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Berry phases in an electric-dipole-moment experiment in an all-electric storage ring

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OUTLINE

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- Spin behavior and Berry phases in an electric-dipole-moment experiment in an all-electric storage ring
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Berry (geometric) phases

In classical and quantum mechanics, the geometric phase, Pancharatnam–Berry phase (named after S. Pancharatnam and M. Berry) or most commonly Berry phase, is a phase difference acquired over the course of a cycle, when a system is subjected to cyclic adiabatic processes, which results from the geometrical properties of the parameter space of the Hamiltonian.

Pancharatnam, S. Generalized theory of interference, and its applications. Part I. Coherent pencils, Proc. Indian Acad. Sci. (1956) **44:** 247**.**

Berry, M. Quantal Phase Factors Accompanying Adiabatic Changes, Proc. Roy. Soc. A (1984) **392:** 45**.**

While one could continue to develop this formalism, it is useful to give an example wherein the formalism can be applied. Perhaps the simplest is that of a spin-$\frac{1}{2}$ particle in an external magnetic field "$\mathbf{R}(t)$" for which the relevant Hamiltonian is[1]

$$h(\mathbf{R}(t)) = -(\mu/2)\,\boldsymbol{\sigma}\cdot\mathbf{R}(t)$$

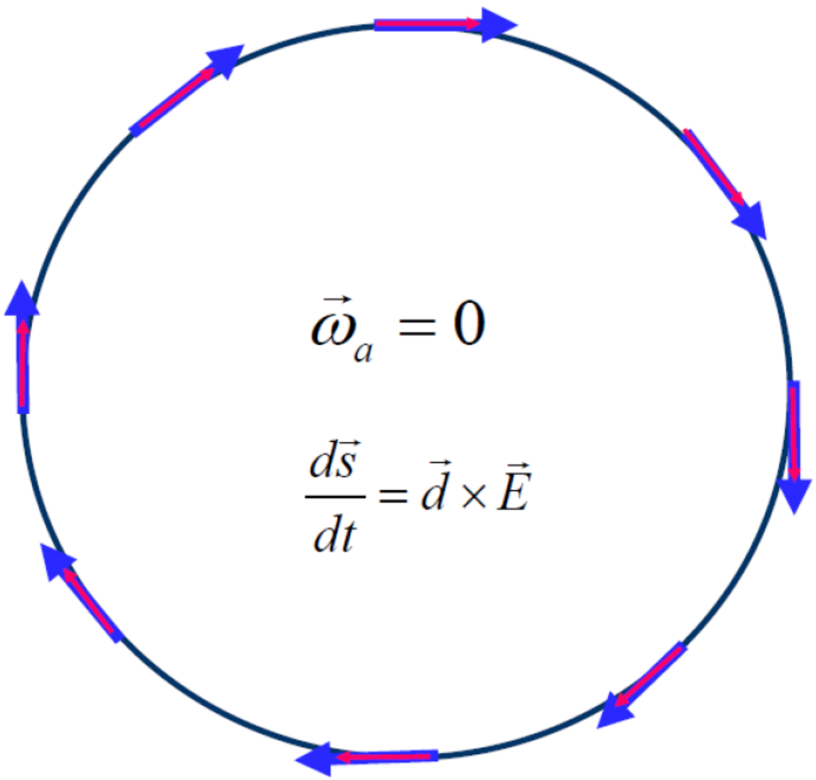
$$= -\frac{\mu}{2}\begin{pmatrix} Z(t) & X(t)-iY(t) \\ X(t)+iY(t) & -Z(t) \end{pmatrix}.$$

Holstein, B.R. The adiabatic theorem and Berry's phase, Am. J. Phys. (1989) **57:** 1079**.**

Berry phases in storage-ring electric-dipole-moment experiments

A Proposal to Measure the Proton Electric Dipole Moment with 10^{-29} $e\cdot$cm Sensitivity

by the Storage Ring EDM Collaboration



$$\vec{\omega}_a = 0$$

$$\frac{d\vec{s}}{dt} = \vec{d} \times \vec{E}$$

October 2011

frozen spin method

Storage Ring EDM Collaboration

V. Anastassopoulos[18], D. Babusci[10], M. Bai[3], S. Baessler[23], M. Berz[17], M. Blaskiewicz[3], K. Brown[3], P. Cameron[3], G. Daskalakis[6], N. D' Imperio[3], M.E. Emirhan[13], F. Esser[24], G. Fanourakis[6], A. Fedotov[3], A. Ferrari[25], W. Fischer[3], T. Geralis[6], Y. Giomataris[21], F. Gonnella[20], M. Gross Perdekamp[11], R. Gupta[3], G. Guidoboni[8], S. Haciomeroglu[13,3], Y. Haritantis[18], G. Hoffstaetter[5], H. Huang[3], M. Incagli[19], D. Kawall[16], B. Khazin[4], I.B. Khriplovich[4], I.A. Koop[4], T. Laopoulos[1], R. Larsen[3], D.M. Lazarus[3], A. Lehrach[9], P. Lenisa[8], P. Levi Sandri[10], F. Lin[3], A.U. Luccio[3], A. Lyapin[15], W.W. MacKay[3], R. Maier[9], K. Makino[17], N. Malitsky[3], W. Marciano[3], S. Martin[26], W. Meng[3], F. Meot[3], R. Messi[20], D. Moricciani[20], W.M. Morse[3], S.K. Nayak[3], Y.F. Orlov[5], C.S. Ozben[13], A. Pesce[8], V. Ptitsyn[3], B. Parker[3], P. Pile[3], V. Polychronakos[3], B. Podobedov[3], D. Raparia[3], F. Rathmann[9], S. Redin[4], S. Rescia[3], G. Ruoso[14], T. Russo[3], N. Saito[22], J. Seele[28], Y.K. Semertzidis[3,*], Yu. Shatunov[4], V. Shemelin[5], A. Sidorin[7], A. Silenko[2], N. Simos[3], S. Siskos[1], A. Stahl[27], E.J. Stephenson[12], H. Stroeher[9], J. Talman[3], R.M. Talman[5], P. Thieberger[3], N. Tsoupas[3], Y. Valdau[9], G. Venanzoni[10], K. Vetter[3], S. Vlassis[18], G. Zavattini[8], A. Zelenski[3], K. Zioutas[18]

Frozen spin method:

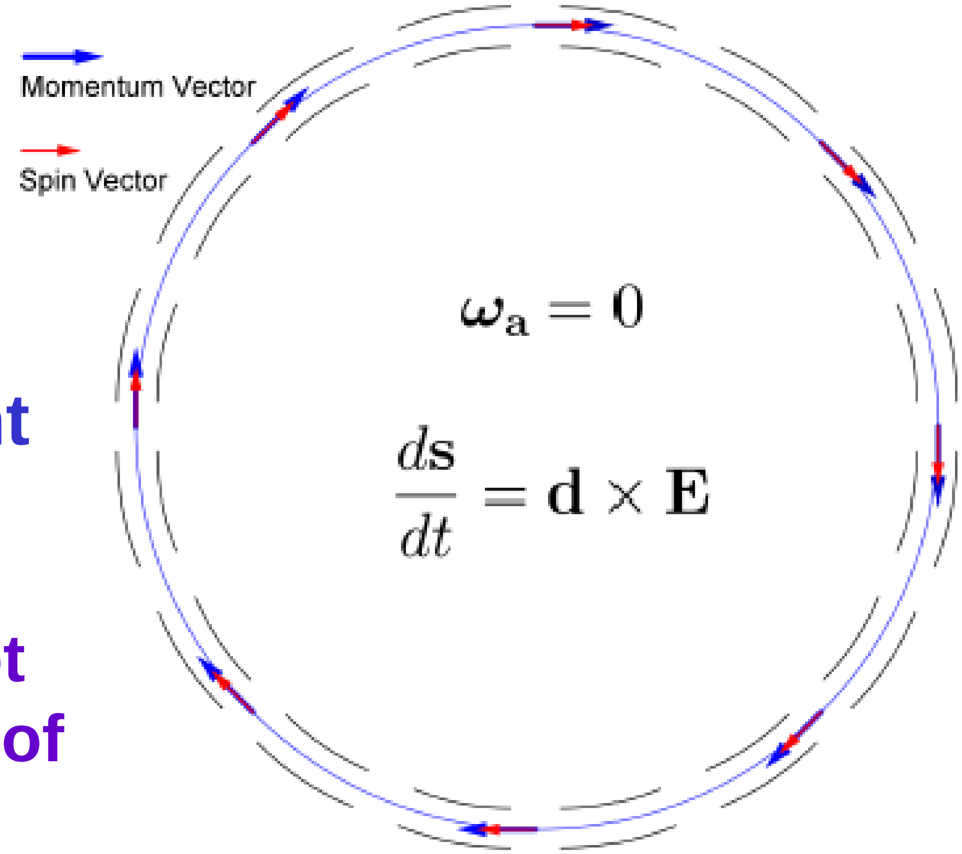
$$\vec{\omega}_a = \frac{e}{m}\left[a\vec{B} + \left(a - \left(\frac{m}{p}\right)^2\right)\vec{\beta}\times\vec{E}\right]$$

Use a radial E_r-field to cancel the g-2 precession

$a=(m/p)^2$, $B=0$:

Proton EDM experiment in an all-electric ring

The proton spins do not precess in the presence of *E-fields alone*



Geometrical phases in storage-ring EDM experiments are caused by the noncommutativity of spin rotations in three dimensions. As a result, alternating spin rotations about two axes can produce a false EDM signal.

Geometrical phases (GP) (e.g., Berry's phase), a major systematic error in neutron EDM experiments.	The GP has a sign depending on the ring azimuthal location. We need two polarimeters to eliminate the effect of the lowest-order GP.	The spurious *B*-fields need to be below μG level. The *E*-field plates need to be aligned to about 30 μm and the *E*-field plane needs to be defined to μrad level. See sub-section 11.3.2 for details.	The specs are easily attainable with present technology. Having two polarimeter locations around the ring provides an extra level of security against this systematic error.

When the focusing system is electric, the main systematic error is a net radial *B-field* around the ring, whereas the main one when magnetic focusing is used is a net vertical (out of plane) *E-field.* These are first-order systematic errors which do not belong to geometric phases.

Spin behavior and Berry phases in an electric-dipole-moment experiment in an all-electric storage ring

The angular velocity of the spin motion in the all-electric storage ring with small perturbations of the azimuthal particle momentum is given by

$$\mathbf{\Omega} = \frac{2e}{mc} G^2 \gamma \left(\boldsymbol{\beta} \times \mathbf{E} \right) \Delta\gamma - \frac{e\eta}{2mc} \mathbf{E},$$

$\Delta\gamma$ **is the deviation from the exact Lorentz factor. The main systematic error caused by the vertical electric field consists in local spin rotations about the radial axis:**

$$\mathbf{E}^{(v)} = \pm \left| \mathbf{E}^{(v)} \right| \mathbf{e}_z \equiv \pm E^{(v)} \mathbf{e}_z \,.$$

Let us consider the simplest case leading to this systematic error. This case is shown in Figs. 1, 2. The main electric field is always antiparallel to the radial axis. Regions with nonzero $\Delta\gamma$ **and** $E^{(v)}$ **bounded by the points A, B, C, and D alternate. We will analyze four cases when both the momentum and the spin are collinear to the azimuthal direction in one of these starting points.**

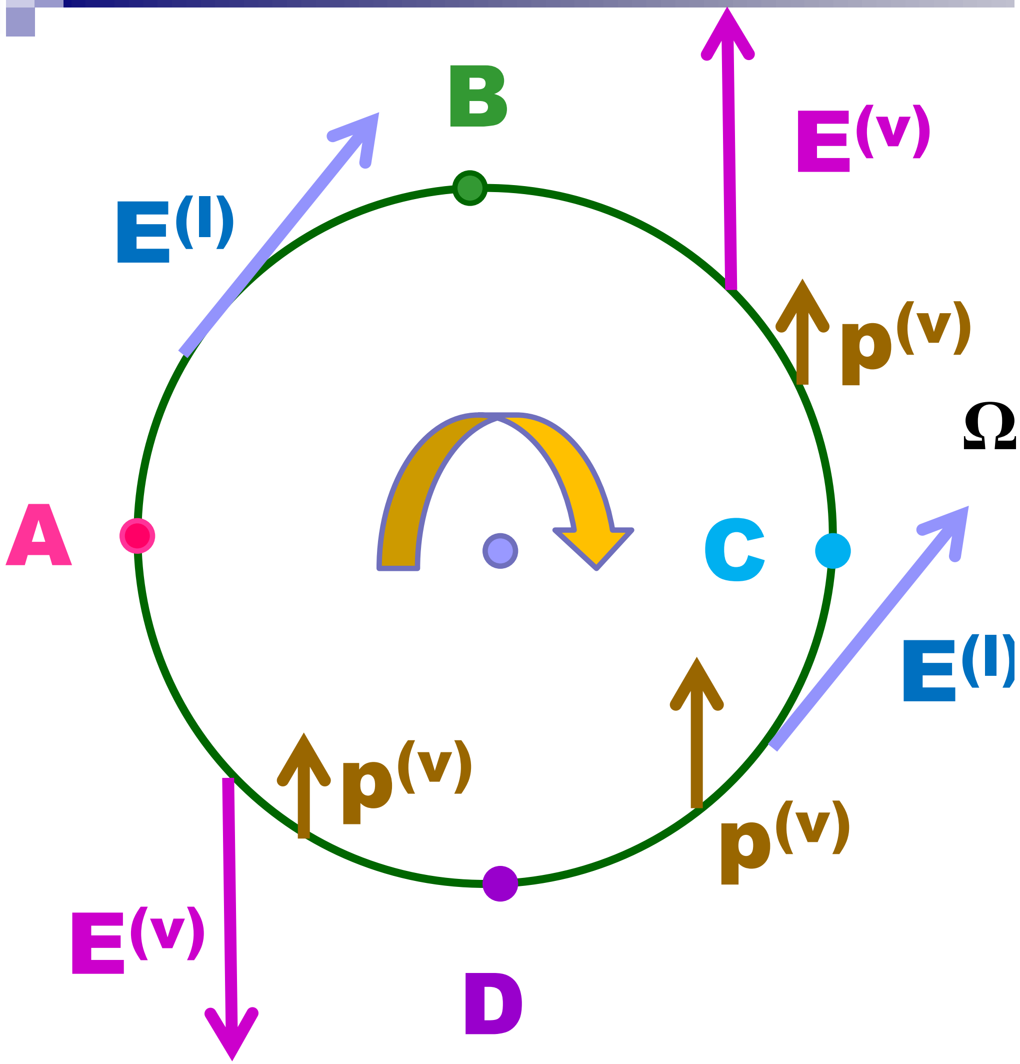


Figure 1. Clockwise beam motion.

The starting point is A. At the points B and C, $\Delta\gamma>0$. The angular velocity of the spin motion on the path BC (without the EDM effect) is

$$\mathbf{\Omega} = \frac{2e}{mc} G^2 \gamma \left(\boldsymbol{\beta} \times \mathbf{E} \right) \Delta\gamma - \frac{e\eta}{2mc} \mathbf{E},$$

At the points D and A, $\Delta\gamma>0$. On the path CA, $\mathbf{\Omega}_{CA}=0$. As a result, the average angular velocity is given by

$$\mathbf{\Omega} = \frac{1}{4} \mathbf{\Omega}_{BC} = -\alpha \mathbf{e}_z,$$

$$\alpha = \frac{e}{2mc} G^2 \beta\gamma E^{(v)} \left| \Delta\gamma \right|.$$

This is a systematic error imitating the EDM effect.

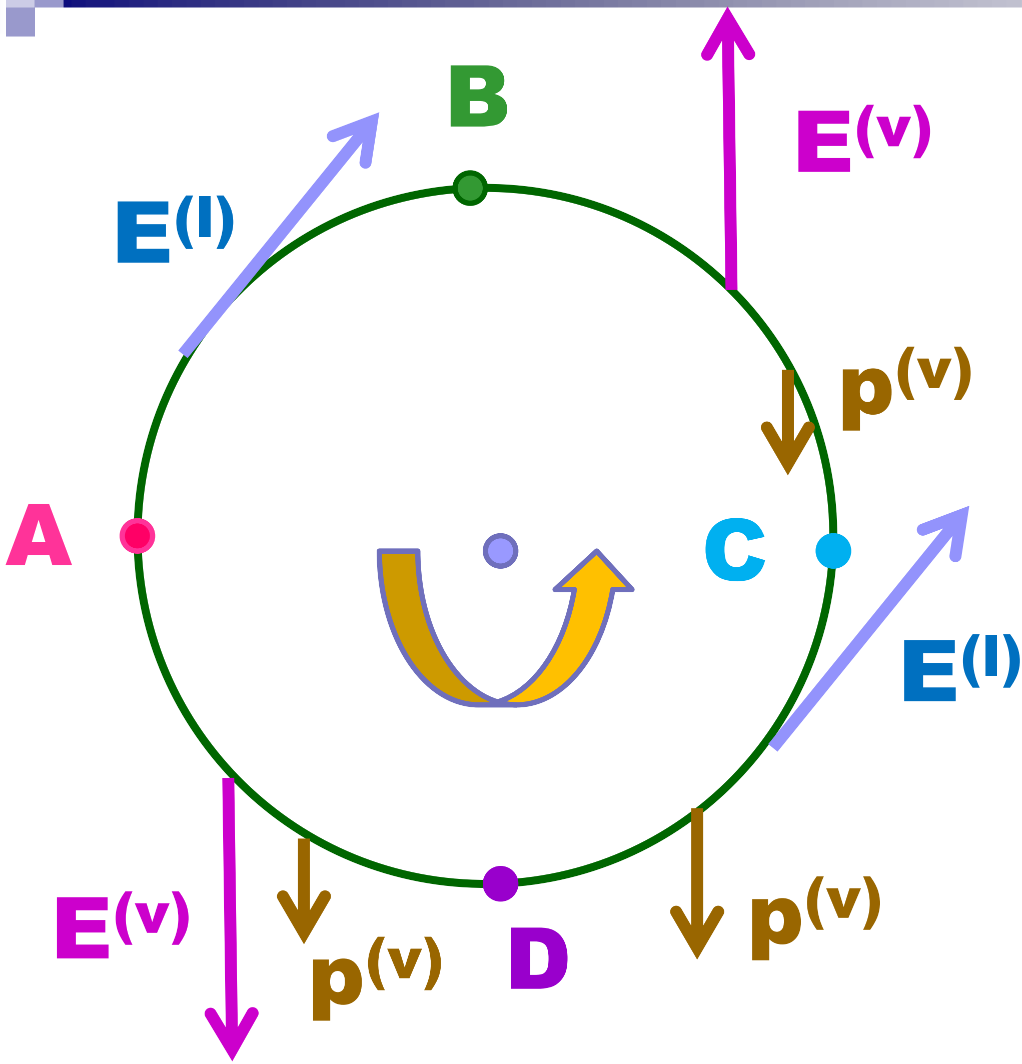


Figure 2. Counterclockwise beam motion.

The starting point is A. At the points D and C, $\Delta\gamma=0$ and $\Delta\gamma<0$, respectively. The false effect is only on the path CB: $\mathbf{\Omega}_{CB}=4\alpha\mathbf{e}_z$. On the path BA, $\mathbf{\Omega}_{BA}=0$. As a result, $\mathbf{\Omega}=\alpha\mathbf{e}_z$.

This is a systematic error imitating the EDM effect.

For clockwise and counterclockwise beams, the false effect does not depend on the starting point.

The systematic error due to the geometric phases has the different signs for the two directions of the beam rotation. The EDM effect remains the same in this case. Thus, the considered systematic error can be canceled with CW and CCW beams.

Spin rotation about the longitudinal direction

There exists also an effect which does not influence spin dynamics of longitudinally polarized beams in all-electric storage rings. This is the spin rotation about the azimuthal axis which is also conditioned by the longitudinal and vertical electric fields. We suppose that the spin precession about the vertical axis is vanished.

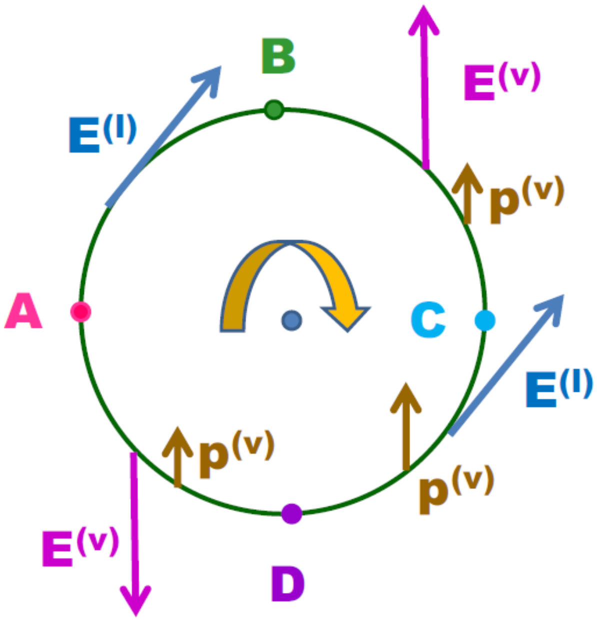


Figure 1. Clockwise beam motion.

The vertical electric field in the sections BC and DA changes the vertical component of the particle momentum. The maximum value of this component is equal to

$$p_{\max}^{(v)}=\frac{eR}{c\beta}\int\limits_{\phi_1}^{\phi_2}E^{(v)}d\phi=\frac{\pi eR}{2c\beta}E^{(v)}.$$

The vertical component of the particle velocity is given by

$$\beta_{\max}^{(v)}=\frac{\pi eR}{2mc^2\beta\gamma}E^{(v)}.$$

When *E* is the main electric field,

$$E=\frac{mc^2\beta^2\gamma}{eR},\qquad \beta_{\max}^{(v)}E=\frac{\pi}{2}\beta E^{(v)}.$$

We analyze the same cases which have been considered in the precedent section. In the current section, we calculate only the angular velocity of the spin motion acting on the radial spin component.

Clockwise beam motion. The momentum distribution shown at this figure is valid when the starting points are A and B.

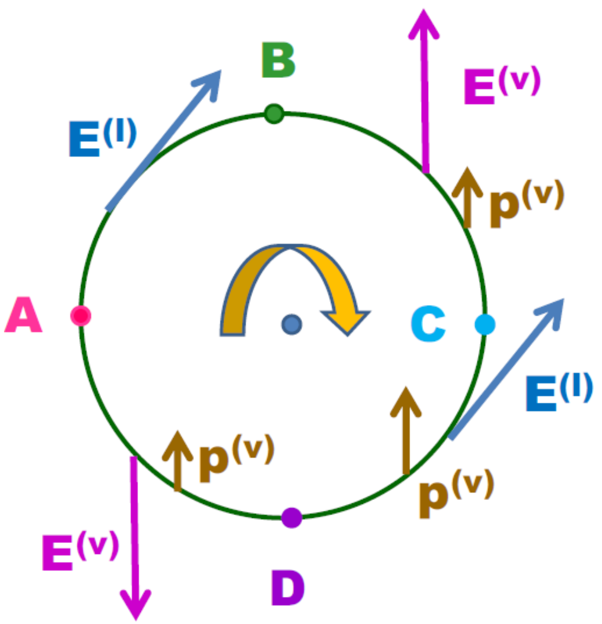


Figure 1. Clockwise beam motion.

The starting point is A. At the points B and C, $\Delta\gamma>0$. The angular velocity of the spin motion on this path reads

$$\boldsymbol{\Omega}_{BC}=-\frac{e}{mc}G^2\beta^{(v)}_{\max}\gamma E\Delta\gamma_{\max}\mathbf{e}_\phi=-\pi\alpha\mathbf{e}_\phi.$$

At the points D and A, $\Delta\gamma=0$. On the path CD, the average value of $\Delta\gamma$ is equal to $\Delta\gamma_{max}/2$. As a result, the angular velocity of the spin motion on this path is given by $\boldsymbol{\Omega}_{CD}=-\pi\alpha\mathbf{e}_\phi$. The average angular velocity of the spin motion takes the form

$$\boldsymbol{\Omega}=-\frac{\pi}{2}\alpha\mathbf{e}_\phi.$$

When the starting point is C, the result is the same. However, the sign is changed when the starting points are B and D:

$$\mathbf{\Omega} = \frac{\pi}{2}\alpha \mathbf{e}_{\phi}.$$

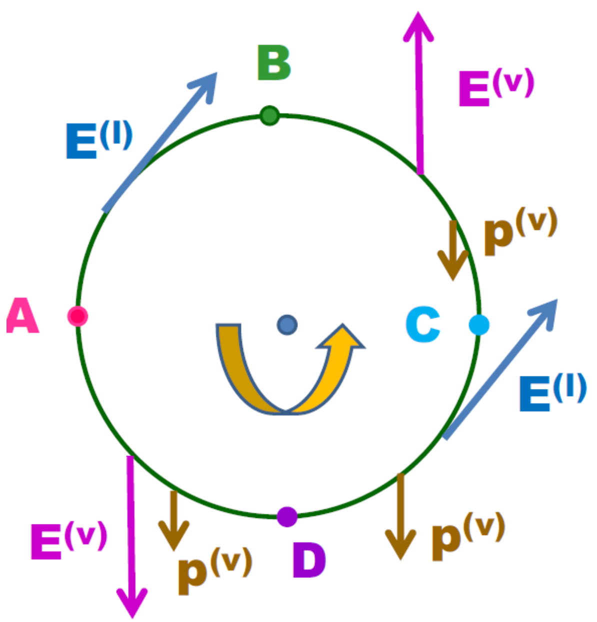


Figure 2. Counterclockwise beam motion.

Counterclockwise beam motion. The momentum distribution shown at this figure is valid when the starting points are A and B.

The starting point is A. At the points C and B, $\Delta\gamma=\Delta\gamma_{max}$. The average value of $\Delta\gamma$ on the path DC is equal to $\Delta\gamma_{max}/2$. The angular velocity of the spin motion on this path reads $\mathbf{\Omega}_{DC}=\pi\alpha\mathbf{e}_{\phi}$. On the path CB, $\Delta\gamma=\Delta\gamma_{max}$ $=const.$ The angular velocity of the spin motion on this path is given by $\mathbf{\Omega}_{CB}=\pi\alpha\mathbf{e}_{\phi}$. The average angular velocity of the spin motion takes the form

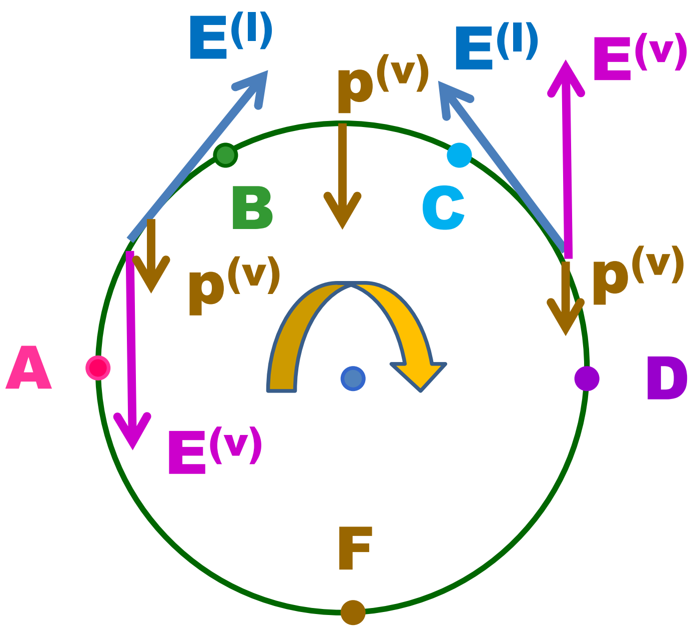
$$\mathbf{\Omega} = \frac{\pi}{2}\alpha \mathbf{e}_{\phi}.$$

When the starting point is C, the result is the same. However, the sign is changed when the starting points are B and D:

$$\mathbf{\Omega} = -\frac{\pi}{2}\alpha\mathbf{e}_{\phi}.$$

Thus, the longitudinal component of the angular velocity of the spin precession has different signs for the CW and CCW directions of the beam rotation. Therefore, this systematic error can be canceled with CW and CCW beams. Even if the spin precession about the vertical axis is not vanished, this property allows one to eliminate the systematic errors caused by the geometric phases. The azimuthal component of **Ω** contrary to the radial one, depends on the starting point where the spin orientation is perfect.

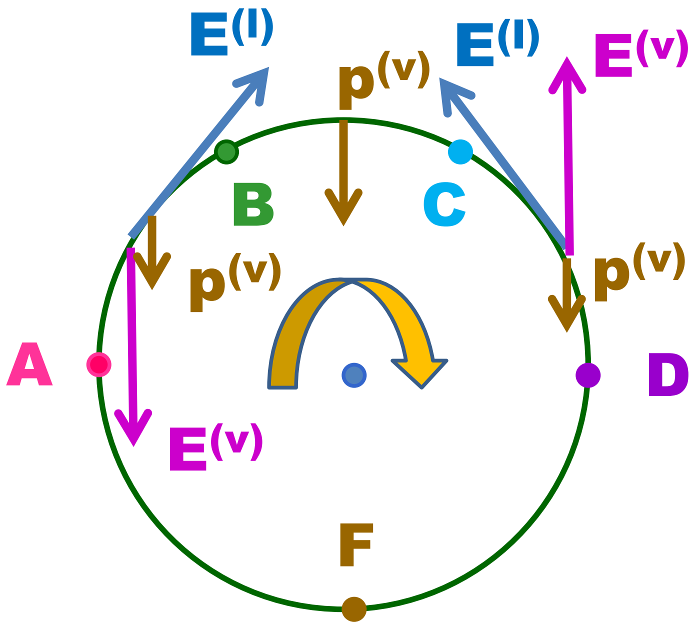
Spin evolution in joined longitudinal and vertical electric fields



In this section, we consider the spin evolution on condition that the longitudinal and vertical electric fields are joined in the same sections without any alternation. This consideration shows that only an alternation of these fields leads to a systematic error mimicking the EDM signal.

Figure 3. Spin motion in joined longitudinal and vertical electric fields.

Let us consider the simple case shown in Fig. 3. In this case, the longitudinal and vertical electric fields are constant into the paths AB and CD and the fields averaged over the ring are equal to zero.



The starting point is F. Since at the points A and D and the beam rotates CW, the average value of *Δγ* on the path AB is positive. The average value of *Δγ* on the path CD is the same. However, the values of $E^{(v)}$ are opposite.

Figure 3. Spin motion in joined longitudinal and vertical electric fields.

As a result, the total spin rotation is equal to zero. The same situation takes place when the beam rotates CCW. Thus, the systematic error due to geometric phases is absent when the longitudinal and vertical electric fields are joined in the same sections without any alternation.

The spin rotation about the azimuthal axis takes place. The angular velocity of the spin rotation has opposite directions for the CW and CCW beams.

Summary

- **Local longitudinal and vertical electric fields in an all-electric storage ring may lead to the systematic errors caused by the geometric phases. This takes place when sections with the longitudinal and vertical electric fields alternate.**
- **While expected systematic errors are not small, they can be canceled with CW and CCW beams.**
- **The sign of the azimuthal component of Ω depends on the starting point (where the spin orientation is perfect) while the radial component of Ω keeps its value for each starting point.**
- **When the longitudinal and vertical electric fields are joined in the same sections without any alternation, the systematic error due to the geometric phases does not appear but another systematic effect of the spin rotation about the azimuthal axis takes place.**


Thank you for your attention