moving in a straight channel undergoes oscillations as the particle advances into the target. Here in the case of the transverse initial polarization of the particle at $\alpha = 1$ linear oscillations occur, and spin rotation only appears at $\alpha \neq 1$, i.e., at the asymmetry of the channel field. Estimate the magnitude of the effect. In axial channeling of a positively charged particle, the values of field inhomogeneity can be as large as of the order of $10^{18}\,\mathrm{V/cm^2}$. In this case, for a bare nucleus ($Q \sim 10^{-24}\,\mathrm{cm^2}$) $\omega \sim 10^9\,\mathrm{s^{-1}}$, i.e., the polarization can change by 1% over the path length of about 1 cm. For an ion passing through a crystal, due to antishielding, the effective field on the nucleus may increase by several orders of magnitude. As a result, the change in the polarization can increase by several orders of magnitude.

A negatively charged elementary particle, for example, an Ω-hyperon in the case of channeling will move inside the atomic layer or in the region of the nuclear tube along the crystallographic axis. Here the electric fields (and their inhomogeneities) are considerably higher than in the interplanar channel, so the appreciable rotation of spin may occur even at quite small values of Q. For example, for a nuclear tube in lead $\varphi_{xx} \sim 10^{20}\,\mathrm{V/cm^2}$ and the value of $\omega \sim 10^9\,\mathrm{s^{-1}}$ is attained at $Q \sim 10^{-26}\,\mathrm{cm^2}$. The measurement of the polarization of Ω^- under such conditions may provide unique information about the hyperon quadrupole moment.

Note also that in a bent channel, for a positively charged particle the magnitude of the spin rotation due to the quadrupole moment, generally speaking, should grow at the cost of trajectory displacement closer to the atomic plane. However, in this case spin rotation due to magnetic moment should be simultaneously taken into account.

9.5 Radiative Self-Polarization of Spin of Fast Particles in Crystals

Let a particle move in a channel bent with the radius of curvature R around the z-axis. The particle motion along the curved path in such a channel means that here the particle is affected by the electric field ε perpendicular to the particle momentum. Therefore the particle in its rest frame is affected by the magnetic filed $H = \gamma\varepsilon$ directed along the z-axis, where γ is the particle Lorentz factor. In the magnetic field spin undergoes radiative transitions between the states with different spin projections on the the field direction. These spontaneous transitions lead to accumulation of particles at a lower energy level, i.e. to the beam polarization along the z-axis, if it has not been polarized when entering the crystal [Baryshevsky (1979c)].

A detailed description of the self-polarization effect can be given, using the equation for spin motion in an external electromagnetic field with due account of radiative damping [Baryshevsky and Grubich (1979a)]. Suppose that the crystal is non-magnetic. In this case the spin polarization vector of a particle satisfies the equation of the form (compare with [Baier *et al.* (1973)], p. 204)

$$\frac{d\vec{\zeta}}{dt} = \frac{e}{m}\left(\frac{\mu'}{\mu_0} + \frac{1}{1+\gamma}\right)[\vec{\zeta}[\vec{v}\vec{E}]] - T^{-1}\left(\vec{\zeta} - \frac{2}{9}\vec{v}(\vec{v}\vec{\zeta}) + \frac{8}{5\sqrt{3}}\frac{[\vec{v}\vec{w}]}{|\vec{w}|}\right), \quad (9.50)$$

where μ' is the anomalous part of the magnetic moment (it depends on the particle energy); μ_0 is the Bohr magneton; $T^{-1} = 5\sqrt{3}\alpha\hbar^2\gamma^5|\vec{w}|^3/8m^2$ is the damping constant; $\alpha = 1/137$; $c = 1$ is the velocity of light; $\vec{v}$ is the particle velocity; m is its mass; $\vec{w}$ is the acceleration. The augend in (9.50) describes the effect of spin rotation in a bent crystal (see (9.1)). The addend leads to the effect of radiative polarization of the beam.

Consider the projection of the polarization vector $\vec{\zeta}$ on the z-axis, around which the crystal is bent. If the particle undergoes planar channeling around the z-axis in xy-plane, then the first term on the right–hand side of (9.50)) has a zero projection onto the z-axis, i.e.,

$$\frac{d\zeta_z}{dt} = -T^{-1}\zeta_z - 8(5\sqrt{3}T)^{-1}[\vec{v}\times\vec{w}]_z|\vec{w}|^{-1}. \quad (9.51)$$

The solution of this equation has the form

$$\begin{aligned}\zeta_z(t) = \zeta_z(0)\exp\left(-\int_0^t T^{-1}(t')dt'\right) - 8(5\sqrt{3})^{-1} \\ \times\exp\left(-\int_0^t T^{-1}(t')dt'\right)\int_0^t dt' T^{-1}(t')\frac{[\vec{v}\times\vec{w}]_z}{|\vec{w}|} \\ \times\exp\left(\int_0^t T^{-1}(t'')dt''\right). \quad (9.52)\end{aligned}$$

Generally speaking, the particle trajectory in the channel is known. For instance, in a bent planar channel $x = \rho(t)\cos\Omega_b t$, $y = \rho(t)\sin\Omega_b t$.

When the potential is harmonic $k(\rho - R)^2/2$, $\rho(t) = \rho_0 + \alpha_1 \cos(\Omega t + \delta)$, where $\rho(t)$ is the radius of the particle orbit; $\Omega_b = c/\rho_0$ is the rotation frequency in a bent crystal; Ω is the oscillation frequency in the channel; α_1 is the oscillation amplitude; δ is the initial phase; ρ_0 is the radius of curvature of the particle equilibrium trajectory in the channel. The value of displacement of the particle equilibrium trajectory from the channel center $\Delta = m\gamma/kR$ is limited by the channel width $\Delta < d/2$.

In the case when $\alpha_1/\Delta \ll 1$, the acceleration equals $1/R$ with high precision. As a result,

$$\zeta_z(t) = \zeta_z(0)e^{-t/T_0} + 8(5\sqrt{3})^{-1}(1 - e^{-t/T_0}), \qquad (9.53)$$

where $T_0^{-1} = (5\sqrt{3}/8)\alpha(\hbar\gamma/m)^2(\gamma/R)^3$. From (9.53) follows that at times $t > T_0$, the value of $\zeta_z = 8(5\sqrt{3})^{-1} \simeq 0.924$ irrespective of the value of the initial polarization (i.e., the beam appears to be polarized along the crystal bending axis z). For example, at channeling of positrons with the energy of 100 GeV and $R \sim 12$ cm, the polarization length T_0 in (110) channel of a single crystal of tungsten is approximately 1 cm. The estimates show that in the case of axial channeling of electrons with the energy of 50 GeV and $R \sim 10$ cm the same polarization length may be attained even in single crystals of relatively light elements (e.g. silicon). The length of self-polarization decreases rapidly with the growth of particle energy. Note also that over the length T_0 the particle emits one photon, i.e., the intensity of this type of radiation is very high.

At $\alpha_1/\Delta > 1$ at the exit from the crystal the magnitude of the projection of the polarization vector $\zeta_z(t)$ as a function of crystal thickness is the sum of a non-oscillating and oscillating with frequency Ω terms. Upon averaging over the initial state of the beam, only non-oscillating part remains, which vanishes with the increase in α_1/Δ. The aforesaid means that even in the limiting (extreme) case of an undeformed crystal ($R \to \infty$, $\Delta \to 0$ for the given particle trajectory there appears a nonzero projection of the polarization vector $\zeta_z(t)$ oscillating with frequency, which vanishes after averaging over all the initial points of particle entrance into the crystal. However, the intensity of electromagnetic radiation accompanying radiation polarization of channeled particles will be high in this case too.

As was mentioned above, the anomalous magnetic moment depends on the particle energy. In the case of channeling of charged particles, the parameter $\chi = e\hbar\varepsilon\gamma/m^2$ (see [Baier *et al.* (1973)]) may be of the order of unity and greater. Therefore the phenomenon of spin precession of charged particles described above opens up possibilities for experimental investigation of the dependence of radiative corrections on particle energy.

It should be emphasized that with χ approaching unity in the spin-flip process, a very hard quantum is emitted. Therefore if in the experiment the electrons are selected by energy as well, then the degree of polarization of the beam will turn out to be higher. In this case the theory based on equation (9.50) is not suitable. The process may be studied, for example, using the density matrix formalism.

Chapter 10

The Influence of Radiative Transitions on Channeling of Charged Particles in Crystals

10.1 Particle Lifetime at the Transverse Motion Level

Radiative transition of a channeled particle from one level to another is accompanied by the change in its energy and momentum. Therefore one should expect that such transitions may affect the character of particle motion in a crystal. In particular, the redistribution of the initial population of transverse motion levels, which will influence the beam divergence at the crystal exit [Baryshevsky *et al.* (1978); Baryshevsky and Dubovskaya (1977a,b)].

Classical theory of the influence of electromagnetic radiation on the motion of channeled particles was first given by Bonch-Osmolovsky and Podgoretsky in [Bonch-Osmolovskii and Podgoretskii (1978, 1979)], quantum theory - by Grubich and the author in [Baryshevsky and Grubich (1978); Baryshevsky *et al.* (1978)]. For example, as shown in [Bonch-Osmolovskii and Podgoretskii (1978, 1979); Baryshevsky and Grubich (1978); Baryshevsky *et al.* (1978)], development of electromagnetic cascade in a crystal is possible in the length considerably smaller than the radiation length. A similar conclusion was later made in [Akhiezer and Shul'ga (1980)].

The possibility in principle to change the angular divergence of a beam under channeling conditions is due to the fact that different quasi-classical transverse momentum corresponds to different levels of transverse motion.

To estimate the rate of the process in question, let us first find the radiation width of excited levels in the model of a rectangular well [Baryshevsky *et al.* (1978); Baryshevsky and Dubovskaya (1977a,b)]. Let us pass to the coordinate system with a zero longitudinal particle momentum. In this case the particle moves between two barriers of height $u' - \gamma u$ ($\gamma = E/m$, u is the height of the potential). To determine the radiation length, apply the dipole approximation:

$$\Gamma_n = \sum_{nn'} \frac{4e^2\omega_{nn'}^3}{3} |x_{nn'}|^2, \tag{10.1}$$

where $\omega_{nn'} = \frac{\pi}{2ma^2}(n^2 - n'^2)$ is the transition frequency; $x_{nn'}$ is the matrix element of the coordinate for the transition between the states n and n'. For a rectangular potential well of the channel the matrix element has the form

$$x_{nn'} = -\frac{2a}{\pi^2}\frac{4nn'}{(n^2 - n'^2)^2}. \tag{10.2}$$

As a result, the expression for the radiation width of the level n, and correspondingly, for the lifetime at the level τ_n in the lab system may be written as follows:

$$\frac{1}{\tau_n} = \frac{A}{\gamma} \sum_{n'<n} \frac{n'^2 n^2}{n^2 - n'^2}, \tag{10.3}$$

where $A = \frac{32}{3}\frac{e^2\pi^2}{m^3a^4}$.

Taking into account that in the high-energy range there are many levels $\left(n_{max} \sim \left(\frac{2ma^2}{\pi^2}\gamma u\right)^{1/2}\right)$ in a well, to obtain accurate enough estimate, in equation (10.3) one may substitute summation for integration, which yields the expression

$$\frac{1}{\tau_n} \simeq \frac{A}{2\gamma} n^3 \ln 2n. \tag{10.4}$$

From (10.4) follows that the lifetime for a particle with maximum probability of residing at level n_{max} (this corresponds to the particle incident on a crystal at the Lindhard angle ϑ_L) is

$$\frac{1}{\tau_{max}} \simeq \frac{e^2}{a\gamma}\left(\frac{\gamma u}{m}\right)^{3/2} \ln ma^2 u\gamma. \tag{10.5}$$

Thus, from (10.5) follows that, e.g., for a positron of energy $E \sim 1$ GeV, the length over which the level population will drop by a factor of e is $l_{n\,max} \sim n \sim 10^{-3} - 10^{-2}$ cm. As also seen from (10.4) the length l_n should grow with the decrease in n. For instance, for particles entering the crystal at the angle one-tenth as large as the Lindhard angle, $n = n_{max}/10$ and $l_n = l_{max} \cdot 10^3 \sim 1$ - 10 cm.

Let the angular divergence of the beam incident on the crystal be $\vartheta \sim 10^{-4}$ rad, i.e., of the same order of magnitude as the critical angle for channeling. In this case for positrons, in fact, all the levels in the potential well of the channel are populated. According to (10.4), (10.5) after a beam of positrons of energy $E \sim 1$ GeV passes through a single crystal with the thickness $L \sim 0.1 - 1$ cm, one should expect an order of magnitude decrease in the angular divergence of the beam.

With the influence of multiple scattering on the beam evolution in the channel ignored, the particle distribution over the levels can be analyzed relatively simply. Let us assume that the initial population over the levels is equally probable. The calculation (Fig. 8) will be carried out using the kinetic equation of the form [Dubovskaya (1978)]

$$\frac{\partial \rho_n}{\partial t} = -\sum_{n'<n} W_{nn'}\rho_n + \sum_{n'>n} W_{nn'}\rho_{n'}$$

Figure 8. The change in the population of the transverse motion levels for a particle passing 0.1-cm-thick crystal target.

The expression for $W_{nn'}$ is obtained from formula (10.3) if summation over n' is ignored, i.e.,

$$W_{n'n} = \frac{A}{2\gamma}\frac{n^2 n'^2}{n^2 - n'^2}.$$

Interestingly enough, to obtain the above estimates, the difference between the real potential of the channel and the harmonic one appears to be of principal importance. In a harmonic well the lifetime at the excited level is easy to find from the classical formula for radiative damping $\tau = \left(\frac{2}{3}\frac{e^2\Omega^2}{m}\right)^{-1}$ $(\hbar = c = 1)$. According it for the length over (in) which the level population will reduce by a factor of e, we get the estimate $l \sim 10$ cm, i.e., in the case of harmonic potential the phenomenon of radiative cooling is practically absent [Dubovskaya (1978)].

It should be noted, however, that the obtained estimates of the the radiative cooling rate do not take into account the processes leading to the increase in the magnitude of the transverse momentum of the channeled particle, such as, for example, multiple scattering in the channel. Allowing for multiple scattering can appreciably affect the features of the motion of a channeled particle. Consider this process in more detail.[1]

[1]The results presented in (10.2) and (10.3) were obtained together with A.O.Grubich.

10.2 Classical Theory of Channeling of Charged Particles with Due Account of Radiation Energy Losses

As the number of the transverse energy levels of a channeled ultra-relativistic charged particle moving in a potential well formed by the crystal axes (planes) is great, we shall use the classical theory as the first step towards the description of the particle motion. In classical thermodynamics the equation of motion of a charged particle in an external field with the account of radiation slowdown has the form [Landau and Lifshitz (1967)]

$$\frac{d\vec{\rho}}{dt} = \vec{F}_L + \vec{F}_{rad}, \tag{10.6}$$

where $\vec{\rho} = m\gamma\vec{v}$ is the particle momentum; $F_L = e\left(\vec{\varepsilon} + \frac{1}{c}[\vec{v}\vec{H}]\right)$ is the Lorentz force;

$$F_{rad} = \frac{2e^2\gamma^2}{2c^3}\left[\vec{w} + \frac{\vec{v}(\vec{v}\vec{w})}{c^2}\gamma^2 + \frac{3\vec{w}(\vec{v}\vec{w})}{c^2}\gamma^2 + \frac{\vec{v}(\vec{v}\vec{w})^2}{c^4}\gamma^4\right] \tag{10.7}$$

is the radiative friction force; $\vec{w} = d\vec{v}/dt; \vec{w} = d\vec{w}/dt$; γ is the Lorentz factor. Using a well known formula of relativistic dynamics [Landau and Lifshitz (1967)]

$$m\gamma\vec{w} = \vec{F} - \frac{1}{c^2}(\vec{v}\vec{F})\vec{v};\ \vec{F} = \vec{F}_L + \vec{F}_{rad}, \tag{10.8}$$

enables one to write equation (10.6) in the form convenient for further analysis ($c = 1$):

$$\vec{w} - \frac{\vec{F}_L - \vec{v}(\vec{v}\vec{F}_L)}{m\gamma} = \frac{2}{3}r_e\gamma[\vec{w} + 3\gamma^2\vec{w}(\vec{v}\vec{w})]; \tag{10.9}$$

$$\gamma - \frac{\vec{F}_L - \vec{v}}{m} = \frac{2}{3}r_e\gamma^4\vec{v}[\vec{w} + 3\gamma^2\vec{w}(\vec{v}\vec{w})]; \tag{10.10}$$

where $r_e = e^2/m$ is the classical electron radius. In a non-magnetic crystal $\vec{F}_L = -\nabla u$, where u is the potential energy of particle interaction with the crystallographic axes (planes), which is averaged over thermal vibrations of the crystal lattice. Recall that equation (10.10) is the corollary to equation (10.9) and formula $\gamma = (1-v^2)^{-1/2}$,and it may be written as, for example,

$$\dot{\gamma} - \gamma^3\vec{v}\vec{w}. \tag{10.11}$$

It is almost impossible to solve equation (10.6) without using numerical methods. Therefore let us dwell on the simplifications that are may be realized in the original equations.

As is known, equation (10.6) is applicable when in one of the reference frames $F_{rad} \ll F_L$. Therefore instead of equation (10.9) an approximate equation is usually used, which is obtained by substitution into the right-hand side of equation (10.9) of the particle acceleration expressed in terms of an external electromagnetic field acting on a particle:

$$\vec{w}^{(0)} = [\vec{F}_L - \vec{v}(\vec{v}\vec{F}_L)]/m\gamma. \qquad (10.12)$$

The condition of smallness of the radiative friction force $\vec{F}_{rad}$ as compared with the external force affecting the charge $\vec{F}_L$ has the form [Landau and Lifshitz (1967)]

$$\gamma \ll \gamma_{quant} = \frac{m}{r_e \vec{F}_L}. \qquad (10.13)$$

However, classical electrodynamics becomes unsuitable due to the production of electron-positron pairs in the external electromagnetic field yet in the range of energies (see [Landau and Lifshitz (1967)], p.267)

$$\gamma \sim \gamma_S = \gamma_{quant}/137, \qquad (10.14)$$

at which the external field ε' acting on the particle in the instantaneous rest frame ($\varepsilon' = \gamma\varepsilon$) attains the value of the Schwinger field $\varepsilon_{quant} = m^2/e\hbar$.

In this regard it is interesting that when the condition

$$\gamma \geq \gamma_{sa} = \frac{m^2|(\vec{v}\vec{\nabla})\vec{F}_L|}{2r_e|F_L^3|} \approx \frac{2m^2 v_\perp}{r_e F_L^2 d} \qquad (10.15)$$

is fulfilled, in the right-hand side of equations (10.9) and (10.10) in the second order perturbation theory, the terms leading to the particle "self-acceleration" prevail. The magnitude of the Lorentz factor γ_{sa} appears to be of the same order of the magnitude as γ_S (implying qualitative estimates we assumed that $|(\vec{v}\vec{\nabla})\vec{F}| \approx v_\perp 4F_L/d$, where d is the channel width).[2]. Indeed, assuming that $v_\perp \approx (F_L x/m\gamma)^{1/2}$, we obtain

$$\gamma_{sa} \approx \gamma_S \alpha^{-1}(r_e x)^{1/3}(d/2)^{-2/3},$$

where x is the amplitude of the particle vibrations in the channel; $\alpha = 1/137$. Thus, at $x = d/4\gamma_{sa} \approx \gamma_S \alpha^{-1}(r_e/d)^{1/3} \sim \gamma_S$.

One might suppose that the application of the Dirac-Lorentz equation in the energy range $\gamma \sim \gamma_S$ (due to quantum effects), though not being quite correct, nevertheless may give a correct qualitative pattern of the motion

[2]When deriving inequality (10.15) it was taken into account that the velocity of the channeled particle is directed at a small angle with the crystallographic axes (planes) forming the channel $|v_\perp| \ll 1$, as well as the fact that the Lorentz force acting on the particle in the channel is transverse.

of particles with the energy $\gamma \sim \gamma_S$ channeled in the crystal. However, according to the estimates obtained, application of the classical equation of motion in the case of a conventionally used approximate equation (10.9) with the radiative force $\vec{F}_{rad} = \vec{F}_{rad}(\vec{w}^{(0)})$ in the range of energies $\gamma \approx \gamma_S$ is quite problematic.

In the range of energies $\gamma \ll \gamma_S$ the principal terms in right-hand sides of equations (10.9) and (10.10) are those proportional to the derivative of the particle acceleration $\vec{w}$. As a consequence, to solve the problem, one may use the approximate equations

$$\vec{w} - \vec{w}^{(0)} = \frac{2}{3} r_e \gamma \dot{\vec{w}}; \tag{10.16}$$

$$\dot{\gamma} - \frac{\vec{F}_L \vec{v}}{m} = 2 r_e \gamma^6 (\vec{v}\vec{w})^2. \tag{10.17}$$

When the crystal thickness is not very large and the time-dependence of factor γ may be neglected, for the simplest forms of the potential u the solution of the equation of motion (10.16) may be found explicitly. For example, at planar channeling in a harmonic potential $u(x) = kx^2/2$ the approximate solution of equation (10.16) for particle transverse vibrations in the channel has the form

$$x(t) = x_m \cos(\Omega t + \varphi) e^{-t/\tau}, \tag{10.18}$$

where $\Omega^2 = k/m\gamma$; φ is the initial phase; $\tau = 3m/r_e k$.

The solution is similar for the case of axial channeling in a two-dimensional harmonic potential $u(\vec{\rho}) = k\rho^2/2$ ($\vec{\rho}$ is the radius-vector of the particle in the plane perpendicular to the crystallographic axes which form axial channels).

Substitution into right-hand sides of equations (10.16), (10.17) of the quantity $\vec{w}$ corresponding to the zero-order approximation (of)(10.12) gives approximate equations of the form [Bonch-Osmolovskii and Podgoretskii (1978, 1979)]:[3]

$$\vec{w}_\perp - \frac{\vec{F}_L}{m\gamma} = \frac{2}{3}\frac{r_e}{m}(\vec{v}\vec{\nabla})\vec{F}_L; \tag{10.19}$$

$$\dot{\vec{\gamma}} = \frac{\vec{F}_L \vec{v}}{m} - \frac{2}{3}\frac{r_e}{m^2}\gamma^2 F_L^2; \tag{10.20}$$

Harmonic potential is often used when considering planar channeling of positively charged particles. Since the real potential may contrast sharply

[3]The equation of motion for a longitudinal component of the radius-vector is not presented, as we are mainly concerned with the transverse motion in the channel.

with the harmonic one, we shall dwell on the quantitative comparison of the features of particle motion in different potentials.

A thorough a review of different model potential is given in [Gemmell (1974)]. In particular, it is shown that in the case of channeling of positively charged particles the potentials of the channels formed by the planes (110) of a single crystal of silicon are well described by the harmonic potential (see [Gemmell (1974)], Fig.9).

Let us consider how a harmonic potential approximates planar channels of single crystals of other chemical elements (Fig. 9).

Figure 9. The potentials of single crystals: planar channels (solid curves), harmonic channels (dashed curves).

The potentials in Fig 9. are depicted in the space region from the channel center to the point located at the distance equal to the shielding radius a from the equilibrium position of atoms of the crystallographic plane, forming the channel wall. The curves are calculated from the formulae

$$u_p^L(x) = u(x) - u(0);\ u(x) = u^L(x + d_p/2) + u^L(x - d_p/2),$$

where the Lindhard potential is

$$u^L(x) = 2\pi n z e^2 a d_p \left[\left(\frac{x^2}{a^2} + 3 \right)^{1/2} - \frac{x}{a} \right]$$

(n is the density of atoms in the crystal; z is the nucleus charge; a is the shielding radius; d_p is the distance between the planes). At point $x = d_p/2 - a$ harmonic potentials $u_p(x) = kx^2/2$ are equal to the potential $u_p^L(x)$.

Interestingly enough, the elasticity constant k_L found from the equality

$$k_L(d_p/2 - a)^2 = u_p^L(d_p/2 - a)$$

is well described by the quantity

$$\tilde{k} = 4\pi n_e e^2, \tag{10.21}$$

used by Bonch-Osmolvsky and Podgoretsky [Bonch-Osmolovskii and Podgoretskii (1978, 1979)] (here n_e is the electron density in the central part of the channel). The magnitudes of the attenuation length $\Lambda = 3mc^2/kr_e$, calculated with the help of the elasticity coefficients k_L and $\tilde{k}$ (electron density $n_e = nz/2.72$) are given in the Table. The accepted expression for n_e corresponds to the uniform distribution of the crystal electrons in the

$Crystal$	$Channel$	$\lambda_L cm$	λcm
Si	(100)	8.8	-
Si	(110)	11.64	11.86
Ge	(100)	5.6	5.88
Cu	(110)	2.92	3.38

central part of the channel with the density which is by a factor of e smaller than the mean electron density $\bar{n}_e = zn$.[4]

The phase trajectory of the transverse motion of a positron (Fig. 10, curve 1) channeled in (110) channel of the single crystal of silicon can be found by means of numerical solution of the system of equations (10.19)-(10.20) in the Moliere potential. The phase trajectory of the particle moving in the Moliere potential is only slightly different from a circle which is the phase trajectory of the harmonic motion (curve 2).

Figure 10. The phase trajectory of the transverse motion of a positron: 1. The trajectory of a particle moving in the Moliere potential; 2 - the trajectory of the harmonic motion.

In the case of planar channeling of positrons, the value of $R = d_p/2a$ serves as a criterion for applicability of the harmonic potential. The smaller the value of R, the better the harmonic potential approximates the channel potential in the range $|x| \leq d_p/2 - a$. So, for the elements given in the Table we have: in silicon for the channel (100) $R \simeq 3.5$, for the channel (110) $R \simeq 4.9$; in tungsten for the channel (111) $R \simeq 4.1$, for the channel (100) $R \simeq 9.9$; the harmonic approximation in this case appears to be of little use.

Now consider the change in the total energy and the attenuation of the transverse velocity. In the initial stage of motion the losses of the particle total energy $\varepsilon(L) = E_0 - E(L)$ and the attenuation of the amplitude of the particle transverse velocity θ_m, are of linear character due to the presence of the radiative friction force (Figure 11):

$$\varepsilon(L) = \alpha_\varepsilon L; \tag{10.22}$$

$$\theta_m(L)/\theta_m(0) = 1 - \alpha_\theta L. \tag{10.23}$$

[4]Henceforth the model of the harmonic potential $u_p(x)$ with the elasticity constant (10.21 is used more than once for quantitative assessments; by the channel width d we shall mean $(d_p - 2a)$.

Figure 11. Radiation energy losses as a function of thickness.

Note that at the energies used in the experiments with channeled particles until recently ($E \leq 10$ GeV), the linear laws (10.22), (10.23) are valid, for example, in diamond and silicon targets up to the crystal thicknesses as large as several centimeters. Indeed, in the case of particle motion in a harmonic potential it follows from equations (10.19), (10.20) that if the conditions $\gamma_0\theta_0 < 1$, or $t \ll \tau/\gamma_0^2\theta_0^2$, $> \gamma_0\theta_0 > 1$ are fulfilled, corresponding to the smallness of radiation energy losses $\varepsilon/E_0 \ll 1$, the particle trajectory is determined by expression (10.18), and the character of the changes in its total energy - by expression

$$\gamma(t) = \frac{\gamma_0}{1 + (1 - e^{-2t/\tau})\gamma_0^2\theta_0^2/2} \tag{10.24}$$

where γ_0 is the Lorentz factor corresponding to the initial energy of a particle E_0; $\theta_0 = x_0\Omega_0$ is the initial amplitude of the particle transverse velocity in the channel. From (10.24) we obtain the below equality for the coefficient α_ε at motion in a harmonic potential

$$\alpha_\varepsilon = \frac{m\gamma_0^3\theta_0^2}{\tau}. \tag{10.25}$$

the coefficient $\alpha_\theta = \tau^{-1}$. From (10.24) is also seen that in the case $\gamma_0\theta_0 < 1$ the approximate solution of (10.24) is applicable for times $t > \tau$ too.

Joint solution of equations (10.19), (10.20) for the harmonic potential $u_p(x)$ was obtained in [Bonch-Osmolovskii and Podgoretskii (1979)]. Therefore the coefficients α_ε, α_θ can certainly be found from equations (62) and (63) of [Bonch-Osmolovskii and Podgoretskii (1979)]. The above analysis shows that the range of energies and crystal thicknesses, where (10.22), (10.23) have linear solutions is quite broad; the correct expression for the coefficient α_ε follows just from the solution of (10.18) and (10.20)[5] at the initial stage of motion ($t \ll \tau/\gamma_0^2\theta_0^2$) $\alpha_\theta = \tau^{-1}$ at any values of $\gamma_0^2\theta_0^2$ (compare [Bonch-Osmolovskii and Podgoretskii (1979)]).

To avoid possible misunderstandings, note that harmonic approximation is not suitable in the cases when even slight nonlinearity of the potential u is of importance, for example, in the case of resonance action of electromagnetic or ultrasonic fields on channeled particles.

Now recall the presence of multiple scattering of a channeled particle by the fluctuating part of the potential of interaction with the grating. Multiple scattering can be taken into account by introducing into the right-hand

[5]Recall that at $\gamma \gg 1$ the augend on the right-hand side of equation (10.20) may be neglected.

side of equation (10.6) of a random force $\vec{F}(\vec{\rho}, \vec{\theta}, t)$ describing the events of inelastic collisions between the particle and the atoms of the crystal lattice. [6] It is known [Rytov (1966)] that from the stochastic equation of motion one may go over to the Einstein-Fokker equation for the probability density $w(\vec{\rho}, \vec{\theta}, t)$ of finding the particle at moment t in the space region $(\vec{\rho}, \vec{\rho} + d\vec{\rho})$ with the velocity $\vec{\theta}$ which is a part of the interval $(\vec{\theta}, \vec{\theta} + d\vec{\theta})$. For the equation of motion (10.19) with the random force $\vec{F}(\vec{\rho}, t)$. The Einstein-Fokker equation has the form

$$\frac{\partial w}{\partial t} + \vec{\theta}\frac{\partial w}{\partial \vec{\rho}} + \frac{\partial}{\partial \vec{\theta}}\left\{ w\left[\frac{\vec{F}_L}{m\gamma} + \frac{2r_e}{3m}(\vec{v}\vec{\nabla})\vec{F}_L\right]\right\} = \sum_{ik} \frac{\partial^2 \mathcal{D}_{ik}(\rho) w}{\partial \theta_i \partial \theta_k}, \tag{10.26}$$

where $\mathcal{D}_{ik}(\vec{\rho}) = c_{ik}(\vec{\rho}, \vec{\rho})/2$; $c_{ik}(\vec{\rho}, \vec{\rho}')\delta(t-t') = \langle \mathcal{F}_i(\vec{\rho}, t) \times \mathcal{F}_k(\vec{\rho}', t)\rangle$; $\mathcal{F}_k(\vec{\rho}, t)$ are normally distributed random fields with zero mean values $\langle \mathcal{F}_i(\vec{\rho}, t)\rangle = 0$.

Radiation energy losses described by equation (10.20) are of a continuous character. Therefore they may be taken into account upon passing from equation (10.26) to the equation for the probability density $w(\vec{\rho}, \vec{\theta}, E, t)$ containing in the right-hand part a differential term $m\frac{d\gamma w}{dE}$ describing the change in the number of particles in the energy range $(E, E+dE)$. However, for a qualitative analysis of the problem it is possible to use directly the set of two differential equations (10.20), (10.26) (see, for example, [Bonch-Osmolovskii and Podgoretskii (1978, 1979)]).

In addition to the electromagnetic radiation, generated by a particle moving in a potential well u, a channeled particle also emits γ quanta through scattering by a fluctuating part of the interaction potential ("ordinary" bremsstrahlung). Large straggling of the radiation energy losses is typical of such bremsstrahlung with the Bethe-Heitler spectrum of the form ω^{-1} [Baryshevskii *et al.* (1977); Heitler (1984)]. The quantitative theory in this case should be based on the kinetic equation with the collision integral describing the bremsstrahlung processes in the right-hand side (see [Baryshevskii *et al.* (1977)]). For not very thick crystals (for example, those of silicon with the thickness of about 1 cm) the usual bremsstrahlung loss may be ignored. [7]

[6] The radius-vector $\vec{\rho}$ describes the particle transverse motion in the (x, y) plane. The velocity $\vec{v}_\perp \equiv \vec{\theta} = \dot{\vec{\rho}}$.

[7] In [Vedel' and Kumakhov (1979)] usual bremsstrahlung loss was taken into account by introducing into the right-hand side of equation of the type of (10.11) of a term equal to the magnitude of the average bremsstrahlung energy loss per unit time, i.e.

In [Bonch-Osmolovskii and Podgoretskii (1979)] it is shown that when a charged particle moves in a one-dimensional harmonic potential, it is possible to obtain from equation of type (10.26) the closed systems of differential equations for the first- and second-order moments. The given statement, generally speaking, is a particular case of the general theorem holding for linear systems [Rytov (1966)].

So, for a harmonic potential, a similar system of closed differential equations may be obtained from the equation of motion (10.16) with a random force $\vec{F}(t)$. Here instead of the system of three equations, we obtain a closed system of six differential equations for the moments $\langle\vec{\rho}^{(k)}\vec{\rho}^{(e)}\rangle$, where $\vec{\rho}^{(k)} = \frac{d^{(k)}}{dt(k)}\vec{\rho}, k, l = 0, 1, 2$.

Below we shall dwell on the analysis of the system of differential equations for two-dimensional moments $\langle\vec{\rho}^{(k)}\vec{\rho}^{(l)}\rangle$ $(k, l = 0, 1)$ which follows from the equation of motion (10.19) with a random force $\vec{F}(t)$ and a harmonic potential u. Implying the qualitative analysis of the problem, we assume here that the random force $\vec{F}(t)$ is independent of $\vec{\rho}, \vec{\theta}$. For the potential $u = k\rho^2/2$ the desired equations are obtained from equations (28) of [Bonch-Osmolovskii and Podgoretskii (1979)] by a simple substitution of one-dimensional moments $\langle x^2\rangle$, $\langle\theta^2\rangle$, $\langle x\theta\rangle$ for two-dimensional ones:

$$\frac{d}{dt}\langle\rho^2\rangle = 2\langle\vec{\rho}\vec{\theta}\rangle;$$

$$\frac{d}{dt}\langle\vec{\rho}\vec{\theta}\rangle = \langle\theta^2\rangle - \frac{2}{\tau}\langle\vec{\rho}\vec{\theta}\rangle - \Omega^2\langle\rho^2\rangle;$$

$$\frac{d}{dt}\langle\theta^2\rangle = -2\Omega^2\langle\vec{\rho}\vec{\theta}\rangle - \frac{4}{\tau}\langle\theta^2\rangle + 4D. \tag{10.27}$$

Then, following the similar lines as in [Bonch-Osmolovskii and Podgoretskii (1979)], supplement equations (10.27) with averaged equation (10.20):

$$\dot{\gamma} = \vec{v}\vec{F} - \frac{2}{\tau}\gamma^3\Omega^2\langle\vec{\rho}^2\rangle. \tag{10.28}$$

The first term on the right-hand side of (10.28) describes the change in the particle energy caused by the work done by the Lorentz force, and in the case $\gamma \gg 1$ it may be dropped. The second term, proportional to $\langle\vec{\rho}^2\rangle$ corresponds to the energy emitted by a relativistic harmonic oscillator per unit time.

If the incursion of the root-mean-square angle of multiple scattering of the channeled particle during the velocity relaxation time of its transverse

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the approximation of the continuous losses was used. Such a method of allowing for bremsstrahlung loss is erroneous [Baryshevskii *et al.* (1977)].

motion in the channel τ is much greater than the angle θ_0^2, then the force $\vec{F}_{rad}$ in the equation of motion can apparently be neglected. In the model under consideration this condition satisfies the following inequality

$$4D\tau \gg \theta_0^2 \tag{10.29}$$

It is also obvious that in the case in question the linear law of motion is valid for not large times t:

$$\begin{aligned} \langle\theta^2\rangle &= 2D(E_0 t + \langle\theta^2\rangle; \\ \langle\rho^2\rangle &= \langle\theta^2\rangle/\Omega^2(E_0), \end{aligned} \tag{10.30}$$

suitable providing that $\varepsilon(t)/E_0 \ll 1$. Then according to (10.28), (10.30), we obtain

$$\gamma(t) = \gamma_0 \left\{ 1 + \frac{2\gamma_0^2 t}{\tau}(\langle\theta_0^2\rangle + Dt) \right\}^{-1} \tag{10.31}$$

(in perfect agreement with formula (10.24)).

Figure 12 exemplifies the comparison between the numerical solution of the system of equations (10.27), (10.28) and approximate solution of (10.30), (10.31).

Figure 12. The root-mean-square angle of multiple scattering and energy losses as a function of thickness.

Planar channeling of positrons in (110) channel of a silicon single crystal at $\langle\theta_0^2\rangle = 0$ (in the case of planar channeling D should be replaced by $1/2D$ in formulae (10.27), (10.30), (10.31)) is considered. As seen from graphs, the two solutions agree well. The diffusion coefficient D, as well as for protons [Kagan and Kononets (1973, 1974)], is taken equal to $d = kD_{chaot}$, where $k = z_v/z^2$ (z_v is the number of valence electrons); $D_{chaot} = E_S^2/4E^2L_R$; $E_S^2 = 4\pi m^2/\alpha$; $\alpha = 1/137$; L_R is the radiation unit of length. Note that in view of the aforesaid characteristics of k_L and k, the value of the coefficient $k = 1/z \cdot 2.72$ is more precise.

At $\langle\theta^2(0)\rangle = 0$ the law of variation of γ/γ_0 which follows from (10.31) is independent of the particle initial energy, as the diffusion coefficient $D \sim \gamma_0^{-2}$. Therefore the corresponding (dashed) curve in Fig. 12 is universal for different initial energies E_0. Dotted curve in Fig. 12 represents the dependence of γ/γ_0 ($E_0 = 10^3$ GeV), used in [Vedel' and Kumakhov (1979)]. The discrepancy with the exact solution ($\langle\theta^2(0)\rangle = 0$)is quite large. As a result, the calculations carried out in the stated work give the incorrect picture of the evolution of angular distributions $\gamma_2(\theta, \theta_0, t)(\theta_0 = 0)$[8]

From (10.29) and (10.31) one can easily find the ranges (γ, t) where the approximation (10.30), (10.31) is applicable:

$$\gamma \ll \gamma_1 = \tilde{\gamma}\frac{\tau}{T}\eta^{-2},\ t < T = \sqrt{\frac{\alpha L_R \tau}{\pi k}};$$
$$\gamma_1 \sim \gamma \ll \gamma_{rad} = 4\gamma_1\frac{\tau}{T},\ t \ll L_E(\gamma) = \tau\frac{\tilde{\gamma}}{\gamma}\eta^{-2}, \qquad (10.32)$$

where $\tilde{\gamma} = m/2u_0$ is the magnitude of the Lorentz factor which corresponds to the particle energy E, at which the critical angle

$$\theta_{cr} = (2u_0/E)^{1/2} = (\gamma\tilde{\gamma})^{-1/2}$$

equals γ^{-1}, $\langle\theta^2(0)\rangle = \frac{1}{2}\theta_0^2$, $\theta_0 = \eta\theta_{cr}$. Note that in the case of planar channeling of positrons $\frac{\tau}{T} = \sqrt{3\mathcal{L}_R}$ (here $\mathcal{L}_R$ is the radiation logarithm ($\mathcal{L}_R \simeq \ln(191z^{-1/3})$ [Ter-Mikaelian (1969, 1972)], $k = \tilde{k}$).

It should also be pointed out that the range (γ, t) determined by the relations (10.32) is rather large. For example, at $\eta = 1/2\, 4\gamma_1\frac{\tau}{T} \sim 10^2\tilde{\gamma}$.

Further make use of the derived relations for seeking the dechanneling length L_D. Define L_D as the pathway where $\langle\rho^2(L_D)\rangle = d^2/4$. As a result,

$$L_D = (1-\eta^2)/2D\gamma\tilde{\gamma} = T\gamma(1-\eta^2)/\gamma_D, \qquad (10.33)$$

where $\gamma_D = 2\tilde{\gamma}\tau/T$. It is easy to see that the expression (10.33) holds true at any $\eta \in [0,1]$ in the energy range $\gamma < \gamma_D$. Thus, the motion of charged particles in a wide range of energies and crystal thicknesses is described by the approximate solution of (10.30), (10.31).

Consider the diffusion coefficient for channeled particles in more detail. The coefficient D_{chaot}, corresponding to the multiple Coulomb scattering of ultra-relativistic electrons (positrons) in an amorphous medium is well known and equal to 1/4 of the root-mean-square angle of particle multiple scattering per unit time: $D_{chaot} = \frac{E_S^2}{4E^2 L_R}$ [Ter-Mikaelian (1969, 1972)].

In the channeling regime the diffusion coefficient D depends on the particle trajectory in the channel (the charge sign and the energy of the particle transverse motion $E_\perp$). For example, at channeling of negatively charged particles moving in the vicinity of nuclei, $D^{(-)} > D_{chaot}$ and vice versa, for positively charged particles moving in the peripheral area of atoms forming a channel, $D^{(+)} < D_{chaot}$. (Hereinafter the superscripts $+(-)$ will be

[8]In [Vedel' and Kumakhov (1979)] in the equation of the the type (10.28) the constant equal to $d_p/4^2$ is used instead of the moment $\langle x^2\rangle$, and thus obtained dependence $\gamma(t)$ is then substituted into the solution of the kinetic equation of the type (10.26).

dropped, unless this leads to misunderstanding). The diffusion coefficient D is normally written as the sum $D = D_{nuc} + D_e$, with the first term corresponding to scattering of channeled particles by a screened potential of nuclei, and the second one corresponding to scattering by valence electrons (conduction electrons).

The concrete form of the coefficient D_e is based on some model of electron distribution in the crystal. In the simplest case the distribution of valence electrons is considered homogeneous. In the case of electron channeling in tubes or layers $D_{nuc} \gg D_e$. We note further that D_{chaot} is proportional to the density of nuclei per unit volume of the crystal, so it is natural to suppose that $D_{nuc}^{(-)}$ is approximately equal to the diffusion coefficient in an amorphous medium, where the density of nuclei is the same as that in tubes or layers, i.e., $D_{nuc}^{(-)} \simeq k_{nuc} D_{chaot}$, where $k_{nucR} = (d_R/a_R)^2$; $k_{nuc\,p} = d_p/a_p$; a_k, a_p is the tube radius and the layer width, respectively; d_R is the distance between the axes along which the channeled particle moves. At channeling of positrons in the central part of the channel $D_e \gg D_{nuc}$, and D_e, as mentioned above, is taken equal to $k_e D_{chaot}$, where $k_e = z_v/z^2$ [Gemmell (1974)]. Hence, the approximate solution of (10.30), (10.31), as well as expression (10.33) for the dechanneling length L_D are also suitable for describing electron motion in tubes or in layers.

Discuss the general pattern of channeling of light ultra-relativistic particles in a harmonic potential we obtained. The domain of applicability of the classical equation of motion is determined by the following two-sided inequality:

$$(40\lambda/d)^2\gamma = \gamma_{min} < \gamma \ll \gamma_S = (d/2\lambda\eta)\tilde{\gamma};\ \lambda = \hbar/mc. \qquad (10.34)$$

If the particle Lorentz factor is $\gamma < \gamma_{min}$, then there are only a few energy levels in a potential well of height U_0, and, as a consequence, the classical description of motion proves to be impossible. On the other hand, in the range $\gamma \sim \gamma_s$ the quantum effects gain importance [Bonch-Osmolovskii and Podgoretskii (1978, 1979)].

The energy range, determined by inequalities (10.34) stretches approximately for four orders of magnitude: from $\gamma \sim 10^2$ to $\gamma \sim 10^6$ ($d \sim 1\,\text{Å}, \eta \sim 1/2$). By the character of motion inside the channel it is helpful to divide the initial energies of channeled particles into three intervals:

I. $\gamma_{min} < \gamma < \gamma_D$;
II. $\gamma_D < \gamma < \gamma_{rad}$;
III. $\gamma_{rad} < \gamma$.

Within interval I the the channeled particle motion is entirely determined by its multiple inelastic scattering by the atoms of the crystal lattice, and the change in the particle energy may be neglected. Within interval II it is also determined by multiple scattering, but considerably affected by the radiation energy losses. Within interval III the character of the particle motion becomes affected by radiation friction (the right-hand side of equation (10.19)).

For energies defining the limits of the stated intervals the following relation holds:

$$\gamma_{min} \ll \gamma_D \ll \gamma_{rad} \sim \gamma_S(d \sim 1\,\text{Å},\, \eta \sim 1/2).$$

Therefore interval III, where the due account of the radiative recoil is important, is practically beyond the applicability of classical description.

At $\gamma \sim \gamma_S$ the characteristic frequency of emitted γ-quanta is $\omega_{eff} \sim E$. But for multiple scattering of particles in the channel, the radiative recoil would also appear to be important in the energy range $\gamma \ll \gamma_S$, when $\omega_{eff} \ll E$.

For particles channeled either in layers or in tubes [Bonch-Osmolovskii and Podgoretskii (1978, 1979)], $\gamma_{min} < \gamma_S(d \ll 1\,\text{Å}$, and, hence, in the cases mentioned above the application of classical description is strongly restricted.

10.3 Quantum Theory of Channeling Electrons and Positrons Allowing for Multiple Scattering and Radiation Energy Losses

The most consistent description of the transmission of relativistic charged particles through crystals may be achieved by means of quantum consideration of the process. This circumstance is due to the fact that in the range of not very high energies (of the order of several megaelectronvolts) there are only several levels for a transverse electron motion in a potential well, while at high energies, the emission of hard photons with the energy of the order of the particle energy is possible, which makes the account of quantum recoil crucial. If the radiation processes are of no importance, then the kinetic equations derived by Kagan and Kononetz [Kagan and Kononets (1973, 1974)] my be used to describe channeling. With the growth of energy of the particles, radiation is gaining greater importance, and in order to describe the behavior of electrons and positrons in crystals we have to introduce into the kinetic equations the collisional term cause by the photon radiation [Baryshevsky and Grubich (1979b)].

For detailed treatment of charged particles in the crystal and the electromagnetic radiation they produce it is necessary to find the density matrix $\rho(t)$ of the system crystal-particles-photons. The sated density matrix satisfies the quantum Liouville equation ($\hbar = c = 1$)

$$i\frac{\partial \rho}{\partial t} = [H_{tot}, \rho] \tag{10.35}$$

with the Hamiltonian

$$H_{tot} = H_e + H_\gamma + H_c + V_{ec} + V_{e\gamma} + V_{\gamma c}, \tag{10.36}$$

where H_e, H_γ are the Hamiltonians of free particles and photons, respectively; H_c is the crystal Hamiltonian; V_{ij} are the operators of interaction between the subsystems i and j ($i, j = e, \gamma, c$).

It is convenient to obtain first from equation (10.35) the equations describing the time change of the diagonal non-diagonal parts of the density matrix [Luttinger and Kohn (1958)]. The equation for the diagonal part of the density matrix describing the time evolution of a certain small subsystem a (the incident particle, and the γ-quanta it produced,) which interacts with a large subsystem β,has the form

$$\frac{\partial \rho_\alpha}{\partial t} = \sum_{\alpha'} \dot{w}_{\alpha'\alpha}\rho_\alpha + \sum_{\alpha'} \dot{w}_{\alpha\alpha'}\rho_{\alpha'}, \tag{10.37}$$

where $\rho_\alpha = (sp_\beta \rho)_{\alpha\alpha}$ is the diagonal matrix element of the matrix $sp_\beta \rho$; sp_β is the trace over the states of subsystem β from the full density matrix. The probabilities of transition per unit time are directly connected with the scattering operator [Berestetsky *et al.* (1968)]. Their explicit form for the processes of photon radiation through radiative transitions and bremsstrahlung was obtained in previous sections. The expressions for $\dot{w}$ describing the process of pair production see in (10.4).

Before passing to a detailed treatment of equation (10.37), it is useful to derive it for the case when the particle interaction with a crystal may be described in term of the perturbation theory. We shall neglect the influence of usual bremsstrahlung on the electron and positron behavior.

Equation for the density matrices of the electron $\rho_e(t) = sp_{\gamma c}\rho(t)$ and photon $\rho_\gamma = sp_{ec}\rho(t)$ subsystems may be found by taking the trace $sp_{\gamma c}$ and sp_{ec} of both parts of equation (10.35):

$$i\frac{\partial}{\partial t}\rho_e = [H_0, \rho_e] + sp_{\gamma c}[V, \rho]; \tag{10.38}$$

$$i\frac{\partial}{\partial t}\rho_\gamma = [H_\gamma, \rho_\gamma] + sp_{ec}[V, \rho], \tag{10.39}$$

where $H_0 = H_e + \overline{V}_{ec}$; $\overline{V}_{ec} = \sum_c \rho_{c(0)}^{c,c} V_{ec}^{c,c}$ is the operator V_{ec} averaged over the crystal states [Kagan and Kononets (1973)]; $\rho_{c(0)}$ is the equilibrium density matrix of the crystal, diagonal in the representation of the eigenfunctions of the Hamiltonian H_c; $H_c|c\rangle = E_c|c\rangle$; $V \equiv W + V_{e\gamma} + V_{\gamma c}$; $W = (V_{ec} - \overline{V}_{ec})$ and responsible for inelastic scattering of channeled particles by the lattice atoms;

$$[A, B] = AB - BA;\ sp_{\alpha\beta}\rho A = \sum_{\alpha\beta\alpha'\beta'} \rho^{\alpha\beta,\alpha'\beta'} A^{\alpha'\beta',\alpha\beta}, |\gamma, c\rangle$$

$$= |\gamma\rangle|c\rangle,\ |e, c\rangle = |e\rangle|c\rangle.$$

Using the integral representation of equation (10.35)

$$\rho(t) = S(t)\rho(0)S^+(t) - i\int_{-t}^{0} d\tau S^+(\tau)[V,\ \rho(t+\tau)]S(\tau), \tag{10.40}$$

int the second order over the operator V from (10.38), (10.39) we obtain the system of integro-differential equations

$$\frac{\partial\rho_e}{\partial t} + i[H_0, \rho_e] = \int_{-t}^{0} d\tau sp_{\gamma c}[S^+(\tau)[V, S(\tau)\rho(t)S^+(\tau)]S(\tau),\ V]; \tag{10.41}$$

$$\frac{\partial\rho_\gamma}{\partial t} + i[H_\gamma, \rho_\gamma] = \int_{-t}^{0} d\tau sp_{ec}[S^+(\tau)[V, S(\tau)\rho(t)S^+(\tau)]S(\tau), V]; \tag{10.42}$$

where $S(\tau) = e^{-iH'\tau}$; $H' = H_0 + H_\gamma + H_c$, $t_0 = 0$, where it is taken into account that at the initial time $t = 0$ of the particle entrance the crystal, the density matrix

$$\rho(0) = \rho_e(0)\rho_\gamma(0)\rho_c(0), \tag{10.43}$$

where the crystal density matrix $\rho_c(0) = sp_{\gamma e}\rho(0)$.

Neglect the interaction of γ-quanta with the crystal ($V_{\gamma c} = 0$). let us also consider that the state of the medium does not change during the particle transmission through the crystal. As a consequence, we may write

$$\rho(t) = \rho_c(0)\rho_{e\gamma}(t), \tag{10.44}$$

where $\rho_{e\gamma}(t)$ is the density matrix of the subsystem particles-photons. As a result

$$\frac{\partial \rho_e}{\partial t} + i[H_0, \rho_e] = sp_{\gamma c} I_1 + sp_\gamma I_2; \tag{10.45}$$

$$\frac{\partial \rho_\gamma}{\partial t} + i[H_\gamma, \rho_\gamma] = sp_e I_2. \tag{10.46}$$

The integral

$$I_2 = \int_{-t}^{0} d\tau [e^{i(H_0+H_\gamma)\tau}[V_{e\gamma}, e^{-i(H_0+H_\gamma)\tau} \rho_{e\gamma}(t) e^{i(H_0+H_\gamma)\tau}] \times e^{-i(H_0+H_\gamma)\tau}, V_{e\gamma}]. \tag{10.47}$$

The integral I_1 is obtained from the integral in (10.47) upon by replacing in it the operator H_γ with H_c, $V_{e\gamma}$ - with W and $\rho_{e\gamma}(t)$ - with $\rho(t)$. Note that as a result of fulfilment of equality $sp_\gamma \rho_{e\gamma}(t) = \rho_e(t)$ the expression $sp_{\gamma c} I_1$ in (10.45), as expected is equal to the right-hand side of equation (2.5) in [Kagan and Kononets (1973)].

As in [Kagan and Kononets (1973)], let the lower limit of integration in the expressions of I_1 and I_2 tend to $-\infty$. In thus obtained integrals of the type

$$\lim_{T\to\infty} \int_{-T}^{0} e^{\pm xt} dt = \pm i \frac{\mathcal{P}}{x} + \pi \delta(x) \tag{10.48}$$

we may neglect the summands with principal values of $\mathcal{P}/x$ which lead to renormalization of the energy spectrum. As a result, the expression $sp_\gamma I_2$ in the representation of the eigenfunctions of the Hamiltonian H_0 has the form

$$\begin{aligned} sp_\gamma I_2^{e,e'} = \pi \sum_{\substack{e''e''' \\ \gamma\gamma'\gamma''}} \Big\{ & \rho_{e\gamma}^{e''\gamma, e'''\gamma''}(t) V_{e\gamma}^{e\gamma', e''\gamma} V_{e\gamma}^{e'''\gamma'', e'\gamma'} [\delta(E_e + \\ & + E_{\gamma'} - E_{e''} - E_\gamma) + \delta(E_{e'} + E_{\gamma'} - E_{e'''} - E_{\gamma''})] \\ & - \rho_{e\gamma}^{\gamma e''', e'\gamma''}(t) V_{e\gamma}^{e\gamma'', e''\gamma'} V_{e\gamma}^{e''\gamma', e'''\gamma} \delta(E_{e''} + E_{\gamma'} - E_{e'''} - E_\gamma) \\ & - \rho_e^{e,e'''}(t) V_{e\gamma}^{e'''\gamma'', e''\gamma'} V_{e\gamma}^{e''\gamma', e'\gamma} \delta(E_{e''} + E_{\gamma'} + E_{e'''} - E_{\gamma''}) \Big\} \end{aligned} \tag{10.49}$$

Due to the symmetry of the integral (10.49) about the operators acting on vectors $|e\rangle$ and $|\gamma\rangle$, the matrix element $sp_e I_2^{\gamma,\gamma'}$ may be obtained from expression (10.49) be substitution of the subscript e into γ, and the subscript γ into e. The matrix element $sp_{\gamma c} I_1^{e,e'}$ appearing in the left-hand side of equation (10.45) is obtained from (10.49) with the operator V replaced by W, and the subscripts γ and c and the density matrix $\rho_{e\gamma}(t)$ -

by $\rho_c(0)\rho_e(t)$ (the explicit form of this matrix element see also in [Kagan and Kononets (1973)], formula (2.7)).

Now use factorization $\rho_{e\gamma} = \rho_e\rho_\gamma$ for the density matrix $\rho_{e\gamma}(t)$. Averaging of the equations obtained over times greater in comparison with the oscillation period of non-diagonal elements $\rho_e^{e,e'}(t)$, $\rho_\gamma^{\gamma\gamma'}(t)$ gives the set of balance equations (10.37). Attenuation of non-diagonal elements of the density matrix $\rho_e(t)$ due to inelastic scattering of channeled particles by the crystal lattice is studied in [Kagan and Kononets (1973)].

Thus, equation (10.37) has the form

$$\frac{\partial}{\partial t}\rho_e^{e,e}(t) = \sum_{e'}\Big\{(w_{e,e}^W + w_{e'e}^S)\rho_e^{e'}(t) - (w_{ee'}^W + w_{ee'}^S)\rho_e^{e,e}(t)\Big\}, \tag{10.50}$$

where

$$w_{ee'}^S = 2\pi\sum_{cc'}|w^{ec,e'c'}|^2\rho_c^{c,c}(0)\delta(E_e - E_{e'} + E_c - E_{c'}) \tag{10.51}$$

is the probability of the transition $e \to e'$ caused by inelastic scattering of channeled particles by the atoms of the crystal lattice;

$$w_{ee'}^S = 2\pi\sum_{\nu}|V_{e\gamma}^{e0,e'1\nu}(\omega_\nu)|^2\delta(E_e - E_{e'} - \omega) \tag{10.52}$$

is the probability of a spontaneous radiative transition $e \to e'$ per unit time; the function $\rho_e^{e,e}(t)$ equals the probability density of finding the channeled particle at moment t in the state $|e\rangle$.

Int the high-energy region it is possible to use impulse approximation when calculating the probability of inelastic scattering of a channeled particle by the atoms of the crystal lattice $w_{ee'}^W$. As a result

$$w_{ee'}^W = 2\pi\delta(E - E')\sum_i(\langle|V_i^{e',e}|^2\rangle_c - |\langle V_i^{e',e}\rangle_c|^2), \tag{10.53}$$

where $\langle A\rangle_c = \sum_{cc'}\rho_c^{c,c'}(0)A^{c,c',c}$. The potential energy of the particle interaction with the i-th atom of the crystal lattice

$$V_i(\vec{r}) = \pm\left(\frac{e^2 z}{|\vec{r} - \vec{r}_i|} - \sum_{j=1}^{z}\frac{e^2}{|\vec{r} - \vec{r}_i - \vec{r}_{ij}|}\right), \tag{10.54}$$

where $\vec{r}_i = \vec{r}_{0i} + \vec{u}_i$ is the radius-vector of the center of inertia of the atom; vector $\vec{r}_{0i}$ determines the equilibrium position of the lattice atom; $\vec{r}_{ij}$ is the radius-vector of the j-th electron with respect to the atom center of inertia.

upon corresponding calculations, we obtain the following expression for the probability $w^W_{ee'}$, for example, in the case of planar channeling [Baryshevsky and Grubich (1979b)]:

$$w^W_{ee'} = \frac{(2\pi)^3\delta(E-E')}{L^3} 4e^4 n_{at} \sum_{\tau_x q_x} \frac{J^{(1)}(q_x) J^{(1)^*}(q'_x)}{(qq')^2} \times \Big\{ z[F(\tau_x) - FF']e^{-w'(\tau_x)} + z^2(1-F)(1-F') \times (e^{-w(\tau_x)} - e^{-w(q)}e^{-w(q')}) \Big\}, \qquad (10.55)$$

where $J^{(1)}(q_x) \equiv J^{(1)}_{nk,n'k'}(q_x) = \int dx \psi_{nk}(x)\psi^*_{n'k'}(x)e^{iq^x_x}$; $\vec{q}_p = \vec{p}'_p - \vec{p}_p$; $\vec{p}_p = (p_y, p_z)$; $\vec{q}' = (q'_x, \vec{q}_p)$; $q'_x = q_x + \tau_x$; $F(q) \equiv F$; $F' \equiv (q')$; $\vec{q} = (q_x, \vec{q}_p)$; $F(q) = z^{-1}\int \rho(\vec{r})e^{-i\vec{q}\vec{r}}d^3r$; $e\rho(r)$ is the electron charge density in the atom; $F(\vec{q})$ is the atomic form-factor; L^3 is the crystal volume; n_{at} is the density of atoms; $e^{-w(q)}$ is the Debye-Waller factor.

If the functions $\varphi(x) = L^{-1/2}e^{ip_x x}$ are introduced into the integrals $J^{(1)}$ instead of the Bloch functions, then expression (10.55) goes over into the probability of particle scattering in a disoriented crystal

$$w^W_{ee'chaot} = (2\pi)^3 L^{-3}\delta(E-E')\frac{4e^4 n_{at}}{q^4}\left\{ z(1-F^2) + z^2(1-F)^2(1-e^{-2q^2u^2}) \right\}, \qquad (10.56)$$

where $q_x = p'_x - p_x$. The augend in (10.56), proportional to the atomic number z is equal to the probability of inelastic scattering of a particle by electron shell of the atoms; the addend, proportional to z^2 is the probabilities of inelastic scattering of the particle by oscillating atoms (photons) without changing their intrinsic state.

For single crystals with $z > 10$ in (10.55), (10.56) scattering by photons of the order of z^2 acts the main part. generally speaking, inelastic scattering of a particle by electron shells of the atoms of the order of z may be neglected. Cooling of the majority of crystals does not lead to considerable suppression of scattering by photons, with the possible exception of the crystals with a low Debye temperature θ_D ($\theta_D \leq 100$ K). The total scattering probability $w^W_e = \sum_{e'} w^W_{ee'}$. In the case of planar channeling the particle state in the crystal is described by a set of quantities $(\vec{p}_p, k, n)$. In view of the completeness condition of the Bloch functions

$$\tilde{F}(\tau_x) = \sum_{n'k'} J^{(1)}(q_x)J^{(1)^*}(q'_x) \int dx|\psi_{nk}(x)|^2 e^{-i\tau_x x}. \qquad (10.57)$$

the integral (10.57) is, in fact, the form factor of the channeled particle. In an axial channeling regime , we get the following form factor instead of (10.57)

$$\tilde{F}(\vec{\tau}_{\perp}) = \int d^2\rho |\psi_e(\vec{\rho})|^2 e^{-i\tau_{\perp}\vec{\rho}}. \qquad (10.58)$$

Next write the total probability of inelastic scattering as a series in terms of the reciprocal lattice vectors

$$w_e^W = \sum_{\vec{\tau}} w_e^W(\vec{\tau}). \qquad (10.59)$$

In the planar channeling regime summation is made over the vectors $\vec{\tau}_x$, while in the case of axial channeling of a particle, vector $\vec{\tau}$ in (10.59) equals $\vec{\tau}_{\perp} = (\vec{\tau}_x, \vec{\tau}_y)$.

At $\vec{\tau} = 0$ the form factor $\tilde{F}(0) = 1$. As a result the first term of series (10.59) corresponds to the probability of inelastic scattering of a particle in a disoriented crystal (10.56). The next following terms of the series determine the correction to $w_{e\,chaot}^W$ which depends on the form factor of the channeled particle. (10.57), (10.58).

The probability of inelastic scattering of a particle in a channeling regime (10.55) differs from the scattering probability in a disoriented crystal (10.56) not only by the presence of a form factor, (10.57) (which reflects the peculiarities of the particle transverse motion) , but also by the temperature dependence. Indeed, $w_e^W(\tau_x) \sim e^{-w(\tau_x)}$. As seen, the terms of the series (10.59) bear the same relationship to the crystal temperature as the probability of elastic scattering of a particle by the crystal, i.e.,proportional to e^{-2w} The appearance of the multiplier e^{-w} becomes obvious, if we recall that with the increase in the crystal temperature, the probability of inelastic scattering of electrons moving in tubes or in layers should tend to the magnitude of inelastic scattering of a particle in a disoriented crystal (amorphous medium) $w_{e\,chaot}^W = w_e^W(0)$.

Introduce the coefficient

$$k = 1 + \sum_{\vec{\tau}\neq 0} w_e^W(\vec{\tau})/w_{e\,chaot}^W. \qquad (10.60)$$

The magnitude of the sum in (10.60) depends on the sign of the form factor $\tilde{F}(\tau)$ which determines the sign of the terms of the series $w_e^W(\vec{\tau}) \neq 0$, and on the cutoff efficiency of the series $\sum_{\vec{\tau}\neq 0} w_e^W(\vec{\tau})$, which is defined by the absolute value of $|\tilde{F}(\vec{\tau})|$ and the multiplier e^{-w}. The sum

$$\sum_{\tau_x\neq 0} \tilde{F}(\tau_x) = 2\sum_{l=1}^{\infty} \int dx |\psi_{nk}(x)|^2 \cos(2\pi l x/d_p), \qquad (10.61)$$

where $\tau_x = 2\pi l/d_p$ $(l = 0, \pm 1, \pm 2, ...)$. Therefore for particles moving in the vicinity of the crystallographic axes (for example, electrons channeled in layers), for which the effective distribution width $|\psi_{nk}(x)|^2$ is much smaller than the channel width, the main contribution to the series (10.61) will come from the summands with l from 1 to $l_{max} \gg 1$. As a result the coefficient $k \gg 1$. In the case of channeling of positrons with transversal energy much smaller than the height of the potential barrier u, the series (10.61) will be alternating, and the coefficient k will turn out to be less than unity.

Consider the cutoff of the series (10.59) caused by the temperature factor e^{-w}. The quantity w included in the Debye-Waller factor, equals $w(\tau) = \vec{\tau}^2\langle u^2\rangle/6$ where $\langle u^2\rangle$ is the mean-square displacement of the atoms of the crystal lattice due to temperature oscillations. Thus, the exponent $e^{-w(\tau_x)}$ leads to the cutoff of the series (10.59), starting with $l_{max} \simeq d_p/\pi\sqrt{\langle u^2\rangle}$.

For most crystals at temperatures $T \sim 300$ K the probability of inelastic scattering of a particle by photons (the term of the order of z^2 in (10.55) is of the same order of magnitude as the probability of scattering a particle by stationary atoms $\sim z^2(1-F)^2$. Therefore in qualitative consideration of channeled particle motion in the space of transverse impulses (within the diffusion approximation) it is possible to apply the diffusion coefficient $D \simeq k_n D_{chaot}$, where the diffusion coefficient $D_{chaot} = \frac{1}{4}(E_S/E)^2 L_R^{-1}$ describes the elastic multiple scattering of electrons and positrons in amorphous substance.

When channeling electrons in layers $k_n \simeq l_{max}$ and, hence, $D \simeq (d_p/\pi\sqrt{\langle u^2\rangle})D_{chaot}$. The estimate of the magnitude of the diffusion coefficient D found here may also be obtained with the help of the following pictorial presentations. The diffusion coefficient D_{chaot} is proportional to the density of the nuclei per unit volume of the crystal. Therefore it is natural to suppose that the coefficient D is approximately equal to the diffusion coefficient in amorphous substance, where the density of the nuclei is the same as that in layers. As a result, $k \simeq d_p/2\sqrt{\langle u^2\rangle}$ [Baryshevsky and Grubich (1979b)]. As seen, both estimates of the magnitude of k differ by a numerical factor equal to $2/\pi$. In the case of electron channeling in tubes, the factor e^{-w} leads to the cutoff of the series (10.59). beginning with $l_{max} \simeq [3d^2/2\pi\langle u^2\rangle]$ (for simplicity,the crystal is assumed to have a simple cubic lattice, with the axes $\langle 100\rangle$, $\langle 010\rangle$, $\langle 001\rangle$ being considered). Thus, according to (10.60) for electron channeled in tubes $k_n \simeq 10^{-1}d^2/\langle u^2\rangle$.

The number of electrons in the electron core of the crystal lattice atom, generally speaking, equals some $z_{cor} \leq z$. Therefore in the general case in

(10.55) in the summand of the order of z due to inelastic scattering of a channeled particle by stationary atoms, z should be replaced by z_{cor} The remained $z - z_{cor}$ electrons are distributed over the entire volume of the unit cell of the crystal. Here the scattering probability $w^W_{ee'}$ will include the the form factor describing the distribution of valence (free) electrons together with the form factor of the atomic cores 9atoms forming the crystal unit cell). If the distribution of the valence electrons is supposed to be homogeneous over the entire volume of the unit cell, then the coefficient k_e corresponding to the contribution to scattering due to elastic collisions with valence electrons, may be assumed equal to the ration z_e/z^2 [Kagan and Kononets (1973, 1974)], where $z_e = (z - z_{cor})$ is the number of valence electrons. as a result, $D = D_n + D_e$ (here $D_n = k_n D_{chaot}$, $D_e = k_e D_{chaot}$).

Obviously, the coefficient $k_e = (2.72\ z)^{-1}$ offered above for describing multiple scattering of positrons in the central region of the planar channel ($|x| \leq d_p/2 - a$) corresponds to the total concentration of crystal electrons in the center of the channel, found ("restored") by the potential $u(x)$. Note that at large z the magnitude of the coefficient is $k_e = z_e/z^2$. For example, for tungsten $z/2.72\ z_e \simeq 14$.

Using the simplest models, let us consider the evolution of the diagonal elements of the density matrix of a charged particle $\rho^{e,e}_e(t)$ in a crystal. Neglecting the change in the total energy E, we obtain a conventional balance equation for the probability $p^{nn}_e(t) = \sum_{\vec{p}_p k} \rho^{e,e}_e(t)$ of finding the particle at moment t at the level n

$$\frac{\partial \rho_n}{\partial t} = \sum_{n'} (w_{n'n}\rho_{n'} - w_{nn'}\rho_n), \tag{10.62}$$

where $\rho_n \equiv \rho^{n,n}_e(t)$; $w_{nn'} = w^S_{nn'} + w^W_{nn'}$ is the probability of transition $n \to n'$.

First suppose that inelastic scattering is absent ($w^W_{nn'} = 0$). Then the evolution of the function $\rho_n(t)$ in a crystal is fully determined by radiative transitions $n \to n'$. In the case of a harmonic potential $u(x)$ the probability of radiative transitions in dipole approximation $w^S_{nn'} = 2\pi\vec{\tau}^{-1}\delta_{n',n-1}$. From equation (10.62) follows that the mean value of the level number $\overline{n}(t) = \overline{n}(0)e^{-2t/\tau}$, where $\tau = 3m/kr_e$. As seen, the relaxation time of the quantum and classical oscillators is the same ($E^{quan}_{\perp} \sim n \sim e^{-2t/\tau}, E^{cl}_{\perp} \sim \theta_0 e^{-2t/\tau}$). The lifetime of the harmonic oscillator $\tau_n = 1/w^S_{nn'} = (3d/2\alpha\eta)\tilde{\gamma}\sqrt{\tilde{\gamma}/\gamma})$ at the level $n(n \gg 1, \gamma \gg 1)$ is much less than the relaxation time $\tau = 3m/kr_e = (3d^2/4r_e)\tilde{\gamma}(\eta = n/n_{max}$, $n_{max} = (d/4\lambda)\sqrt{\gamma/\tilde{\gamma}}$, $\lambda = m^{-1}$, $\gamma = E/m$, $\alpha = 1/137$, $r_e = \lambda\alpha$, d is the channel width).

Here we, certainly, discuss the evolution of only those states of the channeled particles which correspond to the energy levels lying inside the potential well formed by crystal planes (axes). An interesting and complicated problem of the mutual kinetics of sub- and over-barrier states of a channeled particle calls for special study.

The sharper the potential of the channel is the shorter the relaxation time is in comparison with the relaxation time of a harmonic oscillator $\vec{\tau}$. The limiting form of a sharp potential is a rectangular well. Therefore it seems to be of interest to analyze the radiation kinetics of channeled particles in both cases. A detailed treatment of particle motion in a harmonic potential was given in (10.2). Here we shall dwell on the analysis of motion of a particle channeled in an infinite high rectangular potential well.

The probability of a spontaneous radiative transition of a particle channeled in a rectangular potential well in dipole approximation is (see (10.1))

$$w^S_{nn'} = \frac{A}{\gamma}\frac{n^2 n'^2}{n^2 - n'^2}, \tag{10.63}$$

where $n - n' = 1, 3, 5, \ldots$. The particle lifetime at this level is given by (10.4).

Comparison of expressions for the particle lifetime at the level n in a rectangular and harmonic potential wells show that the quantities $\vec{\tau}_n$ appear to be of the same order of magnitude. Nevertheless, due to the fact that in a rectangular potential well far transitions are allowed, unlike those in ia harmonic potential well, the relaxation time $E_\perp$ in a rectangular well is $\vec{\tau}_{rec} \ll \vec{\tau}$. As an illustration of the foregoing, consider two examples.

Substitute into the balance equation

$$\frac{\partial \rho_n}{\partial t} = \sum_{n'} (w_{n'n}\rho_{n'} - w_{nn'}\rho_n), \tag{10.64}$$

the probability (10.63) of spontaneous radiation $w^S_{un'}$ calculated in dipole approximation. Numerical solution of this equation (Fig. 13) was made for a model example, where at the initial time $t = 0$ only one level of the transverse energy of motion is populated

$$\rho_n(0) = \begin{cases} 1, \ n = 49; \\ 0, \ n \neq 49. \end{cases} \tag{10.65}$$

Figure 13. Time change of the distribution $\rho_n(t)$: a - for short times; b - for long times.

The width of the well $d = 1.92$ Å, $n_{max} = 70$, the particle energy $E = 1.0$ GeV (the given figures "correspond" to a rectangular potential well for a positron channeled in the channel (110) of a single crystal of silicon with the initial transverse energy $E_{\perp}(0) = 12$ eV).

At short times of the order of the lifetime at the level n_0 ($t \sim 4.5 \cdot 10^{-2}$ cm) the distribution function $\rho_n(t)$ depends on the parity of the level number due to the dependence of the probability of radiative transition $w^S_{nn'}$ in the model of the rectangular well on the initial and final states (see Fig.13, a). However, with the increase in the time t the difference between the probabilities $\rho_n(t)$ for the levels $n = 2m$ and $n = 2m + 1$ is rapidly smoothed over.

For the times $t \gg 10^{-2}$ cm the function $\rho_n(t)$ may, for the sake of convenience, be approximated by a continuous curve (solid) (Fig.14).

Figure 14. Distribution of $\rho_n(t)$ for large times.

Dotted in Figs. 13 and 14 show the distribution function $\rho_{\overline{n}}(t)$ corresponding to the mean value

$$\overline{n}(t) = \sum_{n=1}^{n_{max}} n\rho_n(t). \tag{10.66}$$

The mean value of the level number $\overline{n}$, as well as the rms value $\overline{n}^2$ and the dispersion $\sigma^2 = \overline{n}^2 - \overline{n}^2$ as the function of time are given in Figure 15.

Figure 15. Time change of $\overline{n}$, $\overline{n}^2$, σ

A high value of dispersion σ, reaching $\overline{n}^2$ (τ_{rec}) at the depth $t \sim \tau_{rec}(c = 1)$, is a characteristic of the particle dynamics in a rectangular potential well caused by radiative transitions between the levels of the transverse energy of motion

Further assume that at the initial time ($t = 0$) the beam incident on the crystal leads to the uniform filling of the entire set of the levels inside the potential well $\rho_n(0) = 1/n_{max}$ ($n \leq n_{max}$). A similar pattern of the level population is likely to occurs when a wide beam with the angular divergence $\theta \sim \theta_{cr}$ is incident on the crystal.

Consider non-radiative transitions. Solving the balance equation (10.62), we obtain the distributions $\rho_n(t)$ (Figure 16, dashed curves).

Figure 16. Distribution in a rectangular well for positrons ($E = 1.2$ GeV (a) and 20 GeV (b)) with account of multiple scattering (dashed curves) and

ignoring it (solid curves): 1 - initial; 2 - for the plate of thickness 10^{-2} *cm; 3 - for the plate of thickness* 10^{-1} *cm; 4 - for the plate of thickness* 0.5 *cm*

The diffusion coefficient $D = 0.16\ z_v z^{-2} D_{chaot}$ (the magnitude of the coefficient D is close to the experimental one). In Figure 16 it is seen that multiple scattering prevails over radiative transitions and fully determines the distribution function $\rho_n(t)$. It is easy to see that the function $\rho_n(t)$ are described by the solutions of an ordinary diffusion equation

$$\rho_n(t) \simeq \rho_n(0)\Phi(\zeta);\ \Phi(\zeta) = \frac{2}{\sqrt{2\pi}} \int_0^{\zeta} e^{-x^2/2} dx;$$
$$\zeta = \frac{n_{max} + 1 - n}{\sqrt{2Dt}}. \qquad (10.67)$$

Note that in view of the accepted model (the particle that has quitted the channel due to non-radiative transition $n \to n'$ ($n' > n_{max}$) never returns into the channel), the distribution $\rho_n(t)$ does not preserve time normalization: $\sum_{n=1}^{n_{max}} \rho_n(t) \leq 1$.

With the growth of particle energy the probability of non-radiative transitions $w^W_{nn'}$ falls rapidly, and the effect of radiative transitions on the change in the distribution function $\rho_n(t)$ is growing.

The process of radiative diminution of the transverse momentum of channeled particles discussed above should be distinguished from from the process of the damping of the amplitude of transverse oscillations of a channeled particle. The rates of relaxation of the transverse momentum and the oscillation amplitude are the same only in a harmonic well. In a steep-walled well relaxation of the transverse momentum is practically not accompanied by the reduction of the amplitude of transverse oscillations. From this follows that in spite of a possible decrease in the transverse momentum, the damping of the oscillation amplitude is not observed: in a steep-walled well it does not exist, and in a harmonic well the relaxation time is large.

10.4 Pair Production by γ-quanta in Crystals Under Channeling Conditions

The cross section for pair production by γ-quanta in crystals which takes account of possible channeling of electrons and positrons has the form

$$d\sigma = 2\pi\delta(\omega - E - E_1)\overline{|M'(\vec{p_1}, \vec{k}, \vec{p})|^2} \frac{d^3p d^3p_1}{8(2\pi)^6 \omega E E_1}\,, \qquad (10.68)$$

where

$$M'(\vec{p_1},\vec{k},\vec{p}) \equiv e\sqrt{4\pi}M'_{fi} = e\sqrt{4\pi}\int d^3r\psi^{(-)*}_{\vec{p}_1}(\vec{r})\vec{\alpha}\vec{e}e^{i\vec{k}\vec{r}}\psi^{(+)}_{-\vec{p}}(\vec{r})\,;$$

$\psi^{(-)}_{p_1}$ is the electron wave function; $\psi^{(+)}_{-p}$ is the wave function with negative energy $(-E)$. The asymptotics of this function should have a form of a diverging spherical wave. The positron wave function (formed from $\psi^{(+)*}_{-\vec{p}}$) will the asymptotics of a converging spherical wave as required for a final particle [Berestetsky *et al.* (1968)].

Similarly to the pair production in a shielded Coulomb potential, all characteristics of pair production in crystals may be found from the expression for the cross section of the bremsstrahlung process, using the transform $(E,\omega,\vec{e}^*) \to (-E,-\omega,\vec{e})$. In the expressions for I_1, $\vec{I}_2$, $\vec{I}_3$ the transmitted momentum $\vec{q} = \vec{p} + \vec{p}_1 - \vec{k}$. As a result, we have

$$M'_{fi} = 2\sqrt{\frac{E}{E_1}}w_1^+\left\{(E_1 - E)(\vec{g}\vec{e}) - i\omega\vec{\sigma}[\vec{g}\times\vec{e}]\right\}w\,. \tag{10.69}$$

Calculation of the trace appearing in (10.68) with due account of polarization of all the final particle gives [Skripka (1974)]

$$d\sigma = e^2\delta(\omega - E - E_1)\mathrm{Tr}\,\rho M'^{+}\rho_1 M'\frac{d^3pd^3p_1}{4(2\pi)^4\omega EE_1}\,, \tag{10.70}$$

where

$$\begin{aligned}
\mathrm{Tr}\,\rho M'^{+}\rho_1 M' = 4\frac{E}{E_1}\Big\{&\frac{\omega^2}{2}|\vec{g}|^2 - 2EE_1(1-\vec{\zeta}\vec{\zeta}_1)|\vec{g}\vec{e}|^2\\
&-\frac{\omega^2}{2}\mathtt{Re}\left\{|\vec{g}|^2\vec{\zeta}\vec{\zeta}_1 - 2(\vec{g}\vec{\zeta})(\vec{g}^*\vec{\zeta}_1)\right\} + \omega E_1\mathtt{Re}\Big\{\Big[|\vec{g}|^2\vec{\zeta}\vec{e}^*\\
&-2(\vec{g}\vec{e}^*)(\vec{g}^*\vec{\zeta})\Big](\vec{\zeta}\vec{e})\Big\} + \omega E\mathtt{Re}\Big\{\Big[|\vec{g}|^2\vec{\zeta}_1\vec{e}^*\\
&-2(\vec{g}\vec{e}^*)(\vec{g}^*\vec{\zeta}_1)\Big](\vec{\zeta}\vec{e})\Big\} + \frac{\omega}{2}|\vec{g}|^2(E\vec{\zeta} + E_1\vec{\zeta}_1)[i\vec{e}\times\vec{e}^*]\\
&-\frac{\omega}{2}\mathtt{Re}\Big\{|\vec{g}|^2(E_1\vec{\zeta} + E\vec{\zeta}_1)[i\vec{e}\times\vec{e}^*] - 2\Big(\vec{g}\Big(E_1\vec{\zeta}\\
&+E\vec{\zeta}_1\Big)\Big)(\vec{g}^*[i\vec{e}\times\vec{e}^*])\Big\} - \frac{\omega}{2}\Big[\omega(1-\vec{\zeta}\vec{\zeta}_1)[i\vec{e}\times\vec{e}^*]\\
&-(E-E_1)[[\vec{\zeta}\times\vec{\zeta}_1]\times[i\vec{e}\times\vec{e}^*]] + (E-E_1)(\vec{\zeta}-\vec{\zeta}_1)\\
&-2\mathtt{Re}\left\{\vec{e}^*((E\vec{\zeta} + E_1\vec{\zeta}_1)\vec{e})\right\}\Big][i\vec{g}\times\vec{g}^*]\Big\}\,.
\end{aligned}$$

Vector

$$\vec{g} = \vec{I}_{2\perp\vec{k}} + \frac{1}{2}\vec{n}_{\vec{p}\perp\vec{k}}I_1 + \frac{m}{2E}\vec{n}_{\parallel}I_1\,,$$

$$\vec{I}_2 = \frac{i}{2E}\int e^{-i\vec{q}\vec{r}}\varphi^{(-)*}_{\vec{p}_1}(\vec{r})\vec{\nabla}_r\varphi^{(+)}_{-\vec{p}}(\vec{r})d^3r\,,$$

$$I_1 = \int e^{-i\vec{q}\vec{r}}\varphi^{(-)*}_{\vec{p}_1}(\vec{r})\varphi^{(+)}_{-\vec{p}}(\vec{r})d^3r\,.$$

Due to mathematical equivalence of (10.70) and (3.27), integration of (10.70) over the variables of one of the particles (an electron or a positron) gives, e.g., for the number of produced positrons, the expression coinciding in form with (3.29):

$$\frac{d^2N_{e^+}}{dEd\Omega_{e^+}} = \frac{e^2E^2}{4\pi^2\omega}\mathtt{Re}\sum_{nfj} Q_{nj}e^{i\tilde{\Omega}_{nj}L}\left[\frac{1-\exp(iq_{zif}L)}{q_{zif}}\right]$$
$$\times\left[\frac{1-\exp(iq_{znf}L)}{q_{znf}}\right]\left\{\frac{\omega^2}{2E_1^2}\vec{g}_{nf}\vec{g}^*_{jf} - 2\frac{E}{E_1}(1-\vec{\zeta}\vec{\zeta}_1)\right.$$
$$\times(\vec{g}_{nf}\vec{e})(\vec{g}^*_{jf}\vec{e}) - \frac{\omega^2}{2E_1^2}\mathtt{Re}\left\{(\vec{g}_{nf}\vec{g}^*_{jf})(\vec{\zeta}\vec{\zeta}_1) - 2(\vec{g}_{nf}\vec{\zeta})(\vec{g}^*_{jf}\vec{\zeta}_1)\right\}$$
$$+\frac{\omega}{E_1}\mathtt{Re}\left\{[(\vec{g}_{nf}\vec{g}^*_{jf})(\vec{\zeta}\vec{e}^*) - 2(\vec{g}_{nf}\vec{e}^*)(\vec{g}^*_{jf}\vec{\zeta})](\vec{\zeta}_1\vec{e})\right\}$$
$$+\frac{\omega E}{E_1^2}\mathtt{Re}\left\{[(\vec{g}_{nf}\vec{g}^*_{jf})\vec{\zeta}_1\vec{e}^* - 2(\vec{g}_{nf}\vec{e}^*)(\vec{g}^*_{jf}\vec{\zeta}_1)](\vec{\zeta}\vec{e})\right\}$$
$$+\frac{\omega}{2E_1^2}(\vec{g}_{nf}\vec{g}^*_{jf})(E\vec{\zeta}+E_1\vec{\zeta}_1)[i\vec{e}\times\vec{e}^*]$$
$$-\frac{\omega}{2E_1^2}\mathtt{Re}\left\{(\vec{g}_{nf}\vec{g}^*_{jf})(E_1\vec{\zeta}+E\vec{\zeta}_1)[i\vec{e}\times\vec{e}^*] - 2\left(\vec{g}_{nf}\left(E_1\vec{\zeta}\right.\right.\right.$$
$$\left.\left.\left.+E\vec{\zeta}_1\right)\right)(\vec{g}^*_{jf}[i\vec{e}\times\vec{e}^*])\right\} - \frac{\omega}{2E_1^2}\left[\omega(1-\vec{\zeta}\vec{\zeta}_1)[i\vec{e}\times\vec{e}^*]\right.$$
$$-(E-E_1)[[\vec{\zeta}\times\vec{\zeta}_1]\times[i\vec{e}\times\vec{e}^*]] + (E-E_1)(\vec{\zeta}-\vec{\zeta}_1)$$
$$\left.\left.-2\mathtt{Re}\left\{\vec{e}^*((E\vec{\zeta}+E_1\vec{\zeta}_1)\vec{e})\right\}\right][i\vec{g}_{nf}\times\vec{g}^*_{jf}]\right\}\,. \tag{10.71}$$

In (10.71) the quantity

$$\vec{g}_{nf} \equiv \frac{1}{2E}(\vec{I}'_{2nf} + \vec{p}_{z\perp\vec{k}}I'_{1nf} + m\vec{n}_{\parallel}I'_{1nf})\,,$$
$$I'_{1nf} = N_\perp\int_S e^{i\vec{k}\vec{\rho}}\psi^*_{f\vec{\kappa}_1}(\vec{\rho})\psi_{n,-\vec{\kappa}}(\vec{\rho})d^2\rho\,,$$
$$\vec{I}'_{2nf} = iN_\perp\int_S e^{i\vec{k}\vec{\rho}}\psi^*_{f\vec{\kappa}_1}(\vec{\rho})\vec{\nabla}_\rho\psi_{n,-\vec{\kappa}}(\vec{\rho})d^2\rho\,, \tag{10.72}$$

the subscript f refers to electron states, subscripts n and j to positron ones; $\vec{\kappa}$ is obtained by reduction of the momentum $\vec{p}_\perp$ to the first Brillouin zone; $\vec{\kappa}_1$ is the same for the momentum $(\vec{k}_\perp - \vec{p}_\perp)$; $E_1 = \omega - E$; $Q_{nj} =$

$c_n(-\vec{p}_\perp)c_j^*(-\vec{p}_\perp)$ (see the definitions in Chapter III); $q_{znf} = p_{zn} + p_{1zf} - k_z$; $p_{zn} = \sqrt{p^2 - 2m\varepsilon_{n\kappa}(E)}$; $p_{1zf} = \sqrt{p_1^2 - 2m\varepsilon_{f\kappa_1}(E_1)}$. It should be pointed out that at pair production, as well as in the process of photon generation (see (3.29) the oscillations of $d^2N_{e^+}$, depending on the thickness L which enter into (10.71) cannot be ignored in the general case. This is due to the fact that there are a lot of closely spaced and even degenerate levels in the structure of transverse levels. For example, at axial channeling there is level degeneracy in the sign of the projection of the orbital moment, and the over-barrier states are, in fact, continuous. The contribution of such states at different energies has not been interrogated yet. For non-degenerate states the features of $1/q_{zjf}$ and $1/q_{znf}$ are not the same, and at $\tilde{\Omega}_{nj}L \gg 1$ the interference terms may be discarded.[9]

If we are not concerned about the polarization of the particles produced, we should assume that in (10.71) $\vec{\zeta}$ and $\vec{\zeta}_1$ are zero, and multiply the result obtained by 4. In consequence, for example, for non-degenerate states we have

$$\frac{d^2N_{e^+}}{dEd\Omega_{e^+}} = \frac{2e^2E^2L}{\pi\omega}\sum_{nf} Q_{nn}\delta(q_{znf})\left\{\frac{\omega^2}{2E_1^2}|\vec{g}_{nf}|^2 - 2\frac{E}{E_1}|\vec{g}_{nf}\vec{e}|^2 - \frac{\omega^2}{2E_1^2}[i\vec{e}\times\vec{e}^*][i\vec{g}_{nf}\times\vec{g}^*_{nf}]\right\}. \tag{10.73}$$

According to (10.71) and (10.73) the cross section of a channeled particle production depends on the photon polarization state. In particular, the cross section of production of a pair undergoing planar channeling is different for photons with polarization vector perpendicular (parallel) to the plane in question. Hence, for such γ-quanta a crystal exhibits dichroism. And flux of non-polarized γ-quanta incident on the crystal will get polarized.

The effect we have considered above will be fundamentally different from the effect of γ-quanta polarization by crystals discussed by Cabibbo (see, e.g., [Ter-Mikaelian (1969, 1972)]). The effect considered by Cabibbo corresponds in a crossed channel with coherent bremsstrahlung; the effect we have considered corresponds with the formation of polarized photons through radiative transitions between the levels of channeled particles. Moreover, e.g., at zero entrance angles with respect to a crystallographic plane, the effect considered by Cabibbo is zero, whereas in our case it

[9] Level degeneracy in the axial case affects attenuation of non-diagonal elements of the density matrix, which should be taken into account when analyzing the bursts of nuclear reactions discussed in [Kagan and Kononets (1970, 1973, 1974)].

is non-zero. From the Kramers-Kroning dispersion relations follows that alongside with absorption, the real parts of refractive indices will be different for different photon polarizations. In other word, the crystal for high-energy γ-quanta appears to be birefringent [Baryshevsky (1979f)].

The general formulae (3.27), (10.70) are also suitable for describing radiation and pair production processes in bent crystals. In particular, in (1.60) the terms proportional to ζ_1 describe the effect of radiation self-polarization of spin in bent crystals if the crystal thickness is less than the length of self-polarization, which was established in [Baryshevsky (1979c)]. The similarity of the formulas for bremsstrahlung and pair formation implies that in bent crystals even non-polarized γ-quanta will produce polarized electrons and positrons. Polarized particles, in turn, will produce polarized (having circular polarization) γ-quanta.

10.5 Nuclear Optics of Crystals at High Energies

We studied the formulae describing photon radiation in crystals in the Sommerfeld-Maue approximation for the wave function of electrons (positrons) and in the two-wave approximation for the wave function of γ-quanta produced. The existence of the rotation effect and particle self-polarization means that in thick crystals we must go beyond the scope of the Sommerfeld-Maue approximation and use the wave functions, which are the solution of the Dirac equation including the anomalous magnetic moment of the electron. Moreover, with the increasing frequency of the produced photon, when the wave length of a γ-quantum appears to be much shorter than the distance between atoms (nuclei), for wave functions of a photon $\vec{A}_{\vec{k}\vec{S}}(\vec{r})$ and other particles (e.g., neutrons) it is possible to apply the approximation similar to that used for describing electron and positron channeling (see §2, [Baryshevsky (1979f)]). When a γ-quantum moves at a small angle with respect to the planes (axes) of a single crystal, one may introduce the averaged over the plane (chain of atoms) dielectric permittivity. In this regard we may talk about the existence of channeling of γ-quanta and any of other particles, (e.g., neutrons, K^0-mesons) [Baryshevsky (1979f)].

It is worthy of mention that the crystal structure of a target is of impact even at very high energies (e.g., gigaelectronvolt and higher) of γ-quanta. In particular, due to channeling of pairs produced by γ-quanta, the intensity of transmitted γ-quanta demonstrates a pronounced dependence on the rotation angle of the crystal with respect to momentum of the incident beam (see also the previous section). Moreover, even at such high energies the crystal proves to be optically anisotropic (the birefringence phenomenon appears) [Baryshevsky (1979f)]. One may theoretically describe this effect, recalling that when a photon moves in an external field (electric, magnetic), due to the vacuum polarization by the field, the area occupied by the field is characterized by the dielectric permittivity tensor of the form [Baier *et al.* (1973)]

$$\varepsilon_{ij} = \delta_{ij} + 2g_1(\kappa)F_i^{(1)}F_j^{(1)} + 2g_2(\kappa)F_i^{(2)}F_j^{(2)},$$

where $\vec{F}^{(1)} = \vec{E}_\perp + [\vec{n}\vec{H}]$; $\vec{F}^{(2)} = [\vec{n}\vec{F}_1]$; $\vec{E}_\perp = \vec{E} - \vec{n}(\vec{n}\vec{E})$; $\vec{n} = \vec{k}/k$ (the definition of functions g_1 and g_2 see in [Baier *et al.* (1973)]). From this, when a γ-quantum moves, for example, at a small angle with respect to the crystallographic plane, the dielectric permittivity of a crystal for photons, whose polarization is parallel to the plane differs from that for γ-quanta, whose polarization is perpendicular to the crystallographic plane. As a result, birefringence arises. As seen from estimates, to transform a linearly polarized photon into a circularly polarized one, the crystal length of the about 1 cm is sufficient. As a consequence, it is definitely impossible to neglect the refraction effects in thick crystals even at high energies of emitted photons. The investigation of birefringence of γ-quanta in electric fields produced by crystallographic planes even now allows studying the effects of vacuum polarization by external fields.

Let us give a brief review of the theory of photon radiation in crystals, which is not restricted by the Sommerfeld-Maue approximation [Baryshevsky (1980c)]. To analyze the photon radiation in crystals, make use of the general quantum theory of reactions.

Let a plane wave of momentum $\vec{p}$ and energy E describing the primary particle be incident on a crystal with the volume V. It is well known that at the distances larger in comparison with the size of the objects, alongside with a primary wave, there are spherical waves describing secondary particles. To find the radiation cross section (the transition probability per unit time, the number of emitted photons) within the framework of the time-independent theory of reactions, it is necessary that the wave function of the primary particle exactly allowing for its interaction with the crystal and having the asymptotic of the diverging spherical wave ($\psi_p^{(+)}(\vec{r})$) should be taken as the initial wave function. The wave functions of final particles have the asymptotic of a converging wave $\psi_p^{(-)}(\vec{r}(A_{\mu\vec{k}}^{(-)}(\vec{r})$ for a photon). Using the general formulae [Berestetsky *et al.* (1968)], we have the following expression for the transition probability per unit time with photon emission ($\hbar = c = 1$):

$$dW = 2\pi\delta(E - E_1 - \omega)|M(\vec{p}_1, \vec{k}; \vec{p})^2| \frac{d^3p_1 d^3k}{8(2\pi)^6 L^3 E E_1 \omega}, \quad (10.74)$$

where $M(\vec{p}_1, \vec{k}; \vec{p}) = \int d^3r \overline{\psi_{\vec{p}_1}^{(-)}}(\vec{r})\gamma^\mu \psi_{\vec{p}}^{(+)}(A_{\mu\vec{k}}^{(-)*}(\vec{r})$; L^3 is the normalization volume; $\psi^{\pm}$ are the exact solutions of the Dirac equation; $(A_{\mu\vec{k}}^{(-)}$ are the exact solutions of Maxwell's equations.

Recall that according to the rules (see [Berestetsky *et al.* (1968)], p. 285) the stated wave functions do not include a factor of the type $1/\sqrt{2EL^3}$. It should be noticed that in view of [Baryshevsky (1976)] a particle in a medium is affected by an effective potential expresses in terms of the elastic scattering amplitude. This amplitude is a complex value. At high energies the contribution to the imaginary part of the amplitude comes from, e.g., bremsstrahlung and the pair production effect. For this reason at large energies the functions $\psi^{\pm}$ satisfy the Dirac equation with a complex potential.

When considering the reactions with polarized particles, it is helpful to represent the solutions of ψ and A in the form explicitly including final polarization of particles too. With this aim in view, write $\psi(\vec{r} = \hat{\psi}(\vec{r})u$ and $A_\mu = B_\mu^\nu e_\nu$, where u is the bispinor characterizing the polarized state of an electron (positron) in a plane wave outside the crystal; e_ν - is the photon polarization vector in the plane wave outside the crystal. As a result the matrix element

$$M = \bar{u}_{\vec{p}_1} G^\nu u_{\vec{p}} e_\nu;\ G^\nu = \int d^3r \overline{\hat{\psi}}_{\vec{p}_1}^{(-)}(\vec{r})\gamma^\mu \hat{\psi}_p^{(+)}(\vec{r}) B_\mu^\nu(\vec{r}). \quad (10.75)$$

Upon introducing polarization density matrices of the initial ρ and final ρ_1 electrons and a photon $\rho_{\mu\nu}$ the squared matrix element entering into (10.74) is:

$$\overline{|M|^2} = sp\, \rho_1 G^\mu \rho \bar{G}^\nu \rho_{\mu\nu} = T^{\mu\nu}\rho_{\mu\nu},$$
$$T^{\mu\nu} = sp\, \rho_1 G^\mu \rho G^\nu \quad (10.76)$$

($T^{\mu\nu}$ is the linear function of the polarization vectors of the initial $\vec{\zeta}$ and final $\vec{\zeta}_1$ electrons (positrons)). Therefore the explicit form of T as a function of $\vec{\zeta}$ and $\vec{\zeta}_1$ may be written as follows:

$$T^{\mu\nu} = \alpha^{\mu\nu} + \vec{\beta}^{\mu\nu}\vec{\zeta} + \vec{\beta}_1^{\mu\nu}\vec{\zeta}_1 + \gamma_{il}^{\mu\nu}\zeta_i\zeta_{1l}. \quad (10.77)$$

Carrying out further analysis, we take into account that integration in (10.75) is made over the he range with linear dimensions exceeding the linear dimensions of a crystal by only the magnitude of the vacuum coherent length of radiation. That is why, when considering the radiation process in a crystal whose lateral dimensions are much larger than its thickness, in order to find the wave functions, one may use the wave functions describing scattering of a plane wave by a crystal plate of finite dimensions. According to (1.1), (1.2) in this case the time-independent wave function of a particle (photon) in the area occupied by the crystal is defined by the superposition of the Bloch functions. Since the Bloch functions may be represented as a superposition of plane waves (their explicit form see in (1.2)), integration in (10.75) with respect to the coordinates in the plane parallel to the crystal surface leads to the δ-function describing the law of conservation of the component of a transmitted momentum, which is parallel to the crystal surface. The stated δ-function together with the energy δ-function enables performing in (10.77) explicit integration with respect to $\vec{p}_1$ (or $\vec{k}$). As a result, we obtain a spectral-angular distribution of emitted photons (or ejected electrons).

From (10.75)-(10.77) follows that all the amplitudes α, β, γ appearing in (10.77) will be the squared absolute values of the superposition of the functions:

$$\sum c_{nn'} \frac{e^{-iq_{znn'}l} - 1}{q_{znn'}},$$

where $q_{znn'}$ is the longitudinal momentum transmitted to the crystal. The stated superpositions oscillate with the crystal thickness.

According to [Baryshevsky (1976)], when particles and γ-quanta move in crystals, a number of polarization phenomena arise (multi-frequency change of polarization characteristics of electrons, positrons and γ-quanta, depending on thickness; the effect of anomalous transmission depending on the external field frequency). All these phenomena also manifest themselves in the case under study. In other words, $T^{\mu\nu}$ and, hence, the intensity and polarization characteristics of emitted photons (electrons) are the oscillating functions of the energy of particles, crystal thickness and the frequency of the variable external field (sound, electromagnetic) imposed on the crystal.

10.6 Surface Channeling of Charged Particles

The experiments [Mashkova *et al.* (1970, 1971); Skripka (1974)] studying refraction of ion beams from crystal surfaces revealed sharp anomalies in the number of particles refracted by the crystal at rotation of the crystal surface about the axes perpendicular to it. The quantum mechanical explanation of this effect is given below [Baryshevskii and Dubovskaya (1977c)].

Let a particle beam be incident on the surface of the crystal which occupies the half-space area $z > 0$ at the glancing angle α. Choose the x-axis perpendicular to a certain family of crystallographic planes, and the y-axis parallel to the stated family of planes. Suppose that particles are incident at a small angle β with respect to the stated planes. Then the general view of the wave reflected from the crystal surface can be written as follows:

$$\psi = e^{i\vec{k}_0\vec{r}} + \sum_{\tau} A_\tau e^{i(k_{0x}+2\pi\tau)x} e^{ik_{0y}y} e^{-ik_z(\tau)z}, \tag{10.78}$$

where $k_z(\tau) = (k_0^2 - (k_{0x} + 2\pi\tau)^2 - k_{0y})^{1/2} = (k_{0z}^2 + k_{0x}^2 - (k_{0x} + 2\pi\tau)^2)^{1/2}$ is found from the condition of the wave energy conservation (preservation) at elastic scattering.

When deriving (10.78), it was taken into account that, due to the periodicity of the potential of the selected family of planes along the x-axis, the parallel x-component of the momentum of the wave reflected from the crystal may only differ from the initial value of k_{0x} by the reciprocal lattice vector $2\pi\vec{\tau}$.

The general solution of the Schrodinger equation describing the particle motion inside the crystal in the case under study has the form

$$\psi = \sum_{nk_x} c_n(k_x)\psi_{nk_x}(x) e^{ik_y y} e^{i\kappa_{zn}(k_x)z}, \tag{10.79}$$

where $\kappa_n(k_x) = (k_{0z}^2 + k_{0x}^2 - q_n^2(k_x))^{1/2}$; $q_n^2(k_x) = \frac{2m}{\hbar^2} E_n(k_x)$; $E_n(k_x)$ is the particle energy in a periodic one-dimensional potential of the family of planes in question in the range n as a function of the wave number k_x; $\psi_{nk_x}(x) = \frac{1}{\sqrt{N_x}} e^{ikx} u_{nk}(x)$ is the Bloch function. Unlike the case of mirror reflection of neutrons and γ-quanta under diffraction conditions [Baryshevsky (1976)], in the the case of diffraction of charged particles we consider the two-wave approximation is not applicable.

Joining (ref(35.1) and (10.79) at the crystal boundary, we may obtain the following expressions for the amplitudes of refracted waves A_τ and coefficients $c_n(k_x)$:

$$A_{\tau=0} = \sum_n \frac{k_{0z} - \kappa_n(k_x)}{k_{0z} + \kappa_n(k_x) + \delta_n\left(\frac{2\pi l}{a}\right)} \times \frac{1}{\Omega} \left| W_n\left(\frac{2\pi l}{a}\right)\right|^2 g_n\left(\frac{2\pi l}{a}\right); \tag{10.80}$$

$$A_{\tau\neq 0} = \sum_n \frac{2k_{0z}}{k_{0z} + \kappa_n(k_x) + \delta_n\left(\frac{2\pi l}{a}\right)}$$
$$\times \frac{1}{\Omega} W_n\left(2\pi\tau + \frac{2\pi l}{a}\right) W_n^*\left(\frac{2\pi l}{a}\right) g_n\left(\frac{2\pi l}{a}\right); \qquad (10.81)$$

$$c_n\left(\frac{2\pi l}{a}\right) = \frac{2\sqrt{N_x} k_{0z} W_n^*\left(\frac{2\pi l}{a}\right)}{k_{0z} + \kappa_n(k_x) + \delta_n\left(\frac{2\pi l}{a}\right)} g_n\left(\frac{2\pi l}{a}\right);$$
$$c_n(k_x \neq k_{0x}) = 0, \qquad (10.82)$$

where $g_n\left(\frac{2\pi l}{a}\right)$ satisfies the set of equations of the form

$$g_n\left(\frac{2\pi l}{a}\right) = 1 + \sum_{n'\neq n}\left[\sum_{\tau\neq 0}(k_{0z} - k_z(\tau))\right.$$
$$\times \frac{1}{\Omega} W_n^*\left(2\pi\tau + \frac{2\pi l}{a}\right) W_{n'}\left(2\pi\tau + \frac{2\pi l}{a}\right)$$
$$\times \frac{W_{n'*}\left(\frac{2\pi l}{a}\right)}{W_n^*\left(\frac{2\pi l}{a}\right)} \frac{g_{n'}\left(\frac{2\pi l}{a}\right)}{k_{0z} + \kappa_{n'}(k_x) + \delta_{n'}\left(\frac{2\pi l}{a}\right)};$$
$$\delta_n\left(\frac{2\pi l}{a}\right) \equiv \sum_{\tau\neq 0}(k_z(\tau) - k_{0z})\frac{1}{\Omega}\left|W_n\left(2\pi\tau + \frac{2\pi l}{a}\right)\right|^2; \qquad (10.83)$$

Integration is made over the unit cell volume Ω; l is the integer part of $\frac{k_{0x}a}{2\pi}$.;

$$W_n(2\pi\tau) = \int e^{-i2\pi\tau x} u_{nk_{0x}}(x)dx.$$

To clarify the structure of expressions (10.80), (10.81), recall that the ordinary amplitude of a mirror reflected wave has the form

$$B = \frac{k_{0z} - k_z}{k_{0z} + k_z}, \qquad (10.84)$$

where $k_z = k_{0z}n$; n is the refractive index.

Comparison of (10.80) and (10.84) shows that the amplitude of a mirror reflected wave A_0 under channeling conditions can be represented as a superposition of the amplitudes describing mirror reflection from the medium with the "refractive index" $n = \kappa_n(k_x)/k_{oz}$ which depends on the number of the zone where the particle incident on the crystal is captured in it.

Consider the dependence of the reflected wave amplitude on the gliding angle α and the azimuth angle β. First note that with the change of the angle β, the magnitude of the component of the particle momentum k_{0x} lso changes, and, hence, the magnitude of the number l. On the other hand, the coefficients $W_n(2\pi l/a)$ as a function of the number of the range n have the maximum distribution at $n = 2l$, which falls rapidly with the increase in the difference $n - 2l$, with the distribution $W_n(2\pi l/a)$ getting sharper at large values of the number l. The features mentioned above result in the fact that at β exceeding a certain critical angle which has the same order of magnitude as the Lindhard angle, $g_n(2\pi l/a) \to 1$, $\delta_n(2\pi l/a) \to 0$. As a consequence, $A_0 \to B$, $A_{\tau\neq 0} \to 0$, and we go over to a known mirror reflection pattern.

Let now the glancing angle of ions α be much greater than the total mirror reflection angle $\vartheta_{cr} = \sqrt{u_{mean}/E}$ (u_{mean} is the mean energy of particle-crystal interaction). In this case , if the angle β is greater than the Lindhard angle, the intensity of reflected particles is small. However, at $\beta \to 0$, one may see (see formulae (4.25), (4.26) and the above mentioned features of the coefficients $W_n(2\pi l/a)$) that though the amplitude A_0 remains small, the amplitudes $A_{\tau\neq 0}$ ($A_{\tau\neq 0} \sim W_0(\tau)W_0^*(0)$) become greater. As a consequence, one may observe the increase in the intensity of reflected particles (as $2\pi\tau/k \ll 1$, for Ar with $E = 30$ keV $k \sim 10^{12}\, cm^{-1}$, the particles with $\tau \neq 0$ move practically in the plane of mirror reflection). The situation is different when ions fall on the crystal at gliding angle comparable with the angle of total mirror reflection. In this case at β greater than the Lindhard angle the total mirror reflection of the wave is observed, i.e., $A_0 \to 1$, and $A_{\tau\neq 0} \to 0$. At the same time at $\beta = 0$ the amplitude A_0 is small, as the here low energy levels for which vector $\kappa_n \sim k_{0z}$ play the leading part. The amplitude $A_{\tau\neq 0}$ also vanishes at $k_{0z} \to 0$, due to the limitation of $\delta_n(2\pi l/a)$. As a result at $\beta = 0$ the minimum in the intensity of reflected particles should be observed. The qualitative picture given here is in good agreement with the experimental results [Mashkova *et al.* (1970)]. Indeed, the total mirror reflection angle for ions A_τ with $E = 30$ keV and the crystal of Cu is about 4°. In the experiment at the gliding angle $\alpha = 10 - 15°$ and the azimuth angle $\beta = 0$ the maximum intensity of scattered particles was observed, at the same time at $\alpha = 5°$ and $\beta = 0$ the minimum was observed. It should be emphasized that the phenomenon discussed is of the general character and it should occur for surface channeling of light particles (electrons and positrons). It is natural that electrons and positrons undergoing surface channeling emit photons due to transitions between the levels (ranges) of transverse motion. Induced transitions caused by a polarized electromagnetic wave result in particle polarization (compare (6.2)). Quantum modulation of a reflected beam also emerges.

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